## **Ratio And Proportion**

**Ratio:** Ratio is the comparison between two quantities in terms of their magnitudes. The ratio of two quantities a and b in some units, is the fraction a/b and we write it as a: b. In the ratio a:b, we call 'a' as the first term or antecedent and 'b', the second term or consequent.

Ex. The ratio 2:3 represents 2/3 with antecedent= 2, consequent = 3

**Example:** A type writer takes 8 hours to type 960 pages. How many pages will it type in 12 hours?

**Sol.**  $\frac{8}{960} = \frac{12}{x}$ x = 1440

## Rule

- The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
- The comparison should always be done of the same quantity (of length, of weight etc.)

Ex.	2	:		3	=	4	:	6	=	8	:	9	=	10	:	15
Exa	<b>Example:</b> The two number are in ratio 2 : 3. If sum of the square two numbers is 20800,															
find								the							numt	pers?
Sol.	4x²					+					9x²=				2	0800
13x <sup>2</sup>	2							=	=						2	0800
X2								=	:							1600
х								:	=							40

Ist no. = 80, IInd no. = 120

**Proportion:** When two ratios are equal, then the four quantities involved in the two ratios are said to be proportional.

When a, b, c, d are in proportion, then a and d are called **EXTREMES** and b and c are called MEANS.

If a: b = c: d, we write, a: b :: c:d and say that a, b, c, d are in proportion. Here a and b are called extremes, and b and c are called means terms.

Thus a:b :: c:d  $\Rightarrow$  (a  $\times$  d) = (b  $\times$  c) Ex. 2:3 :: 5:7  $\Rightarrow$  (2  $\times$  7)= (3  $\times$  5)

**Example:** Which of the following numbers should be added to 13, 43, 23, and 73 So that they are in a proportion?

Sol. by Adding 2  $=\frac{(13+2)}{(43+2)} = \frac{(23+2)}{(73+2)}$   $\frac{15}{45} = \frac{25}{75}$   $\frac{1}{3} = \frac{1}{3}$ So, the number = 2

**Example:** find the value of 'a' in the following proportion 36 : 108 : : x : 12.

**Sol.** 
$$\frac{36}{108} = \frac{x}{12}$$
  
x = 4

**Fourth proportional:** If 2:3 :: 5:7, then 7 is called the fourth proportional to 2,3,5 **Example:** If a : b: c is 2 : 5 : 3 and c : d : e is 2 : 3 : 5 then find a : b : c : d : e ?

Sol. a : b : c : d : e 2 : 5 : 3 : 3 : 3 2 : 2 : 2 : 3 : 5 4 : 10 : 6 : 9 : 15a: b: c : d : e = 4 : 10 : 6 : 9 : 15 Third proportional: If 2:3 :: 5:7, then 5 is called the third proportional to 2 and 3

Example: If A : B is 2 : 5, B: C is 3: 4, then find A : B : C.

Sol. A : B = 2 : 5, B: C = 3 : 4 (given) A : B : C 2 : 5 : 5  $\frac{3 : 3 : 4}{6 : 15 : 20}$ A : B : C = 6 : 15 : 20

**Mean proportional:** Mean proportional between a and b is  $\sqrt{ab}$ 

**Ex**. A= 2, b=3 then mean proportional is  $\sqrt{2 \times 3} = \sqrt{6}$ **Comparison of Ratios:** if we say that (a:b)<(c:d)= $\frac{a}{b} < \frac{c}{d}$ 

**Ex.** (2:3) < (5:7) =  $\frac{2}{3} < \frac{5}{7}$ 

**Compounded ratio:** The compound ratio of the ratios (a:b), (c:d), (e:f) is (ace : bdf)

<b>Ex.</b> (2:3),		(5:7),					
=	(2×5×1	1)	:	(3×7×13)			
=	110	1	:	273			
Duplicate	ratio o	of	(a:b)	is	is (a²:b²)		
Ex.	Duplicate	ratio o	of (2:3)	) is	s (2 <sup>2</sup> :3 <sup>2</sup> )		
= (4:9)							
<b>Example:</b> If $P : Q = 2 : 3$ and $Q : R = 4 : 7$ then $(P + Q)^2 : (Q + R)^2$ is							
Sol.							
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	R 3 7 21						
(8	+12) <sup>2</sup>	:		(12	+21) <sup>2</sup>		

(20) <sup>2</sup>		:			(33) <sup>2</sup>					
400:		1089								
Sub-duplicate ra	<b>tio</b> of ( <u>a:b</u> ) is ( <sub>\</sub>	$\overline{a}:\sqrt{b}$ )								
<b>Ex</b> . Sub-duplicate ratio of (2:3) is $(\sqrt{2}:\sqrt{3})$										
Triplicate	Ratio of	(	a:b)	is	(a³:b³)					
Ex. Triplicate	ratio	of	(2:3)	is	(23:33)					
=		1 1.			(8:27)					
Sub-triplicate ratio of (a:b) is $\left(a^{\frac{1}{3}}:b^{\frac{1}{3}}\right)$										
<b>Ex.</b> (2:3) is $\left(2^{\frac{1}{3}}:3^{\frac{1}{3}}\right)$										
If $\frac{a}{b} = \frac{c}{d}$ , then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (componendo and dividendo)										
<b>Ex</b> . $\frac{2}{3} = \frac{5}{7}$ , then $\frac{2+3}{2-3} = \frac{5+7}{5-7}$										

**Variation:** Two quantities A and B may be such that as one quantity changes in value, the other quantity also changes in value bearing certain relationship to the change in the value of the first quantity.

**Example:** P : Q : R along completed a piece of work in 20, 40 and 30 days respectively. The ratio of the salary of each day is 5: 4: 3 respectively. If the total salary of Q is Rs.144, find total salary of P.

## Sol.

	Р	:	Q	1	R					
work	20	:	40	:	30					
Salary	5	:	4	:	3					
	100	:	160	:	90					
=10: 16: 9										
16x =	144									
$X = \frac{144}{16}$										
x = 9										
P = 10×9 = 90										
Salary of P = Rs. 90										