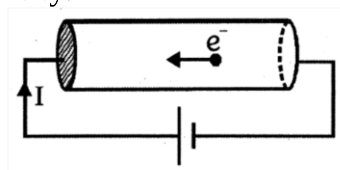
CURRENT ELECTRICITY

1. ELECTRIC CURRENT AND DRIFT VELOCITY

In previous chapters we have dealt largely with electrostatics that is, charges at rest. In this chapter we with focus on electric currents, that is, charges in motion.

1.1 Electric current

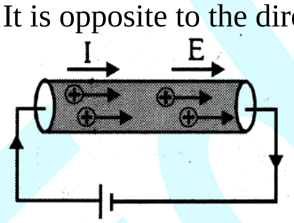
Electric charges in motion constitutes an electric current. Any medium having free electric charges is a conductor of electricity.

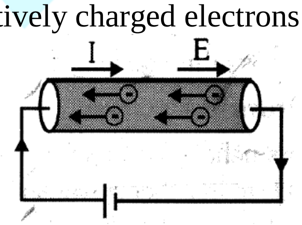


Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminum etc. are good conductors. When charge flows in a conductor from one place to the other, then the rate of flow of charge is called electric current (I). If the moving charges are positive, the current is in the direction of motion of charge. If they are negative, the current is opposite to the direction of motion. If a charge ΔQ crosses an area in time Δt then the average electric current through the area, during this time is

$$\text{Average current } I_{av} = \frac{\Delta Q}{\Delta t}$$

- Instantaneous current $I = \lim\limits_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$
- Current is a fundamental quantity with dimensions $[M^0L^0T^0A^1]$
- Current is a scalar quantity with its SI unit being ampere. (as it doesn't obey vector laws)
  Ampere : The current through any conductor is said to be one ampere if one coulomb of charge is flowing per second through any cross-section of the wire.
- The conventional direction of current is the direction of flow of positive charge or applied field. It is opposite to the direction of flow of negatively charged electrons or ions.

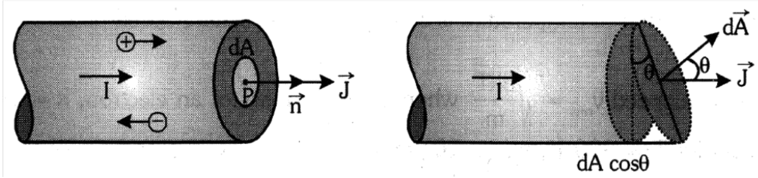




1.2 Current Density (J)

Current is a macroscopic quantity and deals with the overall rate of flow of change through a specify the current with direction in the microscopic level at a point, the term current density is introduced. Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember that surface is normal to the direction of charge flow (or current passing) through that point.

- Current density at point P is given by $\vec{J} = \frac{dI}{dA}\hat{n}$



- If the area is not normal to the current, but makes an angle a with the normal to the direction of current then $J = \dfrac{dI}{dA\cos\theta} \Rightarrow dI = JdA\cos\theta = \vec{J}.d\vec{A} \Rightarrow I = \int \vec{J}.d\vec{A}$
- Current density $\vec{J}$ is a vector quantity. It's direction is same as that of $\vec{E}$. It's SI unit is ampere/m^2 and dimensions [$L^{-2}A$].

1.3 Thermal speed

Conductors contain a large number of free electrons, which are in continuous random motion. Due to random motion, the free electrons collide with positive metal ions with high frequency and undergo change in direction at each collision. So, the thermal velocities are randomly distributed in all possible directions.

Let $\vec{u}_1, \vec{u}_2, ...\vec{u}_N$ be the individual thermal velocities of the free electrons at any given time.

If the total number of free electrons in the conductor = N

Then average velocity $\vec{u}_{ave}\left[\dfrac{\vec{u}_1 + \vec{u}_2 + ...\vec{u}_N}{N}\right] = \vec{0}$

The average thermal velocity is zero but the average thermal speed is non-zero.

1.4 Classification of Materials according to Conductivity

(i) Conductors

In some materials, the outer electrons of each atoms or molecules are loosely bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called conductors.

(ii) Insulators

Materials of another class are called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they cannot leave their parent atoms or molecules and hence cannot move through long distances. Such materials are also called dielectrics.

(iii) Semiconductors

Semiconductors behave like insulators at low temperatures. But "at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its respond is intermediatory to a conductor and an insulator and hence, it is named as semiconductor. A free electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction and are called holes.

1.5 Behaviour of conductor in absence of applied potential difference :-

The free electrons present in a conductor gain energy due to the temperature of surrounding and move randomly in the conductor. The average transport and average velocity is zero for a large number of free. electrons. There is no flow of current due to thermal motion of free electrons in a conductor. In the absence of applied, potential difference, electrons have random motion.

The speed gained by virtue of temperature is called root mean square speed of an electron $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

Sp root mean square speed $v_{rms} = \sqrt{\frac{3kT}{m}}$ where m is the mass o fan electron, k = Boltzmann's constant

At room temperature T = 300 K, $v_{rms} = 10^5$ m/s

- Mean free path (λ) : The average distance travelled by an electron between two successive collisions is the mean free path : $\lambda = \frac{\text{total distance travelled}}{\text{number of collisions}}$, ($l \approx 10$Å in metals)
- Relaxation time (τ) : the average time taken by an electron between tow successive collisions is the relaxation time : $\tau = \frac{\text{total time between two collisions for all the fr ee electrons}}{\text{number of electrons}}$, ($\tau \approx 10^{-14}$s in metals)

1.6 Behaviour of a Conductor in the presence of applied potential difference :

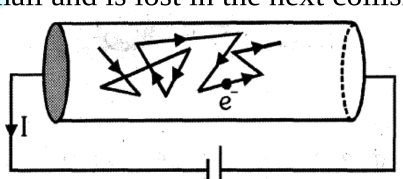
When two ends of a conductors are joined to a battery then one end is at a higher potential and the other at lower potential. This produces an electric field inside the conductor from higher to lower potential.

The field exerts an electric force on free electrons causing acceleration' of each electron.

Acceleration of electron $\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$

1.7 Drift velocity

Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied external electric field. In addition to its thermal velocity, due to acceleration given by applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision.



Under the action of electric field : Random motion of an electron with drift superimposed on it.

At any given time, an electron has a velocity $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$, where $\vec{u}_1$ = the thermal velocity and $\vec{a}\tau_1$ = the velocity acquired by the electron under the influence of the applied electric field.

τ_1 = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are $\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, ...\vec{v}_N = \vec{u}_N + \vec{a}\tau_N$.

The average velocity of all the free electrons in the conductor is equal to the dirft velocity $\vec{v}_d$ of the free electrons

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + ... + \vec{v}_N}{N} = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + ... + (\vec{u}_N + \vec{a}\tau_N)}{N} = \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N}{N} + \vec{a}\left(\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}\right)$$

$$\because \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N}{N} = \vec{0} = \vec{v}_d = \vec{a}\left(\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}\right) \Rightarrow \vec{v}_d = \vec{a}\,\tau = -\frac{e\vec{E}}{m}\tau$$

Note : order of drift velocity is 10^{-4}m/s.

1.8 Relation between Current and, Drift velocity :

Let n = number density of free electrons and A= area of cross-section of the conductor.

Number of free electrons in the conductor of length L = nAL, Total charge on these free electrons Δq = neAL Time taken by drifting electrons to cross the conductor $\Delta t = \frac{L}{v_d}$

$$\therefore \text{ current } I = \frac{\Delta q}{\Delta t} = neAL\left(\frac{v_d}{L}\right) = neAv_d$$

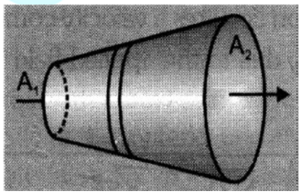
$$I = neAv_d$$

GOLDEN KEY POINTS

- The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second.
- For a given conductor current does not change with change in its cross-section because current is simply the rate of flow of charge. If a steady current flows in a metallic conductor of non uniform cross section :

  (i) I is same along the wire .

  (ii) Current density, electric field strength, drift velocity are inversely proportional to area. Here $I_1 = I_2$ but $A_1 < A_2$ so $J_1 > J_2$, $E_1 > E_2$, $v_{d_1} > v_{d_2}$



- If n particles each having a charge q pass per second per unit area then current associated with cross-sectional area A is $I = \frac{\Delta q}{\Delta t} = nqA$.
- If there are n particles per unit volume each having a charge q and moving with velocity v then current through cross-sectional area A is $I = \frac{\Delta q}{\Delta t} = nqvA$
- If a charge q is moving in a circle of radius r with speed v then its time period is $T = 2\pi r/v$. The equivalent current $I = \frac{q}{T} = \frac{qv}{2\pi r}$

- Order of free e^- density in conductors = 10^{28} electrons/m^3, while in semiconductors = 10^{16} e^-/m^3
- Electric field outside a current carrying conductor is zero but inside a conductor it is $\frac{V}{l}$.
-

Terms	Thermal speed v_r	Mean free path λ	Relaxation time τ	Drift speed v_d
Order of magnitude	10^5 m/s	10Å	10^4 m/s	10^4 m/s

System	Current carriers
A bar made of silver or any metal	Free electrons
Hydrogen discharge tube	Electrons and (+ve) Ions
Voltaic cell	H^+ and SO_4^{-2} (Ions)
Semiconductor	Electrons and holes
Super conductor	Electrons

- If the temperature of a conductor increases, the amplitude of vibrations of positive ions in the conductor also increase. Consequently the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases.
- Mean free path of conduction electrons = Drift velocity × Relaxation time

Illustrations

Illustration 1.

What will be the number of electrons passing through a heater wire in one minute, if it carries a current of 8 A?

Solution

$$I = \frac{ne}{t} \Rightarrow n = \frac{It}{e} = \frac{8\times 60}{1.6\times 10^{-19}} = 3\times 10^{21} \text{ electrons}$$

Illustration 2.

The current through a wire depends on time as i =(2 + 3t) A. Calculate the charge crossed through a cross section of the wire in first 10 seconds.

Solution

$$i = \frac{dq}{dt} \Rightarrow dq = (2 + 3t)dt$$

$$\int_0^q dq = \int_0^{10} (2+3t)\,dt \Rightarrow q = \left(2t + \frac{3t^2}{2}\right)_0^{10} = 2 \times 10 + \frac{3}{2}\times 100 = 20 + 150 = 170 \text{ C.}$$

Illustration 3.

Current through a wire decreases linearly from 4 A to zero in 10 s. Calculate the charge flown through the wire daring this interval of time.

Solution

Charge flown = average current × time

$$= \left[\frac{4+0}{2}\right]\times 10 = 20C$$

Illustration 4.

The current density at a point is $\vec{j} = (2\times10^4\,\hat{j})\ \text{Am}^{-2}$

Find the rate of charge flow through a cross sectional area $S = \left(2\hat{i}+3\hat{j}\right)\text{cm}^2$

Solution

The rate of flow of charge = current = $I\int\vec{J}.d\vec{S} \Rightarrow \vec{J}.\vec{S} = (2\times10^4)\left[\hat{j}\cdot(2\hat{i}+3\hat{j})\right]\times10^{-4}\ \text{A} = 6\text{A}$

Illustration 5.

Find the number of free electrons per unit volume of a metallic wire of density 104 kg/m^3, atomic mass number 100 if the number of free electrons per atom is one.

Solution

Number of free charge particles per unit volume (n) = $\dfrac{\text{total free charge particles}}{\text{total volume}}$

Θ Number of free electron per atom is one implies that total free electrons = total number of atoms = $\dfrac{N_A}{M_W}\times M$

So $n = \dfrac{\dfrac{N_A}{M_W}\times M}{V} = \dfrac{N_A}{M_W}\times d = \dfrac{6.023\times10^{23}\times10^4}{100\times10^{-3}} = 6.023\times10^{28}$

BEGINNER'S BOX-1

1. 10,000 alpha particles pass per minute through a straight tube of radius r. What is the resulting electric current ?
2. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 rnA. What is the number of electrons striking the target per second ?
3. The diameter of a copper wire is 2 mm, a steady current of 6.25 A is generated by 8.5 x 10^{28}/m^3 electrons flowing throught it. Calculate the drift velocity of conduction electrons.
4. A silver wire of 1mm diameter has a charge of 90 coulombs flowing in 1 hours and 15 minutes. Silver contains 5.8×10^{28} free electrons per cm^3. Find the current in the wire and drift velocity of the electrons.

2. OHM'S LAW AND ELECTRIC RESISTANCE

2.1 Relation between Current density, Conductivity and Electric field

Let the. number of free electrons per unit volume in a conductor = n

Cross sectiona1 area = A

Total number of electrons in dx length= n (Adx)

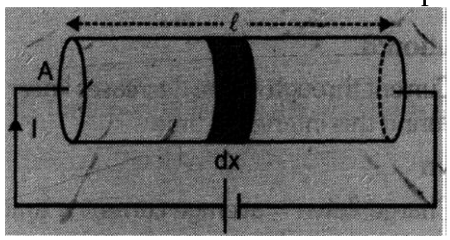
Total charge dQ = n (Adx)

Current $\quad I = \dfrac{dQ}{dt} = nAe\dfrac{dx}{dt} \Rightarrow I = neAv_d$

Current density $J = \dfrac{I}{A} = nev_d \quad \Rightarrow \quad J = ne\left(\dfrac{eE}{m}\right)\tau \qquad \Theta\ v_d\left(\dfrac{eE}{m}\right)\tau$

$J = \left(\dfrac{ne^2\tau}{m}\right)E \qquad \Rightarrow J = \sigma E$, where conductivity $\sigma = \dfrac{ne^2\tau}{m}$

In vector form $\vec{j} = \sigma\vec{E}$ (This is Ohm's law at microscopic level) σ depends only on the material of the conductor and its temperature.



2.2 Ohm's law

$J = \sigma E$ λ = length of conductor

$\frac{I}{A} = \sigma \frac{V}{l}$ V = Voltage applied across conductor

$\frac{I}{A} = \frac{V}{\rho l}$ $\sigma = \frac{1}{\rho}$

$V = \left(\frac{\rho l}{A}\right) I$ ρ = Resistivity or specific resistance of material

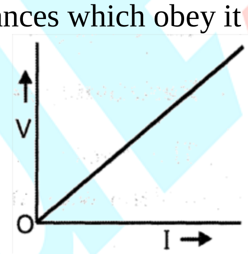
$V = RI$ $R = \frac{\rho l}{A}$

R is constant known as the resistance of the wire whose unit is ohm (Ω).

2.3 V-1 Characteristics

(A) Linear V-1 characteristics

At constant temperature, current is directly proportional to the applied potential difference. This law is called ohm's law and substances which obey it are called ohmic or linear conductors.



(B) Non- Linear V-1 characteristics

- The relation between V and I depends on the sign of V. In other words; if I is the current for a certain V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction (Figure 1).

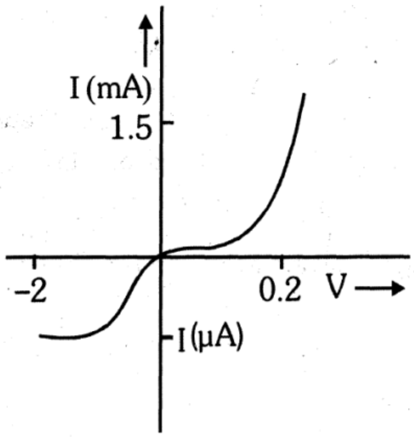


FIGURE 1 : Characteristic curve of a diode, Note the different scales for negative and positive values of the voltage and current

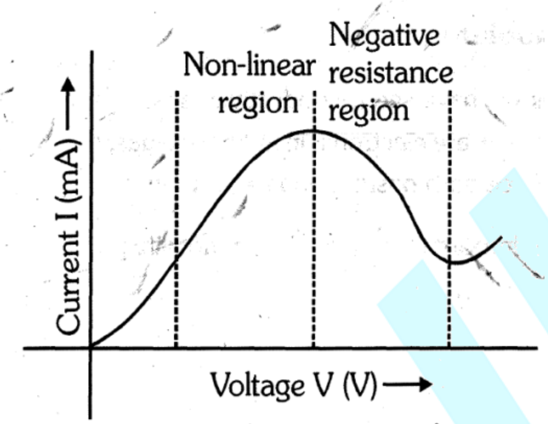


FIGURE 2 : Variation of current versus voltage for GaAs.

- The relation between V and I is not unique, i.e., there is more than one value of V for the same current I (Figure 2). A material exhibiting such a behaviour is GaAs

2.4 Resistance

The resistance of a conductor is the opposition to the flow of charge which the conductor offers. When a potential difference is applied across a conductor, free electrons get accelerated and collide with positive ions and their motion is thus opposed. This opposition offered by the ions is called the resistance of the conductor.

Unit : ohm, volt/ampere, **Dimensions** = $[M\,L^2\,T^{-3}\,A^{-2}]$

Resistance depends on :

γ Length of the conductor $R \propto \lambda$

γ Area of cross-section .of the conductor $R \propto \frac{1}{A}$

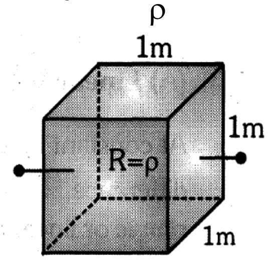
γ Nature of material of the conductor $R = \frac{\rho l}{A}$

2.5 Resistivity and conductivity

Resistivity : $\rho = RA/\lambda$ if $\lambda = 1$ units , $A = 1$ units then $\rho = R$ units

The specific resistance of a material is equal to the resistance of a wire of that material with unit cross sectional area and unit length.

Conductivity is the inverse of resistivity $\sigma = \frac{1}{\rho}$



Resistivity depends on

(i) Nature of material (ii) Temperature of material

Resistivity does not depend on the size and shape of the material because it is a characteristic property of the conducting material.

2.6 Mobility

As we have seen, conductivity arises as a result of mobile charge carriers. In metals, these mobile charge carriers are electrons; in an ionised gas, they are electrons and positively charged ions; in an electrolyte, the can be both positive and negative ions.

An important quantity is the mobility μ defined as the magnitude of the drift velocity per unit electric field :

$$\mu = \frac{|v_d|}{E}$$

The SI unit of mobility is m^2/V-s and is 10^4 times the mobility in practical units (cm^2/V-s). Mobility is positive.

$$v_d = \frac{e\tau E}{m} \qquad \mu = \frac{v_d}{E} = \frac{e\tau}{m}$$

Where τ is the relaxation time for electrons.

2.7 Temperature dependence of resistivity and resistance

The resistivity of a material is found to be dependent on the temperature over a limited range of temperatures (not too large). The resistivity of a metallic conductor is approximately given by

$$\rho_T = p_0[1 + \alpha(T - T_0]$$

Where ρ_T is the resistivity at a temperature T and ρ_0 is the resistivity at a reference temperature T_0. Here, α is called the temperature co-efficient of resistivity.

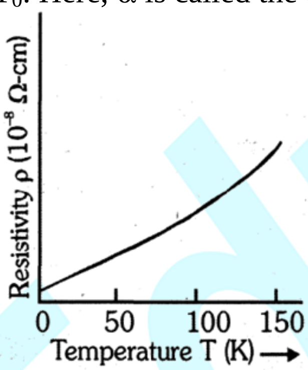


Figure Resistivity ρ_T of copper as a function of temperature T.

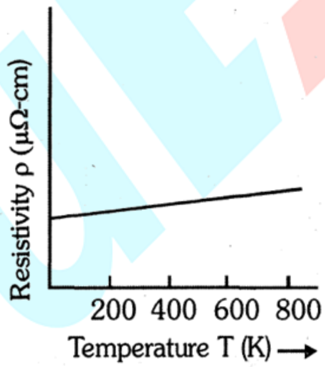


Figure Resistivity ρ_T of nichrome as a function of absolute temperature T.

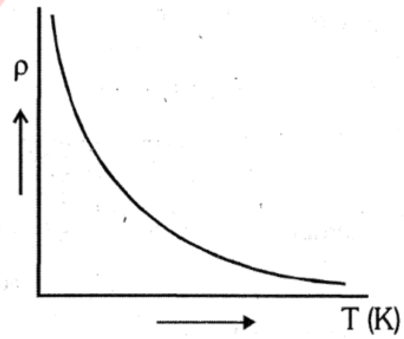


Figure Temperature dependence of resistivity for a typical semiconductor

- Unit of a is $\frac{1}{°C}$
- Resistance corresponding to temperature difference (ΔT) is given as $R_t = R_0(1 + \alpha\Delta T)$
  Where R_t = Resistance at t°C, R_0 = Resistance at 0°C
  ΔT = Change in temperature, a= Temperature coefficient of resistance
  *[For metals : α is positive but for semiconductors and insulators : α is negative]
- Resistance of the conductor decreases linearly with decrease in temperature and. becomes zero at a specific temperature. This temperature is called critical temperature. Below this temperature a conductor becomes a superconductor.

2.8 Colour coding of carbon resistor (For self study)

Resistors in the higher range are made mostly from carbon. Carbon resistors are compact, inexpensive and thus find extensive use in electronic circuits. Carbon resistors are small in size and hence their values are given using a colour code.

TABLE : RESISTOR COLOUR CODES

Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Voilet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold			5
Silver			10
No colour			20

- Here is a mnemonic to memorise the names of colour in a sequence.

BBROY of Great Britain had a Very Good Wife

The resistors have a set of co-axial coloured rings on them whose significance are listed in Table. The first two bands from the end indicate the first two significant figures of the resistance in ohms. The third band indicates the decimal multiplier (as listed in Table). The last band stands for tolerance or possible variation in percentage about the indicated values. Sometimes, this last band is absent and that indicates tolerance of 20%. For example, if the four colours are orange, blue, yellow and gold, the resistance value is $36\times10^4\ \Omega$, with a tolerence value of 5%.

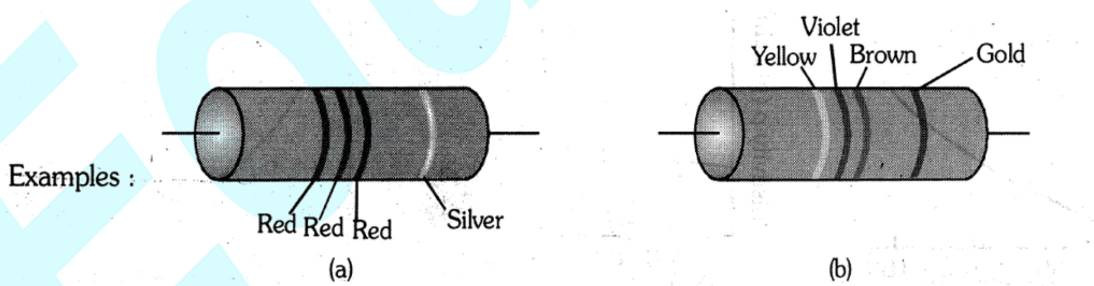
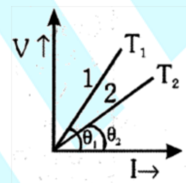


Figure : Colour coded resistors **(a)** $(22\times10^2\ \Omega) \pm 10\ \%$, **(b)** $(47\times10\ \Omega) \pm 5\ \%$

GOLDEN KEY POINTS

- If a wire is stretched ton times of it's original length, its new resistance will be n^2 times.
- If a wire is stretched such that it's radius is reduced to $\frac{1}{n}$th of it's original value, then its new resistance will increase to n^4 times; similarly resistance will decrease to n^4 times if radius is increased ton times by contraction.

- x % change is brought in length of a wire, its resistance will change by 2x %. (if change is very small)
- Ohm's law is not a fundamental law of nature. As it. is possible that for an element :-
  (i) V depends on I nonlinearly (e.g. vacuum tubes)
  (ii)Relation between V and I depends on the sign of V for the same value [forward and reverse bias in diode]
  (iii) The relation between V and I is not unique. That is for the same value of I there is more than one value of V.
- In general:
  (i) $\rho_{alloy} > \rho_{semiconductor} > \rho_{conductor}$
  (ii) Temperature coefficients of alloys are lower than pure metals.
  (iii) Resistance of most of non metals decrease with increase in temperature. (e.g. carbon)
  (iv) The resistivity of an insulator (e.g. amber) is greater than a metal by a factor of 10^{22}.



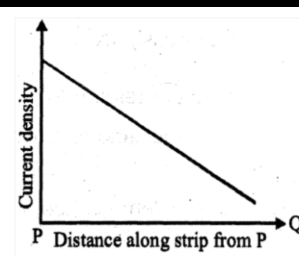
- Temperature coefficient (α) of semi conductor including carbon (graphite), insulator and electroytes is negative.
- At different temperatures V-I curves are different.
  Here $\tan\theta_1 > \tan\theta_2$ So $R_1 > R_2$ i.e. $T_1 > T_2$

- The heating element devices like heater, geyser, electric iron (usual known as press) etc are made of nichrome because it has high resistivity and high melting point. It does not react with air and acquires steady state when red hot at 800°C.
- Fuse wire is made of tin lead alloy because it has low melting point and high resistivity. The fuse is used in series,· and melts to result in an open circuit when current exceeds the safety limit.
- Resistances of resistance box are made of manganin or constantan because they have moderate resistivity and very small temperature coefficient of resistance. The resistivity is nearly independent of temperature.
- The filament of bulb is made up of tungsten because it has low resistivity, high melting point of 3300 K and light at 2400 K. The bulb is filled with inert gas because at high temperatures tungsten reacts with air forming oxide.
- The connection wires are made of copper because it has low resistivity.

Illustrations

Illustration 6.

An electric current flows along an insulated strip PQ of a metallic conductor. The current density in the strip varies as shown in the graph. Which one of the following statements could explain this variation?



(A) The strip is narrower at P than at Q
(B) The strip is narrower at Q than at P
(C) The potential gradient along the strip is uniform
(D) The resistance per unit length of the strip is constant.

Ans. (A)

Solution

The current density at P is higher than at Q. For the same current flowing through the metallic conductor PQ, the cross-sectional area at P is narrower than at Q. The resistance per unit length r is given by r = $\frac{\rho}{A}$ where ρ is the resistivity and A is the cross-sectional area of the conductor PQ. Thus, r is inversely proportional to the cross-sectional area A of the conductor.

Illustration 7.

A constant voltage is applied across a wire of constant length. How does the drift velocity of electrons depends on the area of cross-section of wire.

Solution

$v_d = \mu E$, μ = mobility of free electrons = $\frac{e\tau}{m}$ and E = electric field = $\frac{V}{L}$

so $v_d = \mu\frac{V}{L}$; here μ is constant.

If V and L are constant then v_d does not depend on area.

Illustration 8.

Draw the colour code for 42 kΩ ± 10% carbon resistance.

Solution

According to colour code, colour for digit 4 is yellow, for digit 2 it is red, for digit 3 the colour is orange and 10% tolerance is represented by silver colour. So colour code should be yellow, red, orange and silver.

Illustration 9.

At room temperature (0°C) the resistance of a heating element is 1000. Calculate the temperature of the element if the resistance is found to be 117 Ω (the temperature coefficient of resistance of the material is 1.7×10^{-4} °C^{-1})

Solution

Θ $R_a = R_0(1 + \alpha\theta)$ $\quad\quad$ $\therefore$ $117 = 100(1 + \alpha\theta)$

$$\Rightarrow \quad \theta = \frac{117-100}{100\alpha} = \frac{17}{100\times17\times10^{-4}} = 1000°C$$

BEGINNER'S BOX-2

1. The dimensions of a block is 100m × 1m × 1m. , The specific resistance of the material of the block is 4×10^{-6} Ω-m. Calculate its resistance across its square faces.

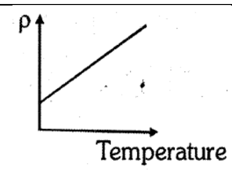
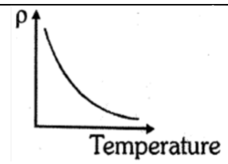
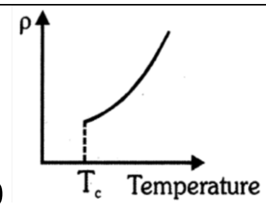
2. If a copper wire is stretched to make its cross-sectional radius 0.1% thinner, then what is the percentage increase in its resistance ?

3. A wire of resistance 5 n is drawn out so that its length is increased to twice its original length. Calculate its new resistance and resistivity.

4. At what temperature would the resistance of a copper wire be double its resistance at 0°C? If $\alpha = \frac{1}{273}$ °C^{-1}.

5. Match the following items-

Graph	Solid

 (A)	(p) Semiconductor
 (B)	(q) Conductor
 (C)	(r) Conductor which exhibits super conductivity

6. Match the column I with column II-

Column I	Column II
(A) Filament of electric bulb	(p) tin-lead alloy
(B) Fuse wire	(q) Manganin
(C) Resistance of resistance box	(r) Nichrome
(D) Filament of heating devices (heater, geyser, press etc)	(s) Tungsten
	(t) Constanton

3. COMBINATION OF RESISTORS AND·KIRCHHOFF'S LAW

3.1 Combination of resistors

- **Series Combination**

γ Same current passes through each resistance

γ Voltage across each resistance is directly proportional to the value of resistance

$V_1 = iR_1$, $V_2 = iR_2$, $V_3 = iR_3$,

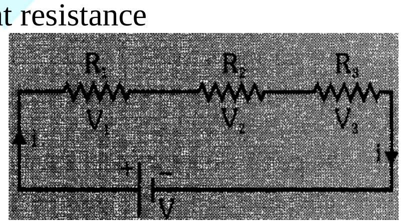
γ Sum of the voltages across all resistances is equal to the voltage applied across the circuit.

i.e., $v = V_1 + V_2 + V_3$

or $iR = iR_1 + iR_2 + iR_3$

or $R = R_1 + R_2 + R_3$

Where R = equivalent resistance



- **Parallel Combination**

γ There is same potential drop across each resistance.

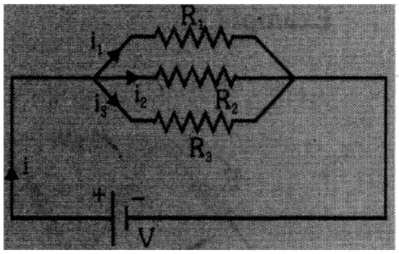
γ Current in each resistance is inversely proportional to the value of resistance.

$$i_1 = \frac{V}{R_1},\ i_2 = \frac{V}{R_2},\ i_3 = \frac{V}{R_3}$$

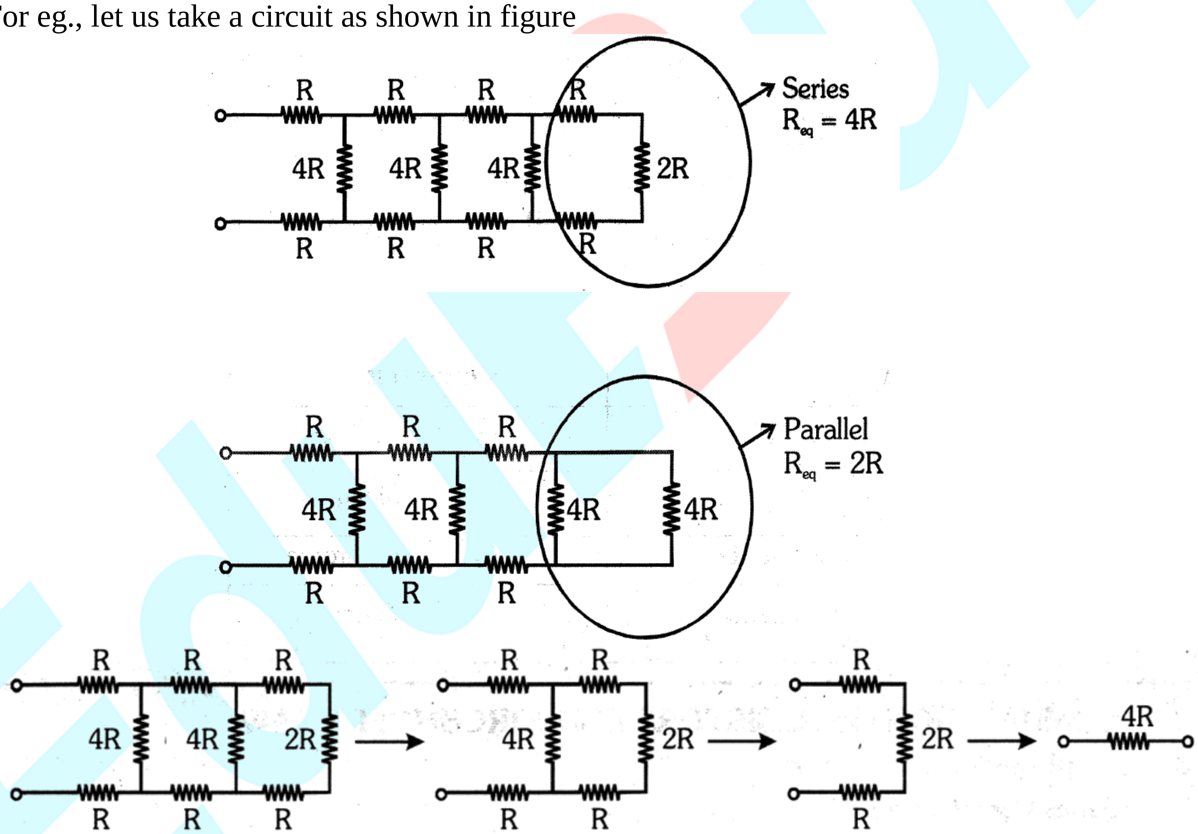
γ Current flowing in the circuit is the sum of the currents in then individual resistance.

$i = i_1 + i_2 + i_3$

$$\frac{V}{R}=\frac{V}{R_1}+\frac{V}{R_2}+\frac{V}{R_3} \quad \Rightarrow \frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$



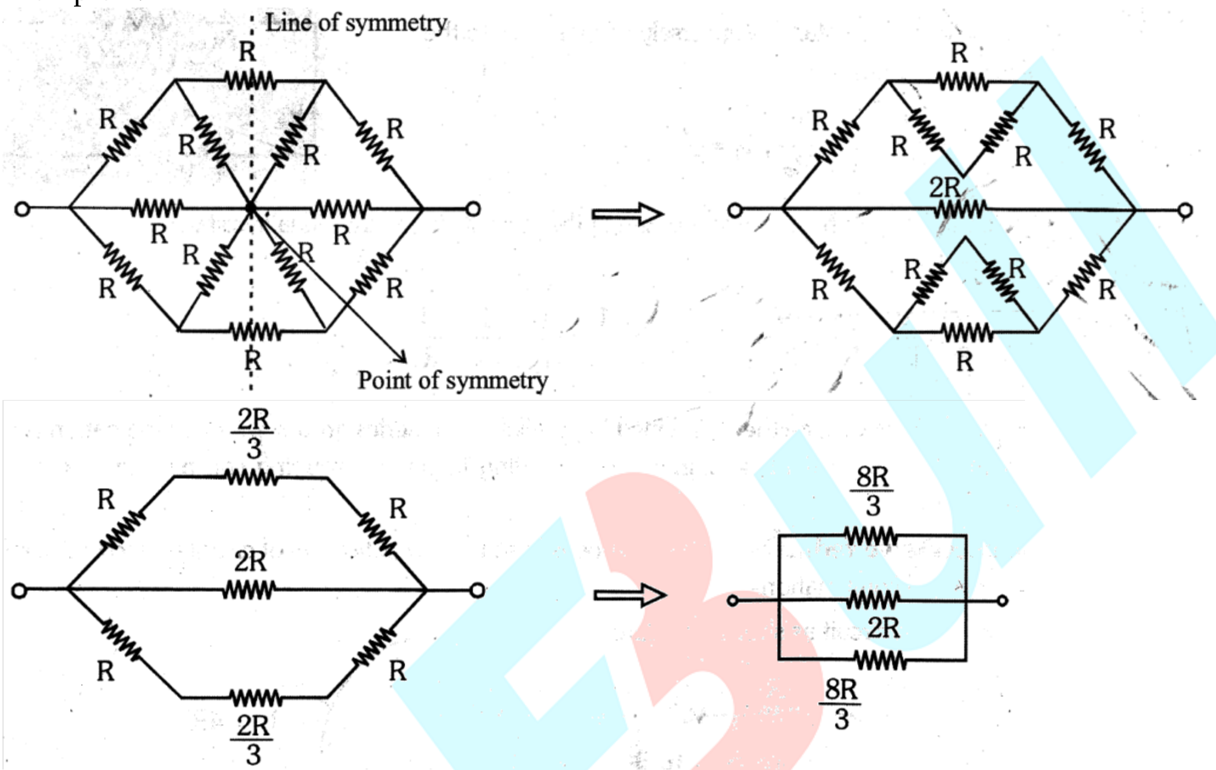
- **Mixed grouping :**
  When a group of resistors is neither combined in parallel nor in series grouping, the combination is called as mixed grouping. The equivalent resistance corresponding to mixed grouping can be found by one of the following methods:
  Method of successive reduction : This method is based on simplification of circuit by successive reduction of series and parallel combinations.
  For eg., let us take a circuit as shown in figure



Method of symmetry :
A line drawn across the circuit perpendicular to the line joining the two points between which resistance is to be found, which divides a circuit in two mirror images is a line of symmetry. This line is an equal potential line. By removing a resistor on equal potential line or by placing a conductor on this circuit is modified without altering its electrical properties. Now the modified circuit is solved by method of successive reduction.

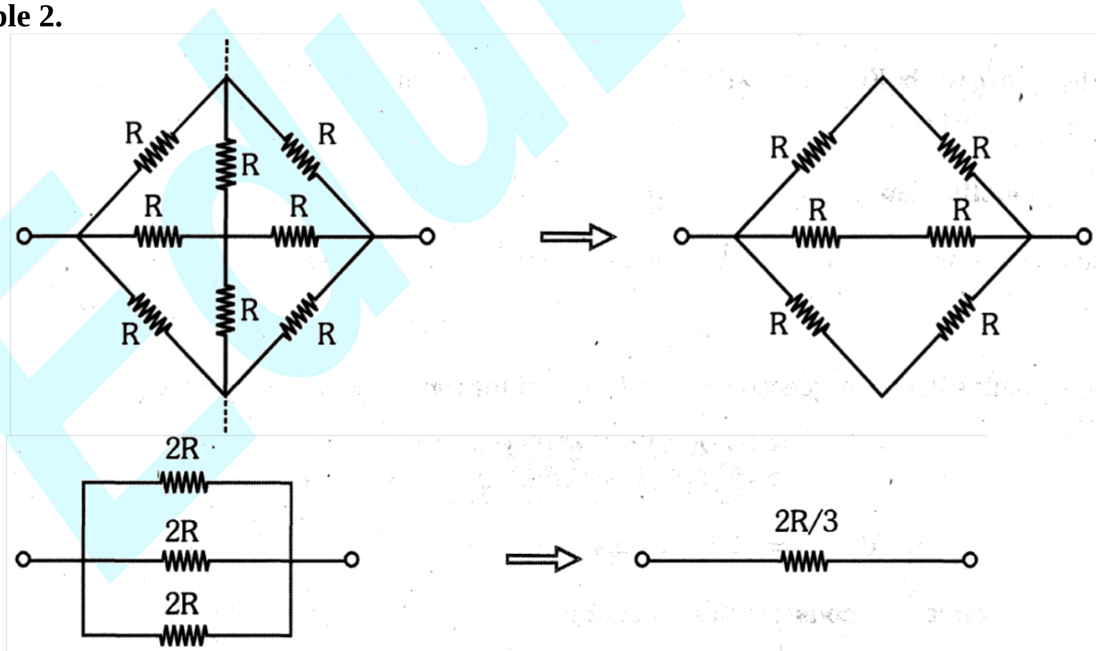
A point of symmetry is a false junction. Current does not get redistributed at this point. By separating the elements carrying the Same current, the circuit is made simple & is then solved by method of successive reduction.

Example 1.



$$\frac{1}{R_{eq}} = \frac{3}{8R} + \frac{1}{2R} + \frac{3}{8R} \qquad \Rightarrow R_{eq} = \frac{4R}{5}$$

Example 2.



Nodal method (For short-circuits)

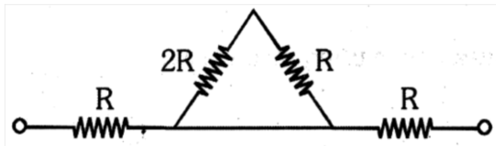
When two points are connected with a conducting wire (having zero resistance, the potential difference across the wire becomes zero i.e. the two nodes will have same' potential. To calculate the equivalent resistance, following steps are taken

1. Identify the nodes with same potential.

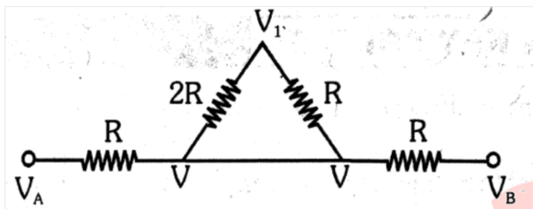
2. Allot the potential on every node.

3. Redraw the circuit and calculate the equivalent resistance.

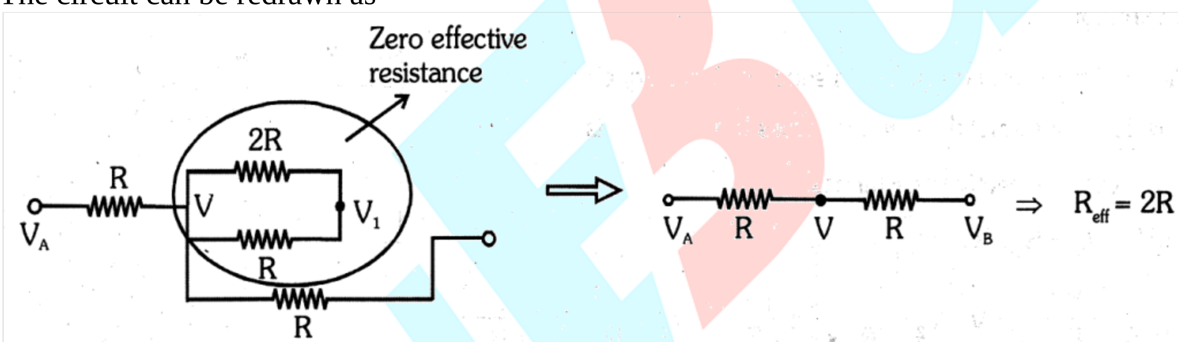
Example :



The circuit can be solved as



The circuit can be redrawn as



3.2 Kirchhoff's Law

There are two laws given by Kirchhoff for determination of potential difference and current in different branches of any complicated network.

- **First law or Junction law**

In an electric circuit, the algebric sum of the currents meeting at any junction in the circuit is zero.

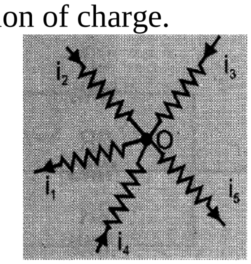
or

Sum of the currents entering the junction is equal to sum of the currents leaving the junction.

$\sum i = 0$ Law of conservation of charge is the basis behind of continuity equation.

$i_1 - i_2 - i_3 - i_4 + i_5 = 0 \quad \Rightarrow i_1 + i_5 = i_2 + i_3 + i_4$

This is based on law of conservation of charge.



- **Second law or Loop law**

  In any closed circuit the algebric sum of e.m.f. s and algebric sum of potential drops is zero.

  $$\sum IR + \sum E = 0$$

  while moving from negative to positive terminal inside a cell, e.m.f. is taken positive while moving in the direction of current in a circuit the potential drop (i.e. IR) across a resistance is taken negative.

  γ This law is based on law of conservation of energy.

  Kirchhoff's method : Working rule is described below :

  γ Assume same current to be entering & leaving the circuit.

  γ Distribute currents and find relationship between them using second law.

  γ Find voltage across the circuit

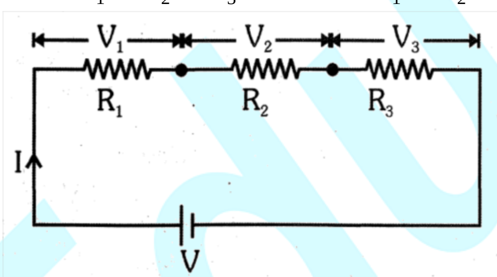
  γ $R_{eq} = \dfrac{\text{Voltage across the circuit}}{\text{Current through the circuit}}$

GOLDEN KEY POINTS

- If n resistance (each R) are connected in series their effective resistance will be nR.
- If n resistances (each R) are connected in parallel then their effective resistance will be $\frac{R}{n}$.
- The equivalent resistance of parallel combination is lower than the value of the lowest resistance in the combination.
- When resistors are connected in series in a battery circuit then the ratio of potential differences across the resistors is equal to ratio of their respective resistances.

  $$V_1 : V_2 : V_3 : R_1 : R_2 : R_3$$

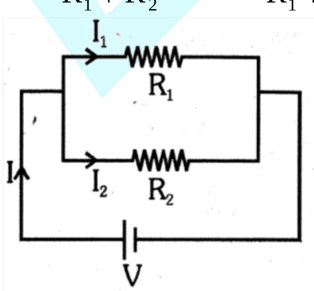
  $V_1 = \dfrac{R_1}{R_1 + R_2 + R_3} V$; $V_2 = \dfrac{R_2}{R_1 + R_2 + R_3} V$; $V_3 = \dfrac{R_3}{R_1 + R_2 + R_3} V$



- When resistors are connected in parallel in a battery circuit then the ratio of currents through them will be the ratio of reciprocals of their respective resistances.

  $I_1 : I_2 : : \dfrac{1}{R_1} : \dfrac{1}{R_2} = R_2 : R_1$

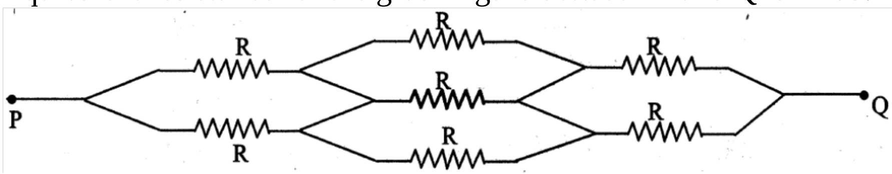
  $I_1 = \dfrac{R_2}{R_1 + R_2} I$; $I_2 = \dfrac{R_1}{R_1 + R_2} I$



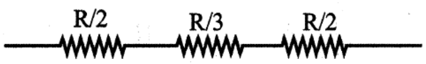
Illustrations

Illustration 10.

Equivalent resistance for the given figure between P and Q is NR/3. Find the value of N.



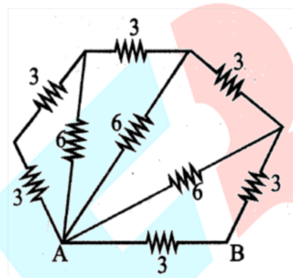
Solution.



$R_{net} = \frac{4R}{3} \Rightarrow N = 4$

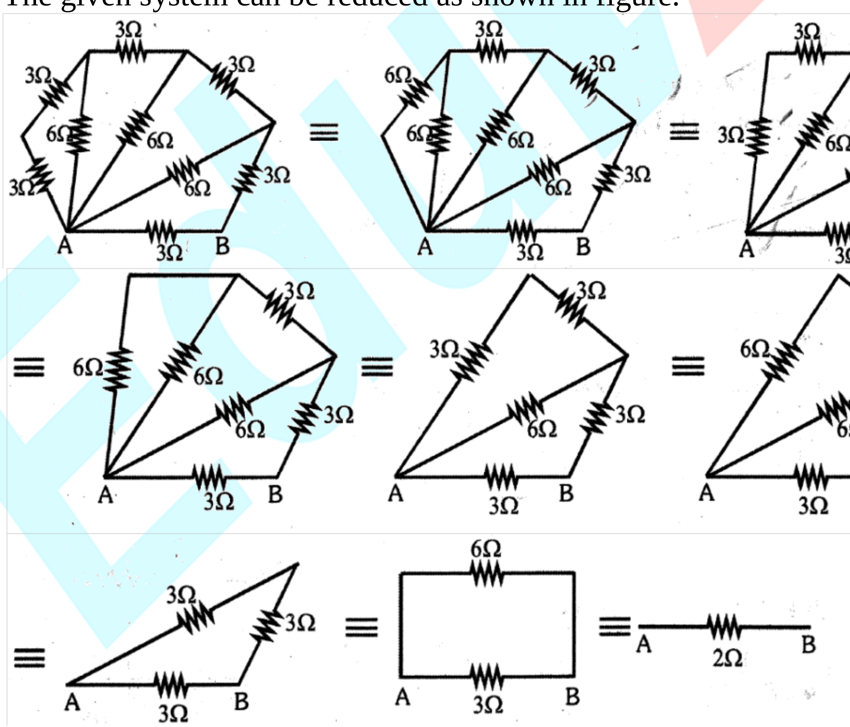
Illustration 11.

All resistances in the diagram below are in ohms. Find the effective resistance between the points A and B (in Ω).



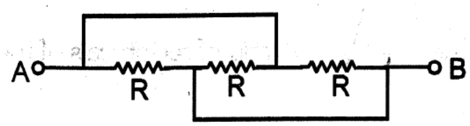
Solution

The given system can be reduced as shown in figure.



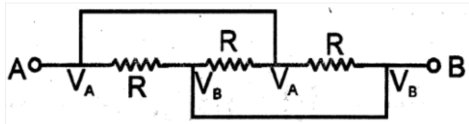
$\therefore R_{AB} = 2\Omega$

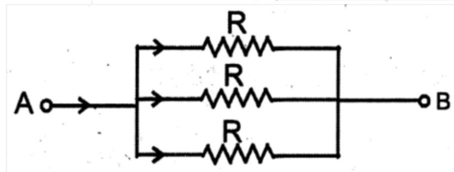
Illustration 12.

Find the equivalent resistance between A and B 

Solution

Here all the resistances are connected between the terminals A and B



Modified circuit is 

So, $R_{eq} = \frac{R}{3}$

Illustration 13.

Resistances R, 2R, 4R, SR ... ∞ are connected in parallel. What is their resultant resistance ?

Solution

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \frac{1}{8R} + ...\infty = \frac{1}{R} = \left[1 + \frac{1}{2} + \frac{1}{4} + ...\infty\right] = \frac{1}{R} = \left[\frac{1}{1 - \frac{1}{2}}\right] = \frac{2}{R} \Rightarrow R_{eq} = \frac{R}{2}$$

Illustration 14.

A wire of $\rho_L = 10^{-6}\ \Omega$/m is turned in the form of a circle of diameter 2 m. A piece of same material is connected as a diameter AB. Then find the resistance between A and B.

Solution

Θ $R = \rho_L \times \text{length}$

∴ $R_1 = \pi \times 10^{-6}\ \Omega$, $R_2 = 2\times10^{-6}\ \Omega$, $R_3 = \pi\times10^{-6}\ \Omega$

$$\frac{1}{R_{AB}} = \frac{1}{\pi\times10^{-6}} + \frac{1}{2\times10^{-6}} + \frac{1}{\pi\times10^{-6}}\ ;\ R_{AB} = 0.88\times10^{-6} \text{ ohm.}$$

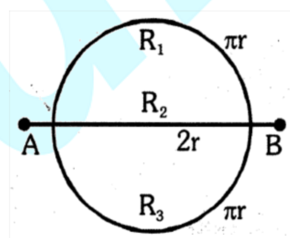


Illustration 15.

Give the nature of V-I graph for

(a) ohmic (b) non-ohmic

circuit elements. Give one example of each type.

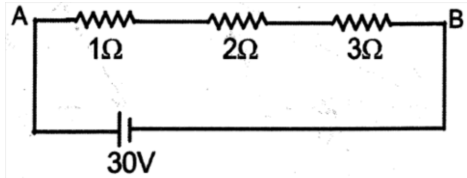
Solution

Linear graph for ohmic. Non-linear graph for non-ohmic circuit elements.

eg : Ohmic - electrical resistance, Non ohmic - diode.

Illustration 16.

Find the current in the circuit



Solution

$R_{eq} = 1 + 2 + 3 = 6\ \Omega$ the given circuit is equivalent to current $i = \dfrac{V}{R_{eq}} = \dfrac{30}{6} = 5A$

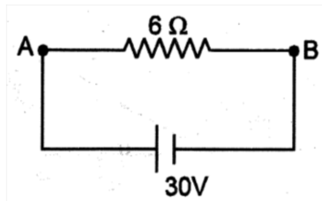
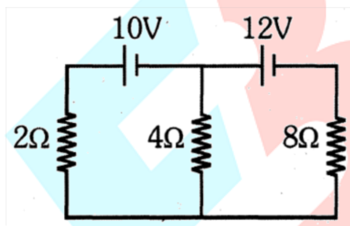


Illustration 17.

For the electric circuit shown here, write the Kirchhoff's circuital law equations, the solutions of which will given currents in the three resistors. Do not solve the equations.

[AIPMT(Mains) 2006]



Solution

$i_1 = i_2 + i_3$

$2i_1 – 10 + 4i_3 = 0$

$8i – 12 – 4i_3 = 0$

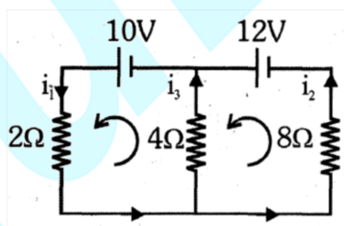
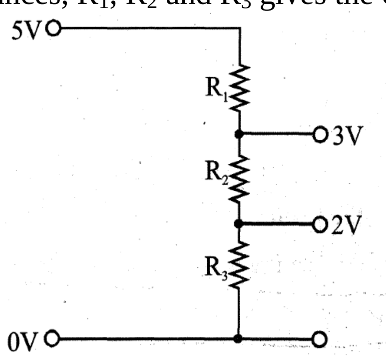


Illustration 18.

A potential divider is used to give outputs of 2V and 3 V from a 5V source, as shown in figure. Which combination of resistances, R_1, R_2 and R_3 gives the correct voltages?



	R_1	R_2	R_3
(A)	1 kΩ	1 kΩ	2 kΩ
(B)	2 kΩ	1 kΩ	2 kΩ
(C)	3 kΩ	2 kΩ	2 kΩ
(D)	3 kΩ	2 kΩ	3 kΩ

Ans. **[B]**

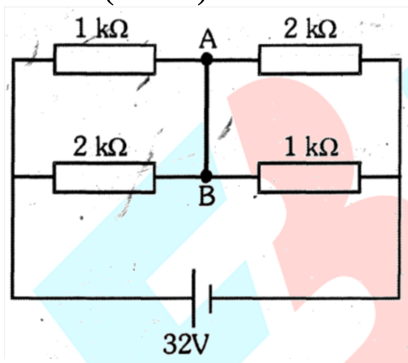
Solution.

For resistors in series connection, current (I) is the same through the resistors. In other words, ratio of the voltage drop across each resistor with its resistance is the same.

That is $I = \frac{5-3}{R_1} = \frac{3-2}{R_2} = \frac{2}{R_3}$ i.e., $R_1 : R_2 : R_3 = 2 : 1 : 2$

Illustration 19.

In the given circuit, find the current (in rnA) in the wire between points A and B.



Ans. **[8]**

Solution

A & B are short circuited

So $R_{eff} = \frac{4}{3}$ kΩ

$I_T = \frac{32}{4/3k\Omega} = 24$ mA

$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{2}{1}$

$\Rightarrow \quad I_1 = \frac{2}{3} \times 24 = 16$ mA $\quad \Rightarrow I_2 = \frac{1}{3} \times 24 = 8$mA

The required current is

$= I_1 - I_2$

$= 16 - 8 = 8$mA

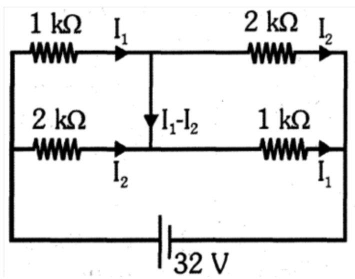
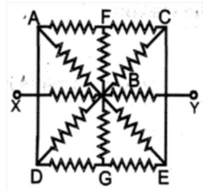


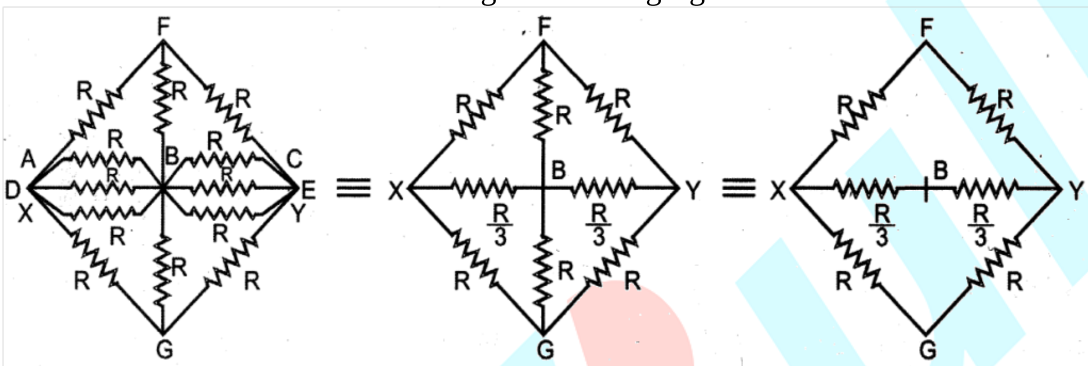
Illustration 20.

Twelve identical resistances each of resistance R are connected as in figure. Find the net resistance between X and Y.



Solution

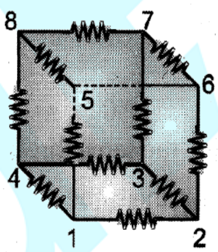
Given circuit can be modified according to following figures :



$$\frac{1}{R_{XY}} = \frac{1}{2R} + \frac{3}{2R} + \frac{1}{2R} = \frac{5}{2R} \Rightarrow R_{XY} = \frac{2R}{5}$$

Illustration 21.

A frame of cube is made by wires each of resistance r then prove that –

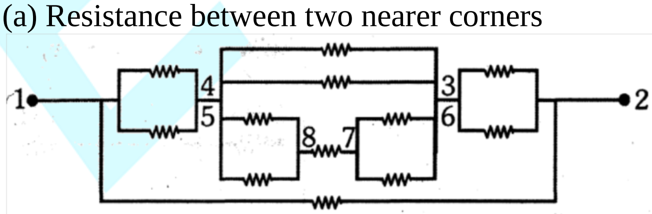


(a) Resistance between two nearer comers $R_{12} = \frac{7}{12}r$

(b) Resistance across face diagonal $R_{13} = \frac{3}{4}r$

(c) Resistance across main diagonal $R_{17} = \frac{5}{6}r$

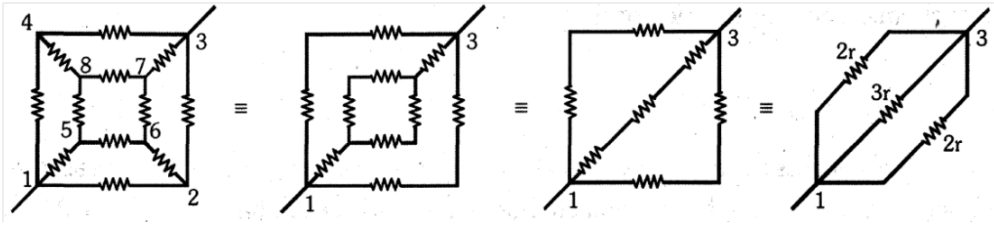
(a) Resistance between two nearer corners



Equivalent resistance between 1 and 2 $\Rightarrow R_{12} = \frac{7}{12}r$

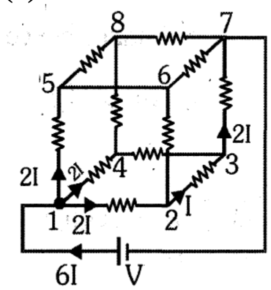
(b) Resistance across face diagonal

Given circuit can be drawn as



Equivalent resistance between 1 and 3 $\Rightarrow R_{13} = \frac{3}{4}r$

(c) Resistance across main diagonal

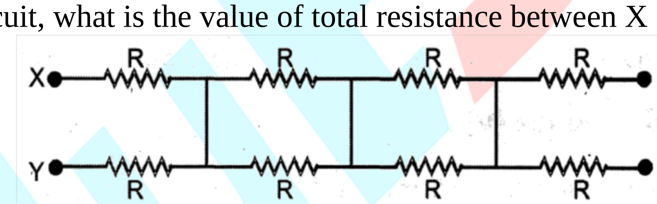


In loop 12371, applying KVL : V = 2Ir + Ir + 2Ir = 5Ir

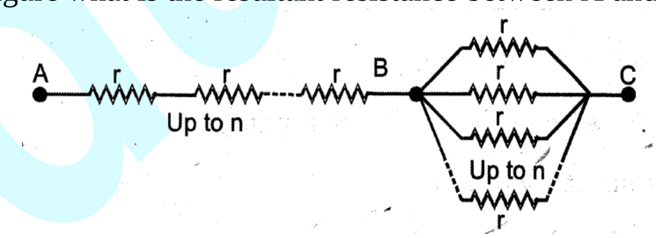
So $R_{17} = \frac{V}{6I} = \frac{5r}{6}$

BEGINNER'S BOX-3

1. For following circuit, what is the value of total resistance between X and Y?



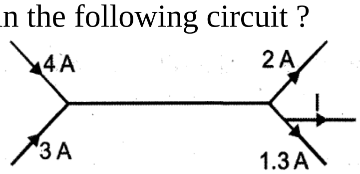
2. In the following figure what is the resultant resistance between A and C?



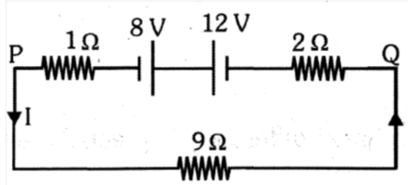
3. What is the smallest resistance that can be obtained by ten $\frac{1}{10}\Omega$ resistors?

4. Two wires of the same metal have the same length, but their cross sectional areas are in the ratio 3 :1. They are joined in series. The resistance of the thicker wire is 10Ω. Then what will be the total resistance of the combination ?

5. What is the value of current I in the following circuit ?



6. In the given circuit, calculate the potential difference between the points P and Q.



4. CELLS, COMBINATIONS OF CELLS, ELECTRICAL. HEATING AND POWER

A cell converts chemical energy into electrical energy.

4.1 Electro motive force (E. M. F.)

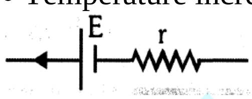
The potential difference across the terminals of a cell when it is not delivering any current is called emf of the cell. The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called the emf of the cell.

• emf depends on :	(i) nature of electrolyte	(ii) metal of electrodes
• emf does not depend on :	(i) area of plates	(ii) distance between the electrodes
	(iii) quantity of electrolyte	(iv) size of cell

4.2 Internal resistance

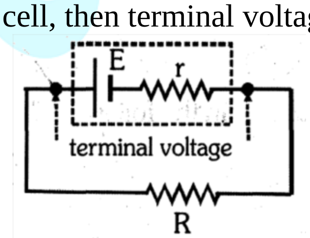
Resistance offered by the electrolyte of the cell when an electric current flows through it, is known as internal resistance.

- Distance between two electrodes increases ⇒ r increases
- Area dipiped in electrolyte increases ⇒ r decreases
- Concentration of electrolyte increases ⇒ r increases
- Temperature increases ⇒ r decreases



4.3 Terminal potential difference (V)

- When current is drawn through a cell or current is supplied to it then, the potential difference across its terminals is called terminal voltage.
- When I current is drawn from cell, then terminal voltage Vis less than it's e.m.f i.e., $V = E - Ir$

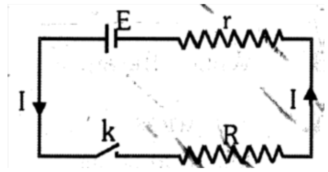


(a) When cell is discharging :

Current inside the cell is from cathode to anode.

Current $I = \dfrac{E}{r + R} \Rightarrow E = IR + Ir = V + Ir \Rightarrow V = E - Ir$

When current is drawn from the cell potential difference is less than the emf of the cell. Greater is the current drawn from the cell smaller is the terminal potential difference. When a large current is drawn from a cell its terminal potential difference is reduced appreciably.

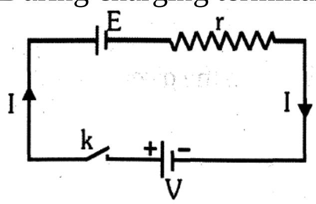


(b) When cell is getting charged :

Current inside the cell is from anode to cathode.

Current $I = \frac{V - E}{r} \Rightarrow V = E + Ir$

During charging terminal potential difference is greater than the emf of the cell.



(c) When cell is in the open circuit :

In open circuit $R = \infty$ $\quad\quad \therefore I = \frac{E}{R + r} = 0 \Rightarrow V = E$

In open circuit, terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

(d) When cell is short circuited :

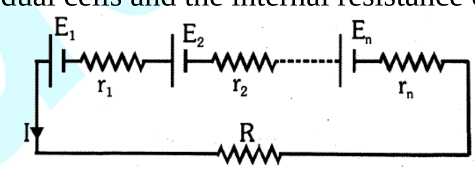
In short circuit $R = 0 \Rightarrow I = \frac{E}{R + r} = \frac{E}{r}$ and $V = IR = 0$

Note : In short circuit, current from the cell is maximum and terminal potential difference is zero.

4.4 Combination of cells

- Series combination

When cells are connected in series the total e.m.f. of the series combination is equal to the sum of the e.m.f. of the individual cells and the internal resistance of the cells will also be in series.



Equivalent internal resistance $r = r_1 + r_2 + r_3 + \ldots$ Equivalent emf $= E = E_1 + E_2 + E_3 + \ldots$

Current $I = \frac{E_{net}}{r_{net} + R}$, If all n cells are identical then $I = \frac{nE}{nr + R}$

- If $nr >> R$, $I = \frac{E}{r} \propto$ current from any cell when shot circuited.
- If $nr << R$, $I = \frac{nE}{R} \propto n \times$ current from any one cell, when connected with the external resistance.

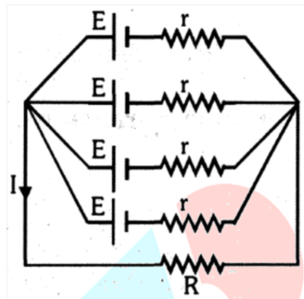
- **Parallel combination**

When the cells are connected in parallel, the total e.m.f. of the parallel combination remains equal to the e.m.f. of a single equivalent cell and internal resistance of the cells will also be in parallel. If m identical cells connected in parallel then total internal resistance of this combination $r_{net} = \frac{r}{m}$.Total e.m.f. of this combination = E

Current in the circuit $I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$

If $r << mR$ $\quad I = E/R$ = Current from any one cell when connected with the external resistance

If $r >> mR$ $\quad I = \frac{mE}{r}$ = m × current from any cell when short-circuited



- Mixed combination of identical cells If n identical cells are connected in series and there are m such branches in the circuit then the total number of cells in this circuit is nm. The internal resistance of the cells connected in a row = nr. Since there are m such rows,

  Total internal resistance of the circuit $r_{net} = \frac{nr}{m}$

  Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row $E_{net} = nE$

  Current in the circuit $I = \frac{E_{net}}{R + r_{net}} = \frac{nE}{R + \frac{nr}{m}}$

  Note : current I in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance of the cells $R = \frac{nr}{m}$.

4.5 Electrical Heating and Power

Cause of Heating

The potential difference applied across the two ends of a conductor sets up an electric field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in the conductor, the energy of electrons transferred to the atoms and ions appear as heat.

- **Joules's Law of Heating**

  When a current I is made to flow through a ohmic resistance R for time t, heat Q is produced such that

  $Q = I^2Rt = P \times t = VIt = \frac{V^2}{R}t$

Heat produced in a conductor does not depend upon the direction of current.

SI unit : joule ; Practical Units : 1 kilowatt hour (kWh)

1kWh = 3.6×10^6 joules = 1 unit 1 BTU (British Thermal Unit) = 1055 J

- Power : $P = VI = \frac{V^2}{R} = I^2R$

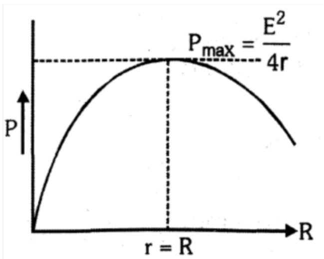
  The watt hour meter placed on the premises of every consumer records the electrical energy consumed.

- Power transferred to load by cell :

  $P = I^2R = \frac{E^2R}{(r+R)^2} \quad \Rightarrow P = P_{max}$ if $\frac{dP}{dR} = 0 \quad \Rightarrow r = R$

  Power transferred by cell to load is maximum when

  $r = R$ and $P_{max} = \frac{E^2}{4r} = \frac{E^2}{4R}$

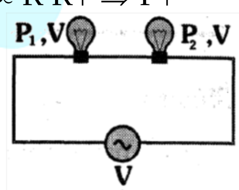


- Series combination of resistors (bulbs)

  (a) Total power consumed $P_{total} = \frac{P_1P_2}{P_1+P_2}$. If n bulbs are identical $P_{total} = \frac{P}{n}$

  (b) In series combination of bulbs, brightness $\propto$ Power consumed by bulb $\propto R \propto \frac{1}{P_{rated}}$. Bulb of lesser wattage will shine more.

  (c) For same current $P = I^2R$ so $P \propto R$ $R\uparrow \Rightarrow P\uparrow$



- **Parallel combination of resistors (bulbs)**

  (a) Total 'power consumed $P_{total} = P_1 + P_2$. If n bulbs are identical $P_{total} = nP$

  (b) In parallel combination of bulbs

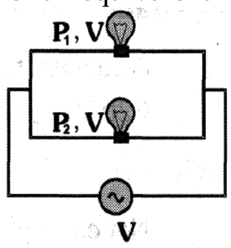
  Brightness $\propto$ power consumed by bulb $\propto I \propto 1/R \Rightarrow$ Bulb of greater wattage will shine more.

  (c) For same V more power will be consumed in smaller resistance $P \propto \frac{1}{R}$

  (d) Two identical heater coils generate a total heat H_S when connected in series and H_P when connected in parallel then, $\frac{H_P}{H_S} = 4$ [In this it is assumed that supply voltage is same]

(e) If a heater boils m kg of water in time T_1 and another heater boils the same amount of water in time T_2, then both connected in series will boil the same amount of water in time $T_s = T_1 + T_2$ and in parallel $T_P = \frac{T_1T_2}{T_1 + T_2}$ [Use, time taken ∝ Resistance]

(f) Instruments based on heating effects of current, work with both A.C. and D.C. Equal values of A.C. (RMS) and D.C. produce, equal heating effect. Due to this reason, the brightness of bulb is same whether it is operated by A.C. or an equivalent D.C.



GOLDEN KEY POINTS

- At the time of charging a cell, when current is supplied to it, the terminal voltage is greater than the e.m.f. E

  $V = E + Ir$ So $V > E$
- Series combination is useful when internal resistance of the cell is less than external resistance.
- Parallel combination is useful when internal resistance of the cell is greater than external resistance.
- Power in R (given resistance) is maximum, if its value is equal to the net resistance of remaining circuit.
- Internal resistance of an ideal cell = 0
- If external resistance is zero, then current delivered by the battery is maximum.

Illustrations

Illustration 22.

A battery of e.m.f. 2 volts and internal resistance 0.1 Ω is being charged with a current of 5 A. Calculate the potential difference between the terminals of the battery.

Solution

Potential drop across internal resistance = Ir = 0.1 × 5 = 0.5 V

Hence potential difference across the terminals= E .t Ir = 2 + 0.5 = 2.5 volts.

Illustration 23.

A battery of six cells each of e.m.f. 2 V and internal resistance 0.5 n is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10 Ω. What will be the charging current?

Solution

Net e.m.f of the battery = 12 V and total internal resistance = 3 Ω

Total resistance of the circuit = 3 + 10 = 13 Ω

$$I = \frac{\text{Net e.m.f.}}{\text{total resistance}} = \frac{220 - 12}{13} = 16\text{A}$$

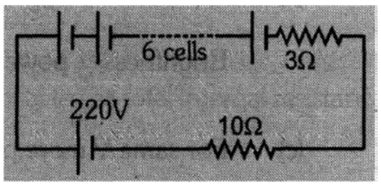


Illustration 24.

A battery of six cells each of e.m.f. 2 V and internal resistance 0.5 n is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10 n. What is the potential difference across the battery ?

Solution

In case of charging of battery, terminal potential V = E + Ir = I_2 + 16×3 = 60 volts.

Illustration 25.

Four identical cells each of e.m.f. 2 V are joined in parallel providing current to an external circuit consisting of two 15 n resistors joined In parallel. The terminal voltage of the equivalent cell as read by an ideal voltmeter is 1. 6 V. Calculate the internal resistance of each cell.

Solution

Total internal resistance of the cell combination $r_{eq} = \frac{r}{4}$

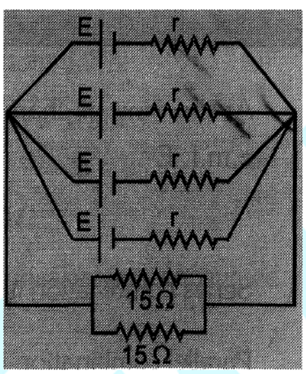
Total e.m.f. $E_{eq} = 2V$

Total external resistance $R = \frac{15\times15}{15+15} = \frac{15}{2} = 7.5\ \Omega$

Current drawn from the equivalent cell $I = \frac{\text{terminal potential}}{\text{external resistance}} = \frac{1.6}{7.5}A$

$\Theta \quad E - I\left(\frac{r}{4}\right) = 1.6 \qquad \therefore \quad 2 - \frac{1.6}{7.5}\left(\frac{r}{4}\right) = 1.6$

$\Rightarrow \quad r = 7.5\ \Omega$



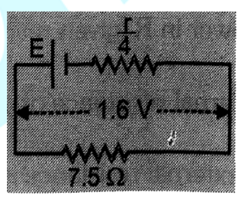


Illustration 26.

The e.m.f. of a primary cell is 2 V; when it is shorted it gives a current of 4 A. Calculate its internal resistance.

Solution

$I = \frac{E}{r+R}$

If cell is shorted ten R = 0

$I = \frac{E}{r} \qquad \therefore \qquad r = \frac{E}{I} = \frac{2}{4} = 0.5\ \Omega$

Illustration 27.

n rows each containing m cells in series, are joined in parallel. Maximum current is drawn from this combination in a 3 Ω resistance. If the total number of cells used is 24 and the internal resistance of each cell is 0.5 Ω, find the values of m and n.

Solution

Total number of cells = mn = 24

For maximum current $\frac{mr}{n} = R \quad \Rightarrow 0.5\,m = 3n$

$\Rightarrow m = \frac{3n}{0.5} = 6n$

∴ $6n \times n = 24 \quad \Rightarrow n = 2$

and $m \times 2 = 24 \quad \Rightarrow m = 12$

Illustration 28.

The e.m.f. of a cell is 6 volts. When 2 amperes current is drawn from it then the potential difference across its terminal remains 3 volts. Find its internal resistance.

Solution

Internal resistance $r = \frac{E - V}{I} = \frac{6-3}{2} = 1.5\Omega$

Illustration 29.

An electric heater and an electric bulb are rated 500 W, 220 V1and 100 W, 220 V respectively. Both are connected in series to a 220 V a.c. mains. Calculate the power consumed by (i) heater (ii) bulb

Solution

$P = \frac{V^2}{R}$ or $R = \frac{V^2}{P}$, For heater, Resistance $R_h = \frac{(220)^2}{500} = 96.8\ \Omega$. For bulb, resistance $R_L = \frac{(220)^2}{100} = 484\ \Omega$

Current in the circuit when both are connected in series $I = \frac{V}{R_L + R_h} = \frac{220}{484 + 96.8} = 0.38$ A

(i) Power consumed by heater $= I^2R_h = (0.38)^2 \times 96.8 = 13.98$ W

(ii) Power consumed by bulb $= I^2R_L = (0.38)^2 \times 484 = 69.89$ W

Illustration 30.

A heater coil is rated 100 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel to the same source of 200 V. Calculate the energy liberated per second in the new combination.

Solution

$\Theta\ P = \frac{V^2}{R} \qquad \therefore R = \frac{V^2}{P} = \frac{(220)^2}{100} = 400\Omega$

Resistance of half piece $= \frac{400}{2} = 200\ \Omega$

Resistance of pieces connected in parallel $= \frac{200}{2} = 100\Omega$

Energy liberated /second $P = \frac{V^2}{R} = \frac{200 \times 200}{100}$ = 400 W

Illustration 31.

The power of a heater is 500 W at 800°C. What will be its power at 200°C. If a = 4×10^{-4} per °C ?

Solution

$$P = \frac{V^2}{R} \qquad \therefore \frac{P_{200}}{P_{800}} = \frac{R_{800}}{R_{200}} = \frac{R_0(1 + 4 \times 10^{-4} \times 800)}{R_0(1 + 4 \times 10^{-4} \times 200)} \qquad \Rightarrow P_{200} = \frac{500 \times 1.32}{1.08} = 611 \text{ W}$$

Illustration 32.

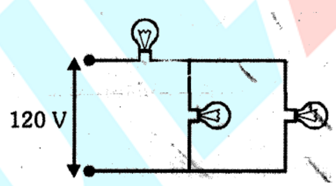
When a battery sends current through a resistance R_1 for time t, the heat produced in the resistor is Q. When the same battery sends current through another resistance R_2 for time t, the heat produced in R_2 is again Q. Determine the internal resistance of the battery.

Solution

$$\left[\frac{E}{R_1 + r}\right]^2 R_1 = \left[\frac{E}{R_2 + r}\right]^2 R_2 \qquad \Rightarrow r = \sqrt{R_1 R_2}$$

Illustration 33.

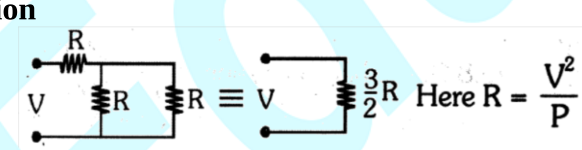
Three 60 W, 120 V light bulbs are connected across a 120 V power source. If the resistance of each bulb does not change with current then find out the total power delivered to the three bulbs.



(A) 180 W (B) 20W (C) 40 W (D) 60W

Ans. [C]

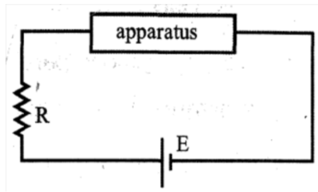
Solution



Total power supplied = $\frac{V^2}{\left(\frac{3}{2}\right)R} = \left(\frac{2}{3}\right)\left(\frac{V^2}{R}\right) = \frac{2}{3} \times 60 = 40\text{W}$

Illustration 34.

An apparatus is connected to an ideal battery as shown in figure. For what value of current, power delivered to the apparatus will be maximum?



(A) $\frac{E}{R}$ (B) $\frac{E}{2R}$

(C) $\frac{E}{4R}$ (D) information is insufficient

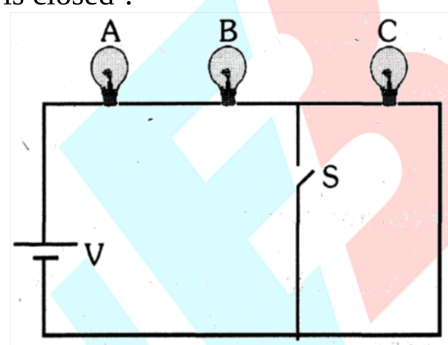
Ans. [B]

Solution

For maximum power $R_{ext} = R_{int} = R$ $\therefore$ current $= \frac{E}{2R}$

Illustration 35.

Three identical bulbs are connected in a circuit as shown. What will happen to the brightness of bulbs A & B if the switch is closed ?



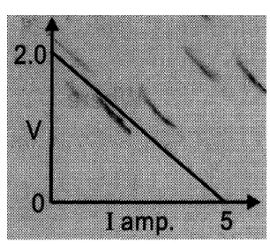
Solution

When switch S is closed, current does not pass through bulb C. Hence total resistance decreases & current passing through bulbs A & B increases. Brightness of bulbs A & B also increases.

BEGINNER'S BOX-4

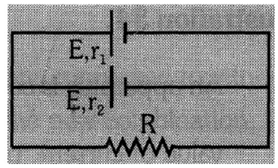
1. A battery of 20 cells (each having e.m.f. 1.8 V and internal resistance 0.1 Ω) is charged by 220 volts and the charging current is 15 A. Calculate the resistance to be put in the circuit.

2. For a cell, the graph between the potential difference (V) across the terminals of the cells and the current I drawn from the cell is as shown in figure. Calculate the e.m.f. and the internal resistance of the cell.



3. In the given circuit, calculate the value of current in the 4.5 n resistor and indicate its direction. Also calculate the potential difference across each cell.

4. Two cells of equal e.m.f. E but of different internal resistances r_1 and r_2 are connected in parallel and the combination is connected with an external resistance R to obtain maximum power in R. Find the value of R.



5. What is the largest voltage you can safely apply across a resistor marked 196 Ω; 1W?

6. Two bulbs whose resistances are in the ratio 1 : 2 are connected in parallel to a source of constant voltage. What will be the ratio of the power dissipation in these?

5. MEASURING DEVICES

5.1 Galvanometer

The instrument used to measure strength of current, by measuring the deflection of the coil due to torque produced by a magnetic field, is known as galvanometer.

Shunt

The small resistance connected in parallel to a galvanometer coil, in order to control the current flowing through the galvanometer, is known as shunt.

Merits of shunt

(i) To protect the galvanometer coil from burning.

(ii) Any galvanometer can be converted into ammeter of desired range with the help of suitable shunt.

(iii) The resistance of ammeter decreases due to shunt, thereby affecting the observed current to a lesser extent.

Demerits of shunt

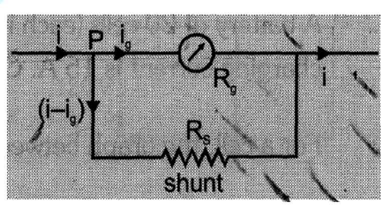
Shunt resistance decreases the sensitivity of galvanometer.

Conversion of Galvanometer into Ammeter

A galvanometer can be converted into an ammeter by connecting a low resistance (shunt) in parallel to its coil.

• The value of shunt resistance to be connected in parallel to galvanometer coil is given by –

$$R_S = \frac{R_g i_g}{i - i_g}$$



Where i = Range of ammeter

i_g =Current required for full scale deflection of galvanometer.

R_g = Resistance of galvanometer coil.

• $R_S = \frac{R_g i_g}{i}$ if $i_g << i$

Where

$$\begin{bmatrix} r = \text{radius of wire} \\ R_S = \text{resistance of shunt wire} \\ \rho = \text{specific resistance of material of wire} \end{bmatrix}$$

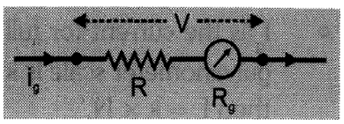
- The length of shunt wire l = $\dfrac{\pi r^2 R_S}{\rho}$
- On connecting shunt in parallel to a galvanometer coil, the equivalent resistance becomes less than even the shunt resistance.

Conversion of galvanometer into voltmeter

- A galvanometer can be converted into a voltmeter by connecting a high resistance in series with its coil.
- The high resistance to be connected in series with galvanometer coil is given by –

$R = \dfrac{V}{i_g} - R_g$

- If R_g is negligible, then $R = \dfrac{V}{i_g}$



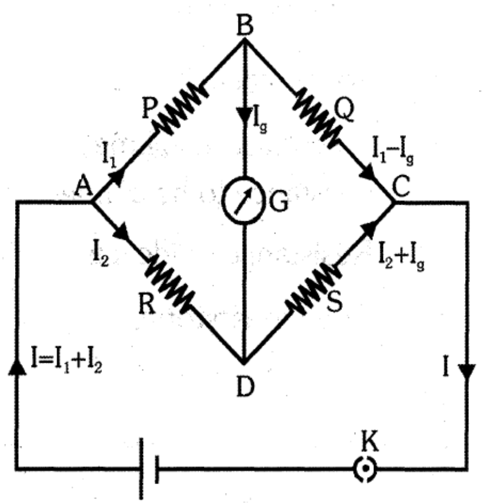
5.2 Wheat stone bridge

Wheatstone bridge devised by Charles Wheatstone in 1843 is an arrangement of four resistances which can be used to determine one of them in terms of the rest.

- The configuration in the adjacent figure is called Wheat stone bridge.
- If current in galvanometer is zero ($I_g = 0$) then bridge is said to be balanced.

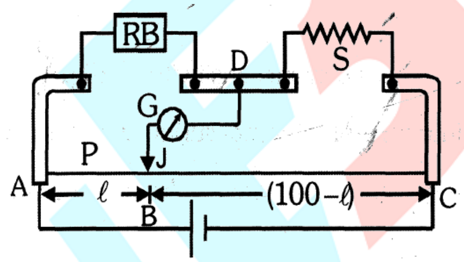
$V_D = V_B \qquad \Rightarrow I_1P = I_2R \;\&\; I_1S \qquad \Rightarrow \dfrac{P}{Q} = \dfrac{R}{S}$

- If $\dfrac{P}{Q} < \dfrac{R}{S}$ then $V_B > V_D$ and current will flow from B to D.
- If $\dfrac{P}{Q} > \dfrac{R}{S}$ then $V_B < V_D$ and current will flow D to B.



5.3 Meter Bridge

It is based on the principle of wheat stone bridge. It is used to find out unknown resistance of wire. AC is a 1 m long uniform wire. R.B. (Resistance Box) is a known resistance and Sis an unknown resistance. A cell is connected across the 1m long wire and Galvanometer is connected between Jockey midpoint D. To find out the unknown resistance the jockey made to touch from A to C and balanced condition is found. Let balance is at B point on wire.



AB = λ cm P = rλ

BC = (100 - λ) cm Q = r (100 – λ) where r = resistance per unit length of wire

In the balanced condition : $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{rl}{r(100-l)} = \frac{R}{S}$

$\Rightarrow \quad S = \frac{(100-l)}{l}R$

GOLDEN KEY POINTS

- The rate of variation of deflection depends upon the magnitude of deflection itself and hence on the accuracy of the instrument.
- A suspended coil galvanometer can measure currents of the order of 10^{-9} amperes.
- I_g is the current for full scale deflection. If the current corresponding to a deflection of one division on the galvanometer scale is k and N is the total number of divisions on one side of the zero of galvanometer scale, then $I_g = k \times N$.
- A ballastic galvanometer is a specially designed moving coil galvanometer, used to measure charge flowing through a circuit for a small time interval.
- To increase the range of an ammeter a shunt is connected in parallel with the galvanometer.

- To convert an ammeter of range I amperes and resistance $R_g\ \Omega$ into an ammeter of range nI amperes, the value of resistance to be connected in parallel will be $\frac{R_g}{n-1}\Omega$.
- To increase the range of a voltmeter a high resistance is connected in series with it.
- To convert a voltmeter of resistance $R_g\ \Omega$ and range V volts into a voltmeter of range nV volts, the value of resistance to be connected in series will be $(n-1)R_g\Omega$.
- Resistance of ideal ammeter is zero & resistance of ideal voltmeter is infinite.
- The wheatstone bridge is most sensitive when the resistances in all its four branches of same order are almost equal.

Illustrations

Illustration 36.

A 100 volts voltmeter whose resistance is 20 kΩ is connected in series with a very high resistance R. When it is joined across a line of 110 volts, it reads 5 volts. What is the magnitude of resistance R ?

Solution

When the voltmeter is connected across 110 volts line, current through the voltmeter $I = \frac{110}{(20\times10^3 + R)}$

Potential difference across the voltmeter $V = IR_V$ $\Rightarrow 5 = \frac{110\times20\times10^3}{(20\times10^3+R)}$

$\Rightarrow \quad 20\times10^3 + R = 440\times10^3 \quad \Rightarrow \quad R = 420\times10^3\Omega$

Illustration 37.

A galvanometer having 30 divisions has current sensitivity of 20 μA division. If has a resistance 25Ω.

(i) How will you convert it into an ammeter measuring currents upto 1 ampere ?

(ii) How will you convert this galvanometer into a voltmeter measuring voltages upto 1 volt ?

Solution

The current required for full scale deflection $I_g = 20\ \mu A\times30 = 600\ \mu A = 6\times10^{-4}A$

(i) To convert it into ammeter, a shunt is required in parallel with it.

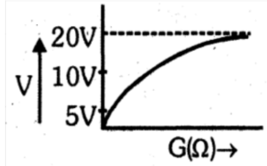
$$\text{Shunt resistance } R_S = \frac{I_gR_g}{(I-I_g)} = \left(\frac{6\times10^{-4}}{1-6\times10^{-4}}\right)25 = 0.015\ \Omega$$

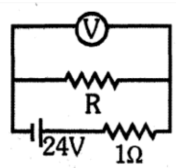
(ii) To convert the galvanometer into a voltmeter, a high resistance is required in series with it.

$$\text{Series resistance } R = \frac{V}{i_g} - R_g = \frac{1}{6\times10^{-4}} - 25 = 1666.67 - 25 = 1641.67\ \Omega$$

Illustration 38.

A cell of internal resistance 1Ω is connected across a resistor R. A voltmeter having variable resistance is used to measure the potential difference across the resistor. The plot of voltmeter reading V against G is shown. What is value of external resistor R? (G = Resistance of galvanometer)





(A) 5Ω (B) 4Ω (C) 3Ω (D) cannot be determined

Ans. **[A]**

Solution

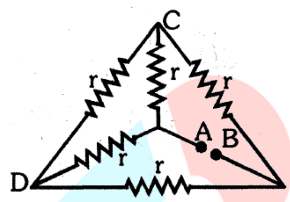
When galvanometer resistance tends to infinity $G \to \infty$,

Potential difference across R is 20V $\Rightarrow 20 = 24 - i \times 1$ $\Rightarrow i = 4$ A

Also $20 = 4 \times R$ $\Rightarrow R = 5$ n

Illustration 39.

In the adjoining network of resistors each has resistance r. Find the equivalent resistance between the points A and B.

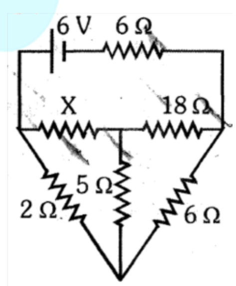


Solution

Given circuit is a balanced Wheatstone Bridge $\therefore \frac{1}{R_{AB}} = \frac{1}{2r} + \frac{1}{2r} = \frac{1}{r}$ $R_{AB} = r$

Illustration 40.

Calculate the magnitude of resistance X in the circuit shown in figure, when no current flows through the 5 Ω resistor?

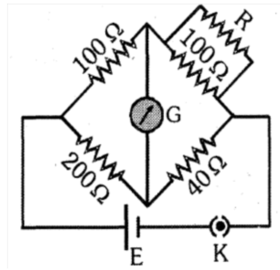


Solution

Since Wheatstone bridge is balanced so $\frac{x}{18} = \frac{2}{6}$ or $x = \frac{18 \times 2}{6} = 6\Omega$

Illustration 41.

In the following diagram the galvanometer shows zero deflection then what is the value of R ?



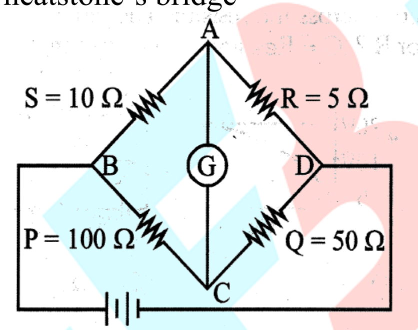
Solution

For a balanced Wheatstone bridge $\frac{100}{\frac{100R}{(100+R)}} = \frac{200}{40}$

$\Rightarrow \quad \frac{100+R}{R} = 5 \quad \Rightarrow \quad 100 + R = 5R \quad \Rightarrow \quad R = \frac{100}{4} = 25\Omega$

Illustration 42.

Figure shows a balanced Wheatstone's bridge

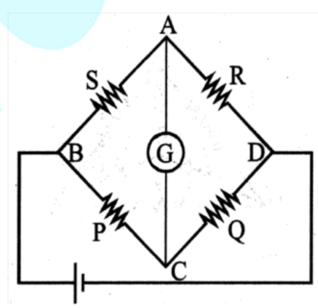


(A) If P is slightly increased, the current in the galvanometer flows from C to A
(B) If P is slightly' increased, the current in the galvanometer flows from A to C
(C) If Q is slightly increased, the current in the galvanometer flows from C to A
(D) If Q is slightly increased, the current in the galvanometer flows from A to C

(1) AB (2) BC (3) CD (4) AD

Ans. [2]

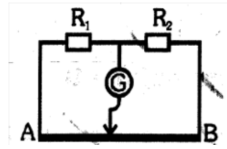
Solution



If P is slightly increased, potential of C will decrease. Hence current will flow from A to C. If Q is slightly increased, potential of C will increase, hence current will flow from C to A.

Illustration 43.

Consider a part of a meter bridge experiment as shown in figure when resistances in the two gaps are R_1 and R_2 then balancing length from end A is 30 cm. Now 10Ω resistances are added in both gaps then balancing length becomes 40 cm. Calculate:

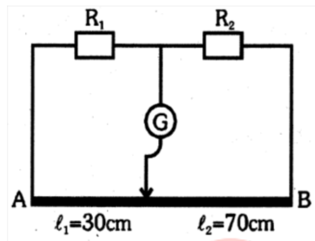


(a) Ratio of $\frac{R_1}{R_2}$

(b) Values of R_2 and R_1

(c) Value of resistance to be added in R_1 so that the balancing point is at the mid-point of the wire AB.

Solution



(a) $\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{30}{70} = \frac{3}{7}$

(b) $\frac{R_1 + 10}{R_2 + 10} = \frac{40}{60} = \frac{2}{3} \Rightarrow 3R_1 + 30 = 2R_2 + 20$

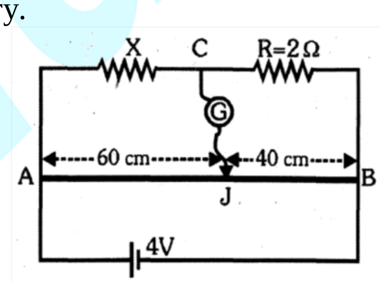
$\Rightarrow \quad 3R_1 + 10 = 2\times\frac{7R_1}{3} \quad \Rightarrow R_1 = 6W \;\&\; R_2 = 14W$

(c) Assume that value of resistance added in R_1 is x

$\therefore \quad \frac{R_1 + x}{R_2} = \frac{50}{50} \Rightarrow 6 + x = 14 \;\Rightarrow x = 8\Omega$

Illustration 44.

If no current flows through the galvanometer then find the value of the unknown resistance X. Assume that the resistance per unit length of the wire AB is 0.02 Ω/cm. Also calculate total current supplied by the battery.



Solution

For balanced condition $\frac{X}{R_{AJ}} = \frac{2}{R_{JB}}$

$R_{AJ} = 0.02\times60 = 1.2\ \Omega$ $R_{JB} = 0.02\times40 = 0.8\ \Omega$ $\quad \Rightarrow \frac{X}{1.2} = \frac{2}{0.8} \;\Rightarrow X = 3\ \Omega$

Current supplied by the battery $I = \frac{4}{R_{eq}}$

where net Resistance $R_{eq} = \frac{(3+2)(1.2+0.8)}{3+2+1.2+0.8} = \frac{10}{7}\Omega$ so $I = \frac{4\times7}{10} = 2.8A$

BEGINNER'S BOX-5

1. A galvanometer of resistance 100 0 gives full scale deflection for 10 mA current. What should be the value of shunt, so that it can measure currents upto 100 mA?

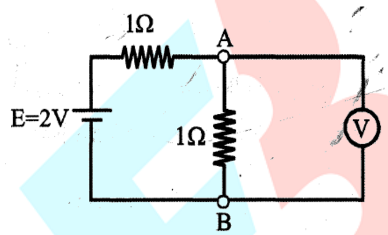
2. The resistance of a moving coil galvanometer is 20 Ω. It requires 0.01 A current for full scale deflection. Calculate the value of resistance required to convert it into a voltmeter of range 20 volts.

3. The resistance of an ammeter of range 5 A is 1.8 0. A shunt of 0.2 Ω is connected in parallel to it. When its indicator shows a current of 2 A then what will be the effective current?

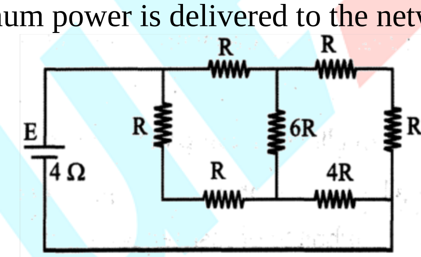
4. A galvanometer of resistance 100 Ω gives full scale deflection for a current of 10^{-5} A. Calculate the shunt required to convert it into an ammeter of 1 ampere range.

5. The shunt required for 10% of main current to be sent through a moving coil galvanometer of resistance 99Ω will be ....

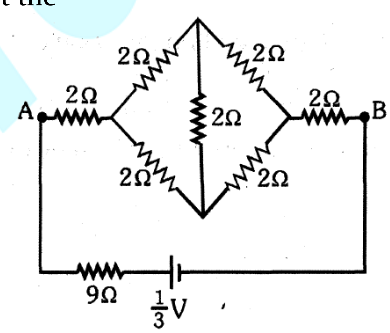
6. In the given circuit, the voltmeter and the electric cell are ideal. Find the reading of the voltmeter (in volts)



7. A battery of internal resistance 4Ω is connected to the network of resistances as shown in figure. In order that maximum power is delivered to the network, the value of R in n should be



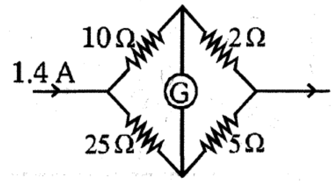
8. For the given circuit find out the-



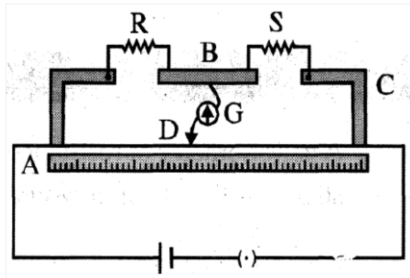
(a) total resistance of the circuit across the battery terminals.

(b) potential difference between A and B.

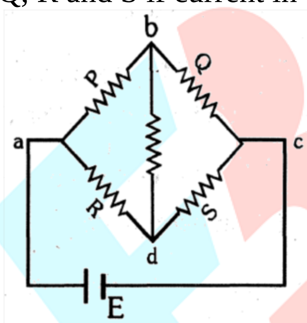
9. Calculate the current flowing through the resistance 2 Ω.



10. Circuit diagram of meter bridge is shown in figure. The null point is found at a distance of 40cm from A If now a resistance of 12 Ω is connected in parallel with S, the null point shifts to 64 cm. Determine the values of R and S.



11. What is the relation between P, Q, R and S if current in the branch bd is



(a) Zero ? (b) From b to d ?

6. POTENTIOMETER

6.1 Necessity of Potentiometer

Practically voltmeter has a finite resistance. (ideally it should be ∞) In other words, it draws certain current from the circuit. To overcome this problem a potentiometer is employed because at the instant of measurement, it draws no current from the circuit.

6.2 Working Principle of Potentiometer

Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over the entire length of a potentiometer wire.

This process is termed as zero deflection or null deflection method.

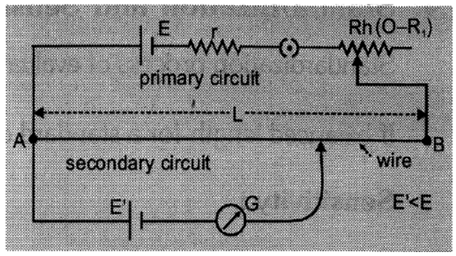
Note:

(i) Potentiometer wire : Made up of alloys of manganin, constantan, Eureka.

(ii) Special properties of these alloys are high specific resistance, negligible temperature co-efficient of resistance (α). This results in invariability of resistance of potentiometer wire over a long period.

6.3 Circuits of Potentiometer

- Primary circuit contains source of constant voltage & rheostat or Resistance Box.
- Secondary circuit contains battery & galvanometer.



Potential gradient (x) (V /m)

(i) Potential difference corresponding to unit length of potentiometer wire is called potential gradient.

(ii) Rate of growth/fall of potential per unit length of Potentiometer wire is equal to potential gradient.

Let r = 0 and R_1 = 0 then V_{AB} = E (max. in the ideal case) then x. = E/L

Unit and dimensions: (V/m ;· $MLT^{-3}A^{-1}$)

(iii) Always $V_{AB} < E$; (Θ $r + R_1 \neq 0$)

$$x = \frac{V_{AB}}{L} \qquad \text{Now } V_{AB} = IR_P \qquad (R_P = \text{resistance of potentiometer wire})$$

Let ρ = Resistance per unit length of potentiometer wire

$$\text{So } x = \frac{IR_P}{L} = I\rho \qquad \rho = \frac{R_P}{L}$$

$$\text{Current in primary circuit } I = \frac{E}{R_1 + r + R_P}; \quad x = \frac{E}{R_1 + R_P + r}\left(\frac{R_P}{L}\right)$$

(iv) If cross sectional radius is uniform ⇒ x is uniform over the entire length of potentiometer wire.

(v) If constant, then $x \propto \frac{1}{(\text{radius})^2}$

(vi) 'x' depends on → ρ, r, σ etc.

Factors affecting 'x'

(i) If V_{AB} = constant and L = constant then for any change → x remains unchanged.

(ii) If there is no information about V_{AB} then always take V_{AB} as constant so ($x \propto 1/L$)

(iii) If V_{AB} and L are constant :

For any change like radius of wire, substance of wire, (σ) there is no change in x.

(iv) Any change in the secondary circuit results in no change in x because x is an element of primary circuit.

Note : $x = \frac{E}{R_P + r + R_1}\left(\frac{R_P}{L}\right)$

x_{max} or x_{min} on the basis of range of rheostat or resistance box (R.B.)

If $R_1 = 0 \Rightarrow x_{max} = \frac{E}{R_P} \times \frac{R_P}{L}$ $(r \propto 0)$

If $R_1 = R \Rightarrow x_{min} = \frac{E}{R_P + R}\left(\frac{R_P}{L}\right)$ then $\frac{x_{max}}{x_{min}} = \frac{R_P + R}{R_P}$

6.4 Standardization and Sensitivity of Potentiometer

Standardization process of evaluating x experimentally :

If balanced length for a standard cell (emf E) is = $\square_0$ then potential gradient x = $\dfrac{E}{l_0}$

Sensitivity :

(i) x is also a measure of the sensitivity of potentiometer.

(ii) If x ↓ ⇒ sensitivity ↑

(iii) To increase sensitivity → R_h ↑ (current in primary circuit should be reduced), L↑

(iv) Any change in secondary circuit has no effect on sensitivity.

(v) Balanced' length for unknown potential difference ↑ ⇒ sensitivity ↑

7. APPUCATIONS OF POTENTIOMETER

(i) To measure the potential difference across a resistance.

(ii) To find out the emf of a cell.

(iii) Comparison of two emfs $\dfrac{E_1}{E_2}$

(iv) To find out the internal resistance of a primary cell.

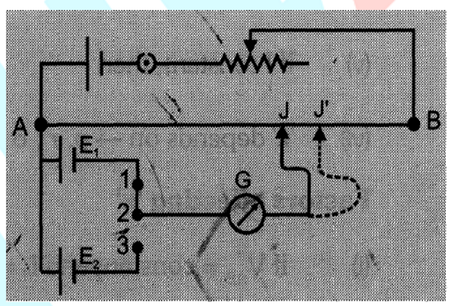
(v) Comparison of two resistances.

(vi) To find out the current in a given circuit.

7.1 Comparison of emfs of Two Cells

plug only in (1–2)	plug only in (2 - 3)
Jockey is at position J	Jockey is at position J'
balance length AJ = $\square_1$	balance length AJ' = $\square_2$
$E_1 = x\lambda_1$	$E_2 = x\lambda_2$

$$\Rightarrow \frac{E_1}{E_2} = \frac{l_1}{l_2}$$



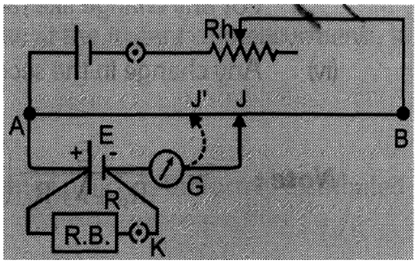
7.2 Internal Resistance of a Given Primary Cell

$$E = V + Ir \qquad \Rightarrow r\ \frac{E - V}{I}$$

$$\text{or } r = \left(\frac{E - V}{V}\right)R$$

Key K open	$E = x\lambda_1$	$(AJ = \lambda_1)$
Key K closed terminal potential difference	$V = x\lambda_2$	$(AJ' = \lambda_2)$

$$r = \left(\frac{l_1 - l_2}{l_2}\right)R$$



7.3 Comparison of Two Resistances

Plug only in (1 – 2)

potential difference across R1 is balanced

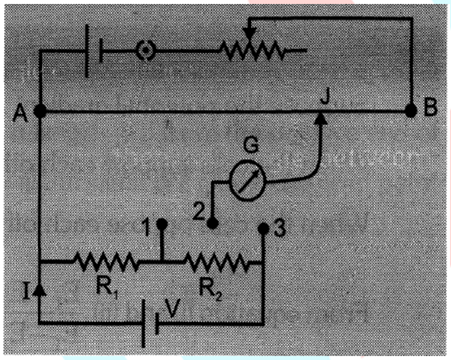
$\therefore$ $IR_1 = x\lambda_1$

Plug onlyin (2 – 3)

potential difference across ($R_1 + R_2$) is balanced

$\therefore$ $I(R_1 + R_2) = x\lambda_2$

$$\frac{R_1 + R_2}{R_1} = \frac{l_2}{l_1}$$



7.4 Measurement of Current

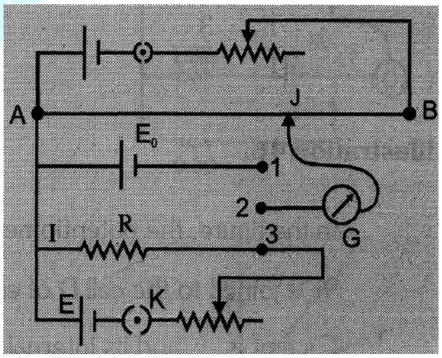
Plug only in (1 – 2)

$E_0 = x\lambda_0$

Plug only in (2 – 3)

$V = IR = x\lambda_1$

$$I = \frac{l_1}{R} \times \frac{E_0}{l_0}$$



GOLDEN KEY POINTS

- The effective resistance of potentiometer is infinite.

- The rate of variation of deflection depends upon the magnitude of deflection itself and hence on the accuracy of the instrument.

Difference between potentiometer and voltmeter	
Potentiometer	**Voltmeter**
• It measures the unknown emf very accurately	It measures the unknown emf approximately.
• While measuring emf it does not draw any current from the driving source of emf.	While measuring emf it draws some current from the source of emf.
• While measuring unknown potential difference the resistance of potentiometer becomes infinite.	While measuring unknown potential difference the resistance of voltmeter is high but finite.
• It is based on zero deflection method.	It is based on deflection method.
• It has a high sensitivity.	Its sensitivity is low.
• It is used for various applications like measurement of internal resistance of cell, calibration of ammeter and voltmeter, measurement of thermo emf, comparison of emf's etc.	It is only used to measured emf or unknown potential difference.

Illustrations

Illustration 45.

There is a definite potential difference between the two ends of a potentiometer. Two cells are connected in such a way that the first time they support each other, and the second time they oppose each other. They are balanced on the potentiometer wire over a 120 cm and 60 cm length respectively. Compare the electromotive forces of the cells.

Solution

Suppose the potential gradient along the potentiometer wire = x and the emf's of the two cells are E_1 and E_2.

When the cells support each other, the resultant emf = $(E_1 + E_2)$ $\quad \therefore E_1 + E_2 = x \times 120$ cm ... (i)

When the cells oppose each other, the resultant emf = $(E_1 - E_2)$ $\quad \therefore E_1 - E_2 = x \times 60$ cm .... (ii)

From equation (i) and (ii) $\dfrac{E_1 + E_2}{E_1 - E_2} = \dfrac{120\,\text{cm}}{60\,\text{cm}} = \dfrac{2}{1}$

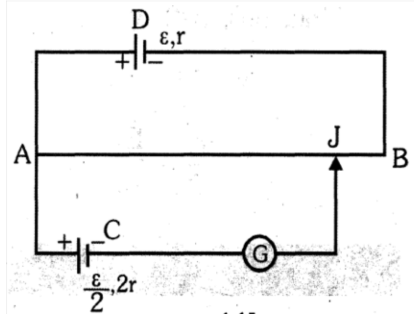
$\Rightarrow \quad E_1 + E_2 = 2(E_1 - E_2)$

$\Rightarrow \quad 3E_2 + E_1$

$\Rightarrow \quad \dfrac{E_1}{E_2} = \dfrac{3}{1}$

Illustration 46.

In the figure, the potentiometer wire AB of length Land resistance 9r is joined to the cell D of emf sand internal resistance r. The cell C's emf is $\dfrac{\varepsilon}{2}$ and its internal resistance is 2r. The galvanometer G will show no deflection when the length AJ is



(A) $\frac{4L}{9}$ (B) $\frac{5L}{9}$ (C) $\frac{7L}{18}$ (D) $\frac{11L}{18}$

Ans. [B]

Solution

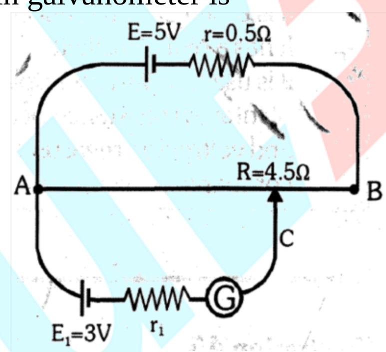
Potential gradient $x = \left(\frac{E}{10r}\right)\left(\frac{9r}{L}\right)$

According to question

$$\frac{E}{2} = \left(\frac{E}{10r}\right)\left(\frac{9r}{L}\right)(\lambda) \Rightarrow \lambda = \frac{5L}{9}$$

Illustration 47.

In the given potentiometer circuit length of the wire AB is 3m and resistance is R=4.5n. The length AC for no deflection in galvanometer is



(A) 2m (B) 1.8m (C) dependent on r_1 (D) None of these

Ans. [A]

Solution

Potential gradient

$$x = \left(\frac{5}{0.5+4.5}\right)\left(\frac{4.5}{3}\right) = 1.5\ Vm^{-1}$$

Here (x) (AC) = 3 $\Rightarrow$ AC = $\frac{3}{1.5}$ = 2m

BEGINNER'S BOX-6

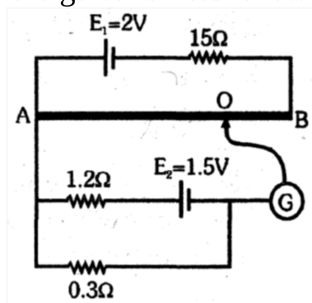
1. The length of a potentiometer wire is λ. A cell of emf E is balanced at a length λ/3 from the positive end of the wire. If the length of the wire is increased by λ/2 at what distance will the same cell give a balanced point

(A) $\frac{2l}{3}$ (B) $\frac{l}{2}$ (3) $\frac{l}{6}$ (5) $\frac{4l}{3}$

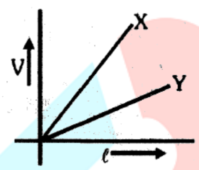
2. AB is 1 meter long uniform wire of 10 ohm resistance. The other data are as shown in the circuit diagram. Calculate

(a) potential gradient along AB and

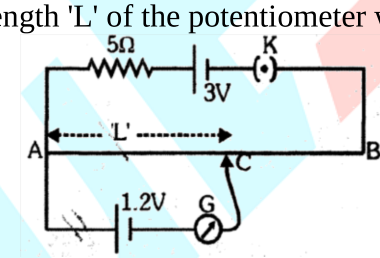
(b) length AO of the wire, when the galvanometer shows no deflection.



3. The variation of potential difference V with length λ in case of two potentiometers X and Y is as shown in the given diagram. Which one of these two will you prefer for comparing e.m.f.'s of two cells and why?



4. The potentiometer wire of length 100 cm has a resistance of 100Ω. It is connected in series with a resistance of 5Ω and an accumulator of emf 3V having negligible resistance. A source of 1.2 Vis balanced against a length 'L' of the potentiometer wire. Find the value of L.



5. A 10m long wire of resistance 20Ω is connected in series with a battery of e.m.f. 3V (negligible internal resistance) and a resistance of 10n. Find the potential gradient along the wire.

ANSWERS

BEGINNER'S BOX-1

1. 0.5×10^{-16} A

2. 2×10^{16}

3. 0.15 mm/s

4. 0.02 A; 2.74×10^{-12} m/s

BEGINNER'S BOX-2

1. 4×10^{-4} Ω

2. 0. 4%

3. R = 20Ω,
Resistivity of the wire remains unchanged as it does not change with change in dimensions of a material without change in its temperature.

4. 273°C

5. (A) - q, (B) - p, (C) - r

6. (A) - s, (B) - p, (C) - q, t, (D) – r

BEGINNER'S BOX-3

1. 2R **2.** $R_{AC} = \left(\frac{n^2+1}{n}\right)r$ **3.** $\frac{1}{100}\Omega$

4. 40Ω **5.** 3.7 A **6.** 3V

BEGINNER'S BOX-4

1. 10.27 Ω **2.** 2 V ; 0.4 Ω

3. I = 0.5 A, anticlockwise CDABC $E_1 = 4.25$ V, $E_2 = 7.5$ V

4. $\frac{r_1 r_2}{r_1 + r_2}$ **5.** 14 volts **6.** $\frac{P_1}{P_2} = \frac{2}{1}$

BEGINNER'S BOX-5

1. 11.11 Ω **2.** 1980 Ω **3.** 20A

4. 10^{-3} Ω **5.** 110 **6.** 1 V

7. 2 **8.** (a) 15 Ω, (b) 0.133 V **9.** 1A

10. $\frac{40}{3}$ W, 20W **11.** (a) $\frac{P}{Q} = \frac{R}{S}$, (b) $\frac{R}{S} > \frac{P}{Q}$

BEGINNER'S BOX-6

1. (B)

2. (a) 0.8 V/m, (b) 37.5 cm

3. A potentiometer circuit in which the potential gradient is smaller will be preferred as it is more sensitive. In the graph, shown $\frac{dV}{dl}$ is smaller for potentiometer Y and it will be preferred.

4. 60 cm

5. 0.2V/m