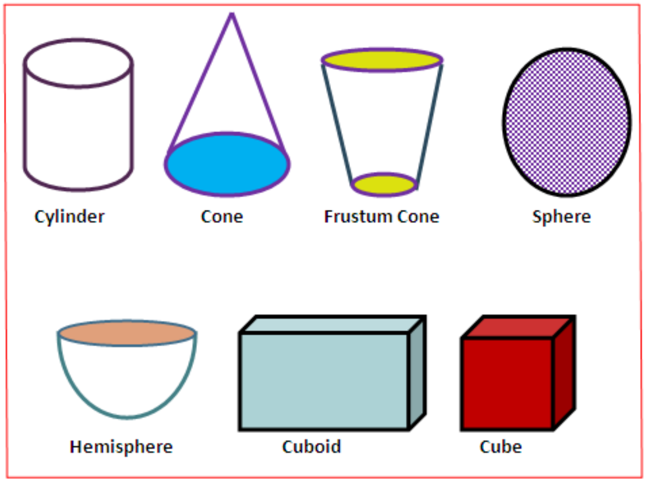
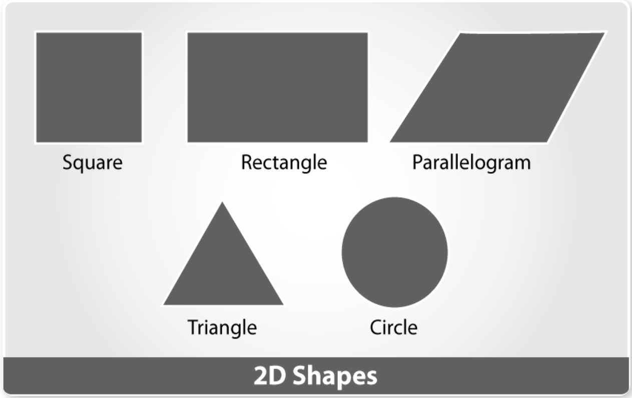
MENSURATION

Introduction- Mensuration is a topic in Geometry which is a branch of mathematics. Mensuration deals with length, area and volume of different kinds of shape- both 2D and 3D. So moving ahead in the introduction to Mensuration, let us discuss what are 2D and 3D shapes and the difference between them.



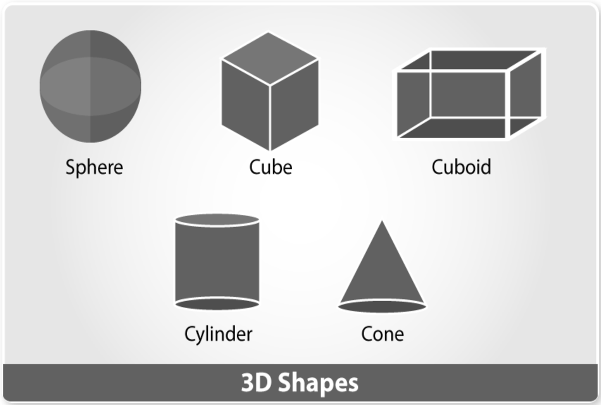
2D Shape -

A 2D shape is a shape that is bounded by three or more straight lines or a closed circular line in a plane. These shapes have no depth or height; they have two dimensions- length and breadth and are therefore called 2D figures or shapes. For 2D shapes, we measure area and perimeter.



3D Shape -

A 3D shape is a shape that is bounded by a number of surfaces or planes. These are also referred to as solid shapes. These shapes have height or depth unlike 2D shapes; they have three dimensions- length, breadth and height/depth and are therefore called 3D figures. 3D shapes are actually made up of a number of 2D shapes. Also, known as solid shapes, for 3D shapes we measure Volume (V), Curved Surface Area (CSA), Lateral Surface Area (LSA) and Total Surface Area (TSA).



Important Terms

Before we move ahead to the list of important mensuration formulas, we need to discuss some important terms that constitutes these mensuration formulas.

Area (A) – The surface occupied by a given closed shape is called its area. It is represented by the alphabet A and is measured in unit square- m^2/ cm^2.

Perimeter (P) – The length of the boundary of a figure is called its perimeter. In other words, it is the continuous line along the periphery of the closed figure. It is represented by the alphabet P and is measures in cm/ m.

Volume (V) – The space that is contained in a three-dimensional shape is called its volume. In other words, it is actually the space that is enclosed in a 3D figure. It is represented by the alphabet V and is measured in cm^3/ m^3.

Curved Surface Area (CSA) – In solid shapes where there is a curved surface, like a sphere or cylinder, the total area of these curved surfaces is the Curved Surface Area. . The acronym for this is CSA and it is measured in m^2 or cm^2.

Lateral Surface Area (LSA) – The total area of all the lateral surfaces of a given figure is called its Lateral Surface Area. Lateral Surfaces are those surfaces that surround the object. The acronym for this is LSA and it is measured in m^2 or cm^2.

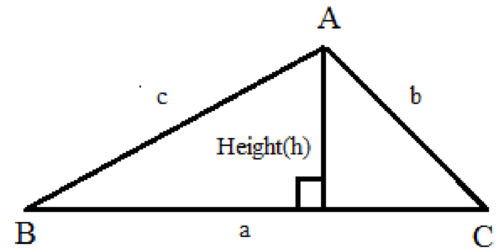
Total Surface Area (TSA) – The sum of the total area of all the surfaces in a closed shape is called its Total Surface Area. For example, in a cuboid when we add the area of all the six surfaces we get its Total Surface Area. The acronym for this is TSA and it is measured in m^2 or cm^2.

Square Unit (m^2/ cm^2) – One square unit is actually the area occupied by a square of side one unit.

Cube Unit (m^3/ cm^3) – One cubic unit is the volume occupied by a cube of side one unit.

TWO - DIMENSIONAL FIGURE(2D)

1. Triangle -



The sides AB, BC and AC are respectively denoted by c, a and b.

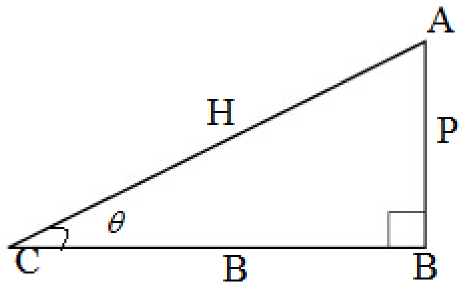
Area of a triangle = $\frac{1}{2} \times$ base $\times$ height

Area by Hero's formula = $\sqrt{s(s-a)(s-b)(s-c)}$

$$s = \frac{a+b+c}{2}$$

Right Angled Triangle -

A triangle having one of its angles equal to 90° is called a right-angled triangle. The side opposite to the right angle is called the hypotenuse.



Right-angled Triangle

AB = Perpendicular (P)

BC = Base (B)

AC = Hypotenuse (H)

In a right angled triangle-

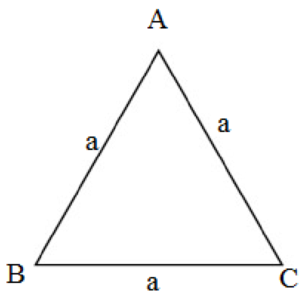
(**Hypotenuse**)^2 = (Perpendicular)^2 + (Base)^2
(Pythagoras theorem)

Area of right angled triangle = $\frac{1}{2} \times P \times B$

Perimeter of right angled triangle = P + B + H

Equilateral Triangle-

A triangle whose all sides are equal is called an equilateral triangle.



Equilateral Triangle

All angle = 60°

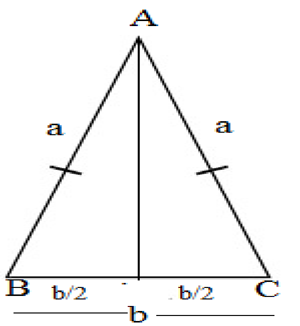
Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

Perimeter of equilateral triangle = 3a

Altitude (Height) = $\frac{\sqrt{3}}{2}a$

Isosceles Triangle

A triangle whose two sides are equal is an isosceles triangle.



Isosceles Triangle

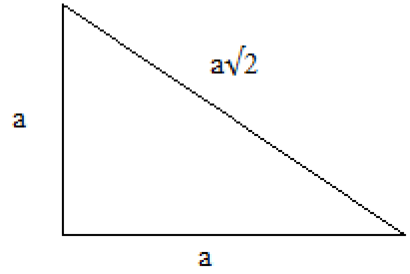
Area of isosceles triangle = $\frac{b}{4}\sqrt{4a^2 - b^2}$

Height of an isosceles triangle = $\frac{1}{2}\sqrt{4a^2 - b^2}$

Perimeter of an isosceles triangle = (2a+b)

Isosceles Right-angled Triangle-

An isosceles right-angled triangle has two sides (a) equal with equal sides making 90° to each other.

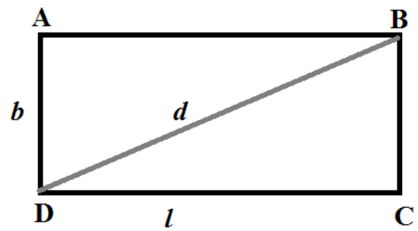


Area of Isosceles Right-angled Triangle = $\frac{1}{2}a^2$

Perimeter of Isosceles Right-angled Triangle = 2a + $\sqrt{2}$ a = $\sqrt{2}$ a($\sqrt{2}$ + 1) = h($\sqrt{2}$ + 1)

Rectangle

A four-sided shape that is made up of two pairs of parallel lines and that has four right angles.



The diagonals of a rectangle bisect each other and are equal.

Area of rectangle = length x breadth = l x b

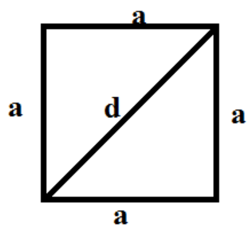
Area of rectangle = $l\sqrt{d^2 - l^2}$

Area of rectangle = $b\sqrt{d^2 - b^2}$

Perimeter (P) of rectangle = 2 (length + breadth)=2(l + b).

Square

A four-sided shape that is made up of four straight sides that are the same length and that has four right angles.



The diagonals of a square are equal and bisect each other at 90^0.

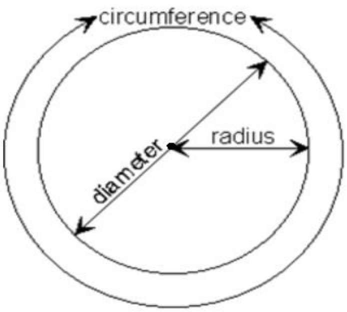
Area of a square $= a^2 = \frac{d^2}{2}$

Perimeter (P) of a square = 4a

Length (d) of the diagonal of a square = $a\sqrt{2}$

Circle

A circle is the path traveled by a point which moves in such a way that its distance from a fixed point remains constant.



Radius = r
Diameter (d)= 2r

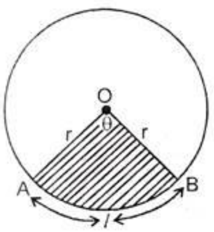
The fixed point is known as center and the fixed distance is called the radius.

Circumference(C) or perimeter of circle = $2\pi r$

Area of circle $= \pi r^2$

Sector-

A sector is a figure enclosed by two radii and an arc lying between them.



In given figure AOB is a sector

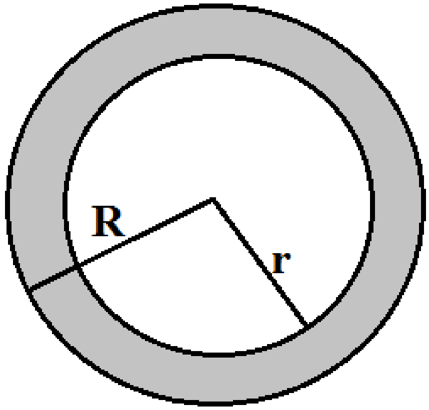
Length of arc AB(l)= $2\pi r \frac{\theta}{360}$

Area of Sector AOB $= \pi r^2 \frac{\theta}{360}$

Ring or Circular Path:

R=outer radius

r = inner radius



Area = π (R^2-r^2)

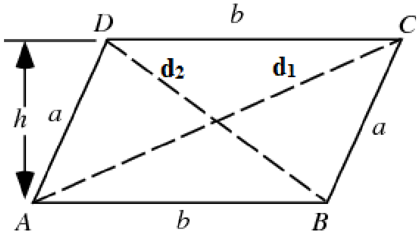
Perimeter = 2π(R + r)

Radius	Perimeter (2πr)	Area (πr²)
7	44	154
14	88	616
21	132	1386
28	176	2464
35	220	3850
42	264	5544

Parallelogram

A quadrilateral in which opposite sides are equal and parallel is called a parallelogram. The diagonals of a parallelogram bisect each other.

Area of a parallelogram = base × altitude = b × h



In a parallelogram, the sum of the squares of the diagonals = 2 (the sum of the squares of the two adjacent sides).

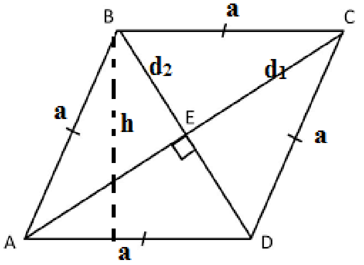
i.e., $d_1^{\ 2} + d_2^{\ 2} = 2(a^2 + b^2)$

Perimeter (P) of a parallelogram = 2 (a+b),

Where a and b are adjacent sides of the parallelogram.

Rhombus

Rhombus is a quadrilateral whose all sides are equal.



The diagonals of a rhombus bisect each other at 90^0

Area of a rhombus = a × h

$$= \frac{1}{2} d_1 \times d_2$$

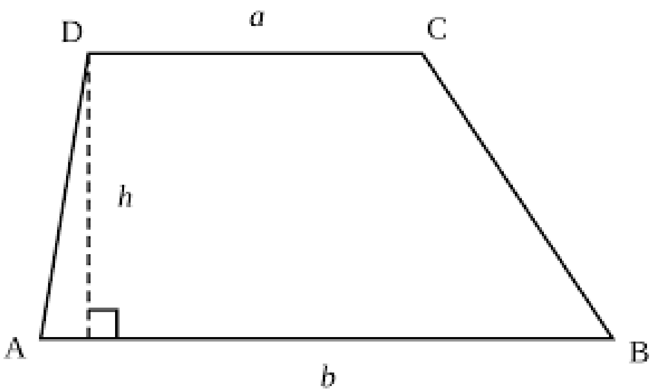
Perimeter of Rhombus = 4a

Relation between side and diagonal of Rhombus;

$$a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$$

Trapezium (Trapezoid)

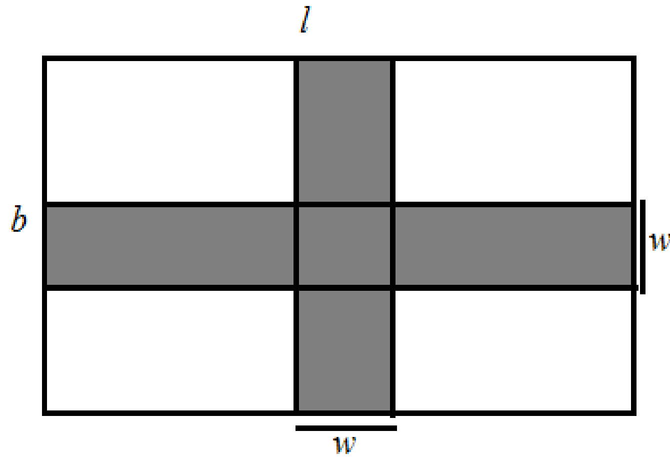
A *trapezium,* also known as a trapezoid, is a quadrilateral in which a pair of sides are parallel, but the other pair of opposite sides are non-parallel.



Area (a) of a trapezium = $\frac{1}{2}$ × (sum of parallel sides)× height = $\frac{1}{2}(a+b)h$

Perimeter of trapezium = AB+ BC + CD + DA

Pathways Running across the middle of a rectangle:

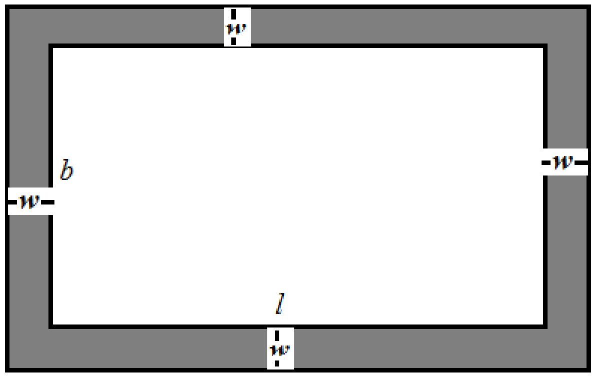


w is the width of the path

Area of path= (l + b - w)w

perimeter= 2(l + b - 2w)

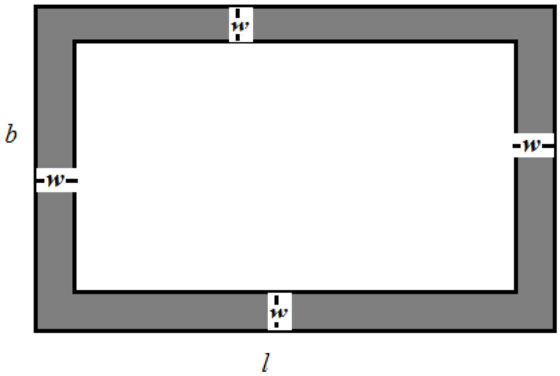
Outer Pathways:



Area = (l+b+2w)2w

Perimeter = 4(l+b+2w)

Inner Pathways:

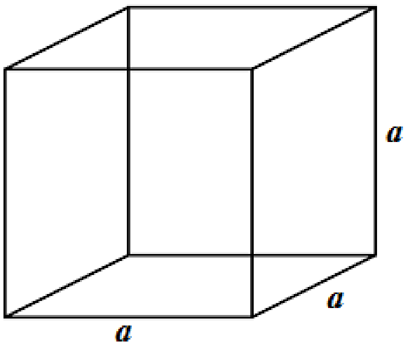


Area = (l+b-2w)2w

Perimeter = 4(l+b-2w)

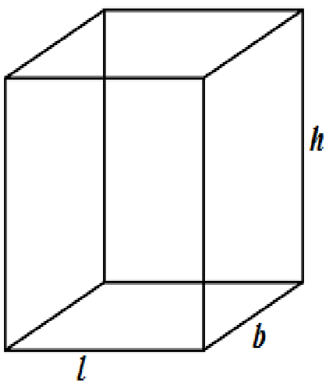
Three - Dimensional figure (3D)

Cube



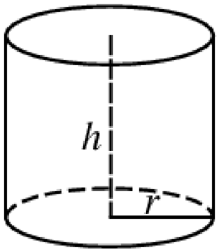
- ✓ **a** = side
- ✓ **Volume: V** = a^3
- ✓ **Lateral surface area** = $4a^2$
- ✓ **Surface Area (S)** = $6a^2$
- ✓ **Diagonal (d)** = $a\sqrt{3}$

Cuboid



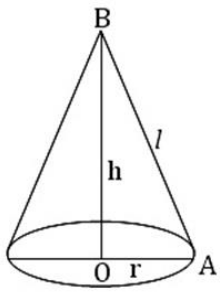
- ✓ **Volume of cuboid** = length × breadth × height= $l \times b \times h$
- ✓ **Lateral surface area** = $2h(l + b)$
- ✓ **Total surface area** = $2\ (lb + bh + hl)$
- ✓ **Longest diagonal** = $\sqrt{l^2 + b^2 + h^2}$

Right Circular Cylinder



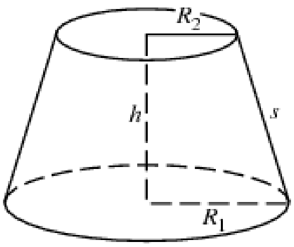
- **Volume of Cylinder** = $\pi r^2 h$
- **Lateral Surface Area (LSA or CSA)** = $2\pi r h$
- **Total Surface Area** = TSA = $2 \pi r (r + h)$

Right Circular Cone



- $l^2 = r^2 + h^2$
- **Volume of cone** = $\frac{1}{3}\pi r^2 h$
- **Curved surface area(CSA)**= πrl
- **Total surface area (TSA)** = $\pi r(r + l)$

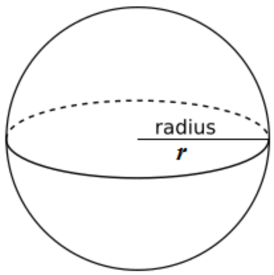
Frustum of a Cone



- **r** = Top radius,
- **R** = base radius,
- **h** = height, ***l*** = slant height
- **Volume**: $V = \frac{1}{3}\pi(r^2 + rR + R^2)h$
- **Surface Area**

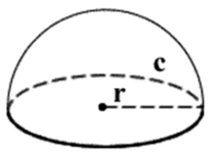
  $S = \pi(R+r)l + \pi r^2 + \pi R^2 \quad = \pi(R+r)l + \pi(r^2 + R^2)$

Sphere



- **r** = radius
- **Volume: V** = $\frac{4}{3}\pi r^3$
- **Curved Surface Area** = **Total Surface Area** = $4\pi r^2$

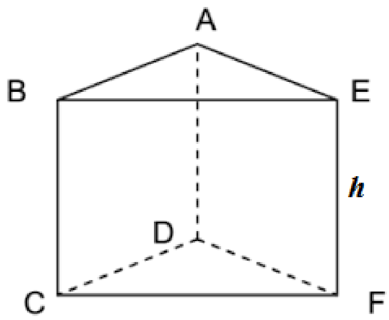
Hemisphere



- **Volume** = $\frac{2}{3}\pi r^3$
- **Curved surface area** = $2\pi r^2$
- **Total surface area** = $3\pi r^2$

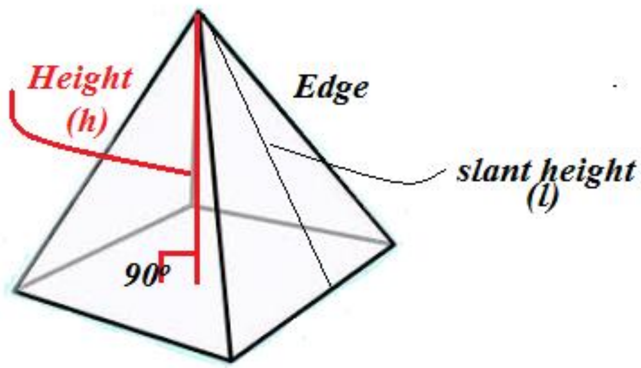
Prism

- **Volume** = Base area x height



- Lateral Surface area = perimeter of the base x height
- Total Surface area = LSA + 2(Area of base)

Pyramid



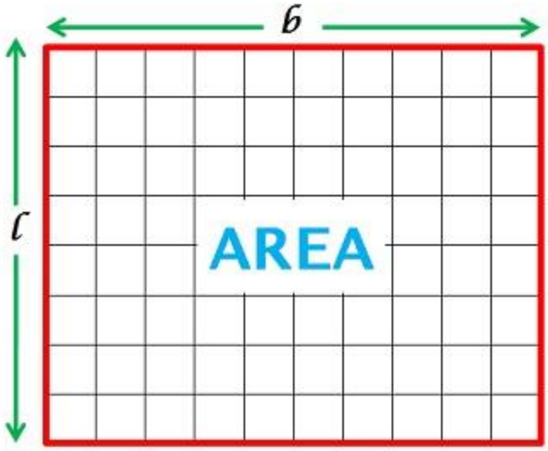
- **Volume of a right pyramid**

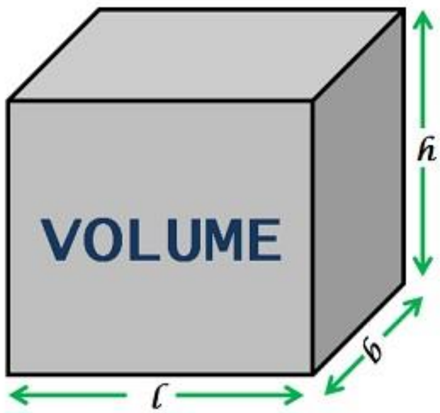
  $= \frac{1}{3} \times \textit{area of the base} \times \textit{height}$

- **Area of the lateral faces of a right pyramid**

  $= \frac{1}{2} \times \textit{perimeter of the base} \times \textit{slant height}$

- **Area of whole surface of a right pyramid** = Area of the lateral faces + Area of the base.





QUESTION WITH SOLUTION

Q.1. Three cubes of metal whose edges are in ratio of 3:4:5, are melted and one new cube is formed. If the diagonal of the cube is $18\sqrt{3}$ cm, then find the edge of the largest among three cubes.

(A) 18 cm **(B)** 24 cm

(C) 15 cm **(D)** 12 cm

Q.2. What is the area of the largest triangle that can be fitted into a rectangle of length 'a' units and width 'b' units?

(A) $\frac{ab}{3}$ unit^2 **(B)** $\frac{ab}{2}$ unit^2

(C) $\frac{a+b}{2}$ unit^2 **(D)** $\frac{a+b}{2}$ unit^2

Q.3. Find the perimeter and area of an isosceles triangle whose equal sides are 5 cm and the height is 4 cm.

(A) 24 cm^2 , 13 cm **(B)** 18 cm^2 , 16 cm

(C) 12 cm^2 , 13 cm **(D)** 12 cm^2 , 16 cm

Q.4. At each corner of a triangular field of sides 20m, 28m, and 36m, a cow is tethered by a rope of length 7m. Find the approximate area not grazed by the cow.

(A) 77 m^2

(B) 200.2 m^2

(C) 157.5 m^2

(D) 172.8 m^2

(A) $\pi/2$ **(B)** $2\pi/3$

(C) $\pi/3$ **(D)** $\pi/4$

Q.6. Find the volume of a sphere whose radius is $\sqrt{7}$ times the radius of another sphere which exactly fits in a cube of side 9 cm?

(A) $2673\sqrt{7}$ cm^3 **(B)** $4876\sqrt{7}$ cm^3

(C) $2486\sqrt{7}$ cm^3 **(D)** $3286\sqrt{7}$ cm^3

Q.7. The outer radius of a hemispherical iron tank is 14 cm and the in radius is 13.3 cm. It is to be painted inside as well as outside. Find the cost of painting it at the rate of ₹15 per 28 cm².

(A) Rs. 1855.65 **(B)** Rs. 1255.65

(C) Rs. 1355.65 **(D)** Rs. 1755.65

Q.8. Find the area of a trapezium whose parallel sides are length 58 meters and 42 meters and whose non-parallel sides are equal each being 20m.

(A) $300\sqrt{21}$ **(B)** $264\sqrt{21}$

(C) $232\sqrt{21}$ **(D)** $200\sqrt{21}$

Q.9. What is the area of the sector whose central angle is 90° and radius of the circle is 14 cm?

(A) 308 sq cm **(B)** 77 sq cm

(C) 154 sq cm **(D)** 231 sq cm

Q.10. If the ratio of Area and diameter of a circle is 35 : 14 then find the circumference of the circle.

(A) 7 **(B)** 10

(C) 12 **(D)** 18

Q.11. A parallelogram has area A sq. mts. A second parallelogram is formed by joining the mid-points of its sides. A third parallelogram is formed by joining the mid-poinds of the sides of the second parallelogram. This process is continued upto infinite. What is the sum of area (in sq. mts) of all the parallelograms so formed?

(A) A **(B)** $\frac{3A}{2}$

(C) 2A **(D)** $\frac{A}{2}$

Q.13. From the four corners of a rectangular sheet of dimensions 25 cm x 20 cm, square of side 2 cm is cut off from four corners and a box is made. The volume of the box is-

(A) 828 cm^3 **(B)** 672 cm^3

(C) 500 cm^3 **(D)** $1000cm^3$

Q.17. The base of a pyramid is a rectangle 30 m × 20 m and its slant height to the smaller side of the base is 17 metre. Find its volume of pyramid ?

(A) 12 **(B)** 9/2

(C) 4 **(D)** 9

Q.18. The radius of wheel is 21 cm. How many rounds will it take to cover the distance of 792?

(A) 600 **(B)** 200

(C) 300 **(D)** 400

Q.19. If the base of a right pyramid is triangle of sides 5 cm, 12 cm, and 13 cm and its volume is 330 cm, then its height (in cm) will be-

(A) 33 **(B)** 32

(C) 11 **(D)** 22

Q.21. A rectangular paper sheet of dimensions 22 cm × 12 cm is folder in the form of a cylinder alongs its length. What will be the volume of this cylinder.

(A) 460 cm^3 **(B)** 462 cm^3

(C) 624 cm^3 **(D)** 400 cm^3

Q.22. A copper rod of 1 cm diameter and 8 cm length is drawn into a wire of uniform diamter and 18 m length. The radius (in cm) of the wire is-

(A) $\frac{2}{15}$ **(B)** $\frac{1}{15}$

(C) $\frac{1}{30}$ **(D)** 15

Q.23. A hollow iron pipe is 21 cm long and its exterior diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weights 8 gm/cm^3, then the weight of the pipe is-

(A)3.696 kg **(B)**3.6 kg

(C) 36 kg **(D)** 36.9 kg

Q.24. Water flows at the rate of 10 meters per minute from cylindrical pipe 5 mm in diamter how long it will take to fill up a conical versel whose diameter at the base is 30 cm and depth 24 cm ?

(A) 28 minutes 48 seconds

(B) 51 minutes 12 seconds

(C) 51 minutes 24 seconds

(D) 28 minutes 36 seconds

Q.25. The radii of the base of two cylinders A and B are in true ratio 3 : 2 and their height in the ratio x : 1. If the volume of cylinder A is 3 times that of cylinder B, the value of x is-

(A) $\frac{3}{4}$

(B) $\frac{3}{2}$

(C) $\frac{4}{3}$

(D) $\frac{2}{3}$

Q.26. If the radius of a right circular cylinder is increased by 50% and its height is increased by 60%. Its volume will be decreased by ?

(A) 100% **(B)** 60%

(C) 40% **(D)** 20%

Q.27. Side of a tetrahedron is $3\sqrt{2}$ Cm. Find its volume.

(A) 12 Cm^3 **(B)**15 Cm^3

(C) 9 Cm^3 **(D)** 6 15 Cm3

Q.28. A hollow cylindrical iron pipe is of length 28 cm. It's outer diameter is 8 cm. and thickness of the metal is 2 cm. Find the weight of the pipe, if 1 $cm.^3$ of iron weights 7 gm.

(A) 13.248 kg. **(B)** 13.648 kg.
(C) 16.528 kg. **(D)** 17.248 kg.

Q.29. A field is in triangular shape whose sides are 26m, 28m, and 30 m. At each corner a cow is tied by 7m rope. Find ungrazed area.

(A) 259 $m.^2$ **(B)** 154 $m.^2$
(C) 306 $m.^2$ **(D)** 186 $m.^2$

Q.30. ABCD is a rectangle of dimensions 8 units and 6 units. AEFC is a rectangle drawn in such a way that diagonal AC of the first rectangle is one side and side opposite to it, is touching the first rectangle at D. What is the ratio of the area of rectangle ABCD to that of AEFC?

(A) 2 : 1 **(B)** 3 : 2
(C) 1 : 1 **(D)** 9

Solution

Q.1.(C) Total volume = $(3x)^3+(4x)^3+(5x)^3 = 216x^3$

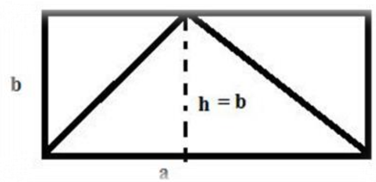
Then side of new cube/= 6x

Then diagonal of new cube = $6x\sqrt{3} \longrightarrow 18\sqrt{3}$

x = 3

Then side of larger cube = 5×3 = 15

Q.2.(B)



Area of triangle = (1/2) base × height

= (1/2) ab $unit^2$

Q.3.(D) 12 cm^2 , 16cm

Q.4.(B) Area of triangle = $84\sqrt{11}$ = 84 × 3.3 = 277.2

Area covered by cow

$= \pi r^2\frac{\theta_1}{360}+\pi r^2\frac{\theta_2}{360}+\pi r^2\frac{\theta_3}{360}=\frac{22}{7}\times(7)^2\left[\frac{\theta_1+\theta_2+\theta_3}{360}\right]$

= 22 × 7 × 0.5 = 77

Required Area = 277.2 - 77 = 200.2 m^2

Q.6.(A) Radius of sphere situated in cube = 9/2 cm

Radius of new sphere = $\frac{9}{2}\sqrt{7}$

Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times\frac{729}{8}\times 7\sqrt{7}$ =

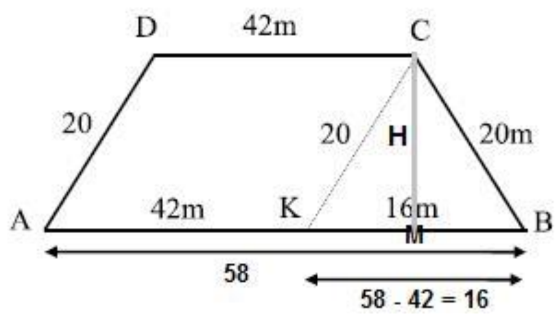
2673 $\sqrt{7}$ cm^3

Q.7.(B) Total Surface Area

$= 2\times\frac{22}{7}(14\times14+13.3\times13.3)$ = 2343.88

Total cost = 2343.88 × (15/28) =Rs. 1255.65

Q.8.(D)



h = 20^2 - 8^2

h = $4\sqrt{21}$

Area = Area of parallelogram + area of triangle = 42 × $4\sqrt{21}$ + (1/2) ×16×$4\sqrt{21}$

= $200\sqrt{21}$

Q.9.(C) Required area

$= \pi r^2 \frac{\theta}{360}$

$= \frac{22}{7}\times14\times14\times\frac{90}{360}$

= 154 sq. cm.

Q.10.(B)

$$\frac{\pi r^2}{2r} = \frac{35}{14}$$

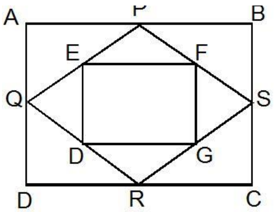
$$\frac{22}{7\times 2} r = \frac{35}{14}$$

$$r = \frac{35}{22}$$

Hence circuference of circle

$= 2\pi r = 2\times\frac{22}{7}\times\frac{35}{22} = 10$ cm

Q.11.(C)



sum of areas

$$= A + \frac{1}{2}A + \frac{1}{4}A + - - - \infty$$

$$= \frac{A}{1-\frac{1}{2}} = 2A$$

Q.13.(B) Volume of the box

$= 16\times21\times2$

$= 672\ cm^3$

Q.17.(D)

$$\frac{a}{2\sqrt{3}} = 3$$

$$a = 6\sqrt{3}$$

$$h = \frac{\sqrt{3}a}{2} = 9cm$$

Q.18.(A) Circumference/परिध $= 2\pi r$

$= 2\ \frac{2\times22\times21}{7} = 132$

Number of revalution $= \frac{79200}{132} = 600$

Q.19.(A)

$$v = \frac{1}{3}(\text{BaseArea})\times h = 330$$

$$\frac{1}{3}(30)\times h = 330$$

$$h = 33cm$$

Q.21.(B)

$$2\pi r = 22$$

$$R = \frac{7}{2}\ cm$$

$$v = \pi R^2 h$$

$$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times12$$

$$= 462\ cm^3$$

Q.22.(C)

$$\pi r^2 h = \pi R^2 h = \frac{1}{4}\times8 = 2$$

$$r^2 = \frac{2}{1800} = \frac{1}{900}$$

$$r = \frac{1}{30}\ cm.$$

Q.23.(A)

$$r = \pi\left(R^2 - r^2\right)\times h$$

$$= \frac{22}{7}\times\left(4^2-3^2\right)\times21 = 462\ cm^3$$

$$\therefore 1cm^3 = 8g$$

$$462\ cm^3 = 462\times8g$$

$$= 2696\ g = 3.696\ kg.$$

Q.24.(A) Volume of water flowing from the pipe in 1 minute

$$\pi\times.25\times.25\times1000$$

$$r = \frac{1}{3}\pi\times15\times15\times24$$

$$T = \frac{1}{3}\times\frac{\pi\times15\times15\times24}{\pi\times.25\times.25\times1000} = 28\frac{4}{5}$$

28 minutes 48 seconds

Q.25.(C)

$$\frac{\pi R^2 H}{\pi r^2 h} = \frac{3}{1}$$

$$\frac{H}{h} = \frac{4}{3}$$

$$\frac{x}{1} = \frac{4}{3}$$

$$x = \frac{4}{3}$$

Q.26.(B) Radius 2⟶1

Height 5⟶ 8

Volume 20⟶ 8

$$\frac{12}{20} \times 100 = 60\%$$

Q.27.(C)

$$V = \frac{a^3}{6\sqrt{2}} = \frac{(3\sqrt{2})^3}{6\sqrt{2}} = 9$$

Q.28.(D) Volume of cylindrical pipe

$$= \frac{22}{7} \times (8^2 - 6^2) \times 28$$

= 2464 $cm.^3$
1 $cm.^3$= 7 gm.
Weight of pipe = 2464 × 7
= 17248 gm.
= 17.248 gm.

Q.29.(A)

$$S = \frac{a+b+c}{2} = \frac{26+28+30}{2} = 42 \text{ m.}$$

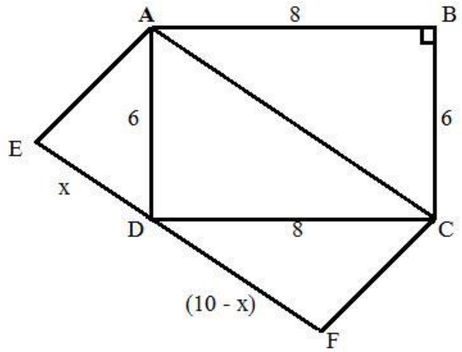
$$\Delta = \sqrt{42(42-26)(42-28)(42-30)}$$

= 336 m^2.

$$\text{Grazed Area} = \frac{\pi(7)^2 (Q_1 + Q_2 + Q_3)}{360}$$

= **77**
Ungrazed area = 336– **77** = **259** m^2.

Q.30.(C)



In ΔADC
$AC^2 = 6^2 + 8^2 = 36 + 64 = 100$
AC = 10
Now Let ED = x and DF = 10-x
[As AC = EF]
Using pythagoras theorem in ΔAED and ΔDFC
$6^2 - x^2 = 8^2 - (10 - x)^2$
$36 - x^2 = 64 - 100 - x^2 + 20x$
20x = 72
x = 3.6 units
$AE^2 = 36 - 12.96 = 23.04$
AE = 4.8 units
Required ratio = 8 × 6 : 10 × 4.8
= 48 : 48 = 1 : 1