WORK, ENERGY & POWER

1. WORK

Whenever a force acting on a body, displaces it, work is said to be done by the force.

Work done by a constant force is equal to the scalar product of the force $\vec{F}$ applied and the displacement $\vec{d}$ of the point of application, W = $\vec{F}.\vec{d}$

Work is a scalar quantity.

1.1 Work done by a constant force

Lf the direction and magnitude of a force applied on a body is constant then the force is said to be constant.

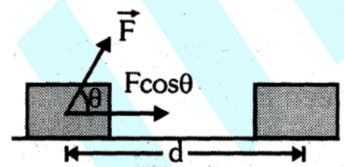
Work done by a constant force, W = Force x component of displacement along the force

= displacement x component of force along the displacement.

i.e., work done will, be $W = (F\cos\theta)\, d$

$= F\,(d\cos\theta)$

In vector form, $W = \vec{F}\cdot\vec{d}$



Note: The force of gravity (within small altitudes) is an example of constant force; consequently work done by it is an example of work done by a constant force.

1.2 Work done by a variable force

If the force applied on a body is changing in direction or magnitude or both, the force is said to be variable. Suppose a variable force causes displacement of a body from position P_1 to position P_2. To calculate the work done by the force, the path from P_1 to P_2 can be divided into infinitesimal elements; each element is so small that during the displacement of the body through it, the force is supposed to be constant. If $d\vec{r}$ be the small displacement of point of application and $\vec{F}$ be the fo.rce acting on the body, the work done by force is $dW = \vec{F}\cdot d\vec{r}$

The total work done in displacing the body from P_1 to P_2 is given

$$\int dW = \int_{P_1}^{P_2}\vec{F}.d\vec{r} \Rightarrow W = \int_{P_1}^{P_2}\vec{F}\cdot d\vec{r}$$

If $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the points P_1 and P_2 respectively then the total work done

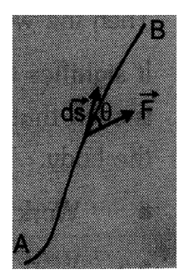
$$W = \int_{P_1}^{P_2}\vec{F}\cdot d\vec{r}$$

Cartesian Form

When magnitude and direction of the force varies with position, work done by the force for infinitesimal displacement $d\vec{s}$ is $dW = \vec{F}\cdot d\vec{s}$

The total work done for the displacement from position A B is $W_{AB} = \int_A^B \vec{F}.d\vec{s} = \int_A^B (F\cos\theta)ds$

In terms of rectangular components

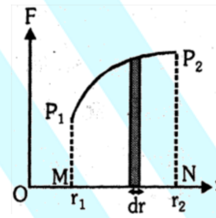


$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\,,\ d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W_{AB} = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k}) = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

1.3 Calculating work from graph

Suppose a body, whose initial position is $\vec{r}_1$, is acted upon by a variable $\vec{F}$ and consequently the body acquires its final position $\vec{r}_2$. From position $\vec{r}$ to $\vec{r}+d\vec{r}$ or for small displacement $d\vec{r}$, the work done will be $\vec{F}\cdot d\vec{r}$ whose value will be the area of the shaded strip of width $d\vec{r}$. The work done on the body in displacing it from position $\vec{r}_1$ to $\vec{r}_2$ will be equal to the sum of areas of ill such strips



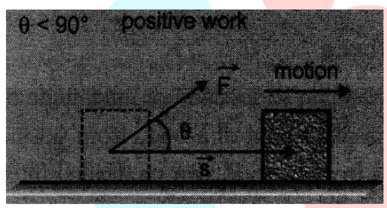
Thus, total work some, $W = \sum_{r_1}^{r_2} dW = \sum_{r_1}^{r_2} \vec{F}.d\vec{r}$ = Area of P_1P_2NM

The area of the graph between curve and displacement axis is equal to the work done.

Note : To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between F cosθ and r.

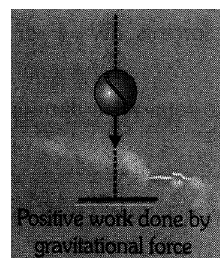
1.4 Nature of Work Done

(i) Positive work



$W = Fs\cos\theta$

If the angle θ is acute (θ < 90º) then the work is said to be positive. Positive work signifies that the external force favours the motion of the body



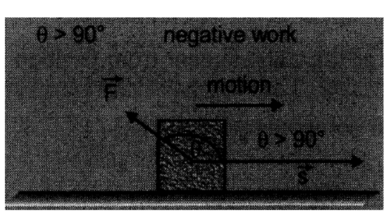
- When a body falls freely under the action of gravity (θ = 0º), the work done by gravity is positive.
- When a spring is stretched, stretching force and the displacement both are in the same direction. So work done by stretching force is positive.

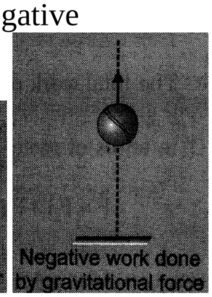
(ii) Negative work

$W = Fs\cos\theta$

If the angle θ is obtuse (θ > 90º). Then the work is said to be negative. It signifies that the direction of force is such that it opposes the motion of the body.

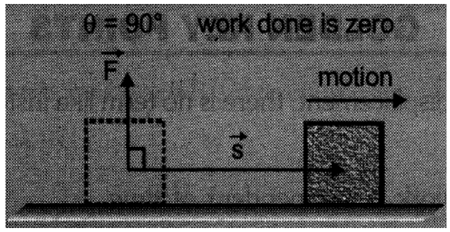
- Work done by frictional force is negative when it opposes the motion.
- Work done by braking force on the car is negative



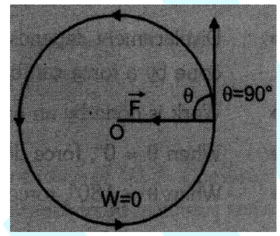


(iii) Zero work

$W = Fs\cos\theta$



Work done will be zero if (F = 0 or s = 0 or θ = 90º)



- body moving with uniform velocity.
- net force on the particle is zero.
- we push the wall and it remains at rest.
- a pendulum is oscillating, work done by tension.
- electron is moving round the nucleus.
- satellite is moving around the earth.
- coolie "Sahayak" is carrying buggage on a horizontal plateform (W.D.) by gravitational force is zero)

1.5 Dependency on Frame of Reference

A force does not depend on frame of reference and assumes same value in all frame of references, but displacement depends on frame of reference and may assume different values relative to different reference frames. Therefore, work of a force depends on choice of reference frame. For example, consider a man with a suitcase standing in a lift that is moving up. In the reference frame attached to the lift, the man applies a force equal to the weight of the bag but the displacement of the bag is zero, therefore work due to this force on the bag is zero. However, in a reference frame attached to the ground the bag has displacement equal to that of the lift and the force applied by the man does a non-zero work.

1.6 Unit & Dimension

- **Unit :**

SI Unit : joule (J).

Joule : One joule of work if said to be done when a force of one newton displaces a body by one meter in the direction of force.

1 joule = 1 newton × 1 meter = 1 kg-m^2/s^2

erg : One erg of work is said to be done when a force of one dyne displaces a body by one centimeter in the direction of force.

1 erg = 1 dyne × 1 cm = 1 g-cm^2/s^2

Other Units : (a) 1 joule = 10^7 ergs (b) 1 erg = 10^{-7} joules
(c) 1 eV = 1.6×10^{-19} joules (d) 1 joule = 6.25×10^{18} eV
(e) 1 MeV = 1.6×10^{-13} J (f) 1 J = 6.25×10^{12} MeV
(g) 1 kilo watt hour (kWh) = 3.6×10^6 joules

- **Dimensions :** [Work] = [Force] [Displacement] = $[MLT^{-2}][L] = [ML^2T^{-2}]$.

GOLDEN KEY POINTS

- Work is defined for an interval or displacement; there is no term like instantaneous work similar to instantaneous velocity.
- For a particular displacement, work is independent of time.
- When several forces act, work done by a force, for a particular displacement, is independent of Other forces.
- Displacement depends on reference frame so work done by a force is reference frame dependent. Work done by a force can be different in different reference frames.
- Work is done by an energy source or agent that applies the force.
- When $\theta = 0°$, force does maximum positive work.
- When $\theta = 180°$, force does maximum negative work.

Illustrations

Illustration 1.

A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2 kg and displaces it from x = 0 to x = 5 m. Calculate the work done in joules.

Solution:

$$W = \int_{x_1}^{x_2} F dx = \int_0^5 (7 - 2x + 3x^2)dx = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = 135 \text{ J.}$$

Illustration 2.

Calculate the amount of work done in raising a glass of water weighing 0.5 kg through a height of 20 cm. ($g = 10 \text{ m/s}^2$)

Solution:

Work done W = Force × displacement = mgh = 0.5 × 10 × 0.2 = 1 J

Illustration 3.

A body of mass 20 kg is at rest. A force of 5 N is applied on it. Calculate the work done in the first second.

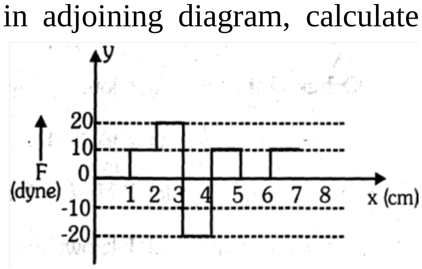
Solution:

Displacement in the direction of force is $d = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}t^2$ $\quad (\because s = ut + \frac{1}{2}at^2)$

so, work done is $W = \frac{1}{2}\frac{F^2}{m}t^2 = \frac{1}{2}\frac{5^2}{20} \times 1^2 = \frac{5}{8}$ J.

Illustration 4.

Corresponding to the force-displacement diagram shown in adjoining diagram, calculate the work done by the force in displacing the body from x = 1 cm to x = 5 cm.



Solution:

Work = Area between the curve and displacement axis

= 10 + 20 – 20 + 10 = 20 ergs.

Illustration 5.

Calculate the work done to move a body of mass 10 kg along a smooth inclined plane ($\theta = 30^\circ$) with constant velocity through a distance of 10 m.

Solution:

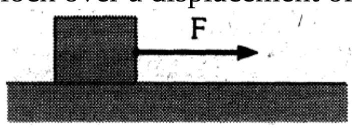
Here the motion is not accelerated, the resultant force parallel to the plane must be zero. So

$$F - Mg\sin 30^\circ = 0 \Rightarrow F = Mg\sin 30^\circ \;\&\; d = 10\text{ m}$$

$$W = Fd\cos\theta = (Mg\sin 30^\circ)\,d\cos 0^\circ = 10 \times 10 \times \frac{1}{2} \times 10 \times 1 = 500\text{ J.}$$

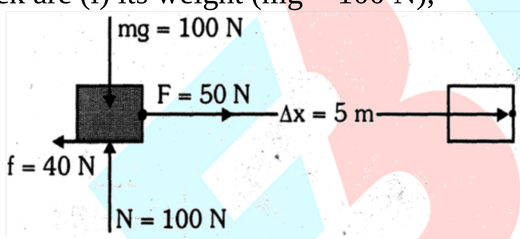
Illustration 6.

A 10 kg block placed on a rough horizontal floor is being pulled by a constant force of 50 N. Coefficient of kinetic friction between the block and the floor is 0.4. Find the work done by each individual force acting on the block over a displacement of 5 m.



Solution:

Forces acting on the block are (i) its weight (mg = 100 N),



(ii) normal reaction (N = 100 N) by the ground, (iii) force of kinetic friction (f = 40 N) and (iv) the applied force (F = 50 N). All these force and the displacement of the block are shown in the figure.

All these forces are constant forces, therefore we use equation $W_{i\to f} = \vec{F}\cdot\Delta\vec{r}$.

Work done W_g by the gravity i.e. weight of the block $W_g = 0\ \left(\because m\vec{g} \perp \Delta\vec{x}\right)$

Work done W_N by the normal reaction $W_N = 0\ \left(\because \vec{N} \perp \Delta\vec{x}\right)$

Work done W_F by the applied force $W_F = 250\ \left(\because \vec{F} \parallel \Delta\vec{x}\right)$

Work done W_f by the force of kinetic friction $W_f = -20$ J. $\left(\because \vec{f} \uparrow\downarrow \Delta\vec{x}\right)$

Illustration 7.

Calculate the work done by the force $\vec{F} = \left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$ N in carrying a particle from point (–2 m, 1 m, 3 m) to point (3 m, 6 m, –2 m).

Solution:

The force $\vec{F}$ is a constant force, therefore we can use equation $W_{i\to f} = \vec{F}\cdot\Delta\vec{r}$.

$$W = \vec{F}\cdot\Delta\vec{r} = \left(3\hat{i} - 2\hat{j} + 4\hat{k}\right).\left(5\hat{i} + 5\hat{j} - 5\hat{k}\right) = -15\text{ J.}$$

Illustration 8.

A particle is shifted from point (0,0,1 m) to point (1 m, 1 m, 2 m), under the simultaneous action of several forces. Two of them are $\vec{F}_1 = (2\hat{i} + 3\hat{j} + \hat{k})$ N and $\vec{F}_2 = (\hat{i} - 2\hat{j} + 2\hat{k})$ N . Find the work done by the resultant of these two forces.

Solution:

Work done by a constant force equals to dot product of the force and displacement vectors.

$$W = \vec{F} \cdot \Delta\vec{r} \Rightarrow W = (\vec{F}_1 + \vec{F}_2) \cdot \Delta\vec{r}$$

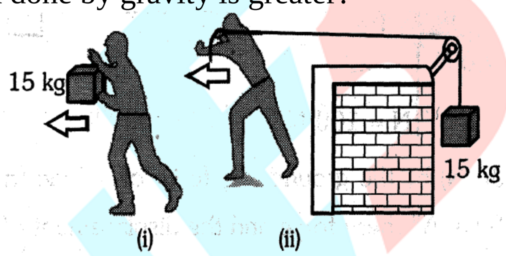
Substituting the given values, we have

$$W = (3\hat{i} + \hat{j} + \hat{k}).(\hat{i} + \hat{j} + \hat{k}) = 3 + 1 + 1 = 5 \text{ J.}$$

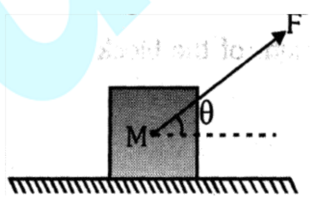
BEGINNER'S BOX - 1

1. A body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done is 25 joules, find the angle which the force .makes with the direction of motion of the body.

2. In Fig. (i), the man walks 2 m carrying a mass of 15 kg on his hand. In Fig. (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end.
In which case is the work done by gravity is greater?



3. Calculate the work done in pulling an object with a constant force F as shown in figure, so that the block moves with a constant velocity through a distance 'd'. (given that the ground is rough with coefficient of friction μ)



4. A particle moves along the x-axis from $x = x_1$ to $x = x_2$ under the influence of a force given by $F = 2x$. Find the work done in the process.

5. A force F = (10 + 0.5x) N acts on a particle in x direction, where x is in meters. Find the work done by this force during a displacement from x = 0 to x = 2.

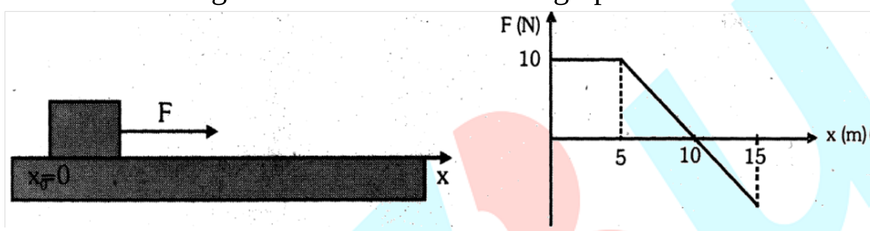
6. A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2 m ?

7. A particle moves from position $\vec{r}_1 = (3\hat{i} + 2\hat{j} - 6\hat{k})$ m to position $\vec{r}_2 = (14\hat{i} + 13\hat{j} + 9\hat{k})$ m under the action of force $(4\hat{i} + \hat{j} + 3\hat{k})$ N. Find the work done.

8. $\vec{F} = K(x\hat{i} + y\hat{j})$ is working on a particle which moves from (0, 0) to (a, 0) along x axis and (a, 0) to (a, a) along y axis. Find the work done by the force.

9. Three forces $3(\hat{i} + 3\hat{j} + \hat{k}), \frac{5}{7}(-2\hat{i} + 9\hat{k})$ and $11\,(2\hat{i} + \hat{j} + 6\hat{k})$ are acting on a particle. Calculate work done in displacing the particle from point (4, –1, 1) to point (11, 6, 8).

10. A horizontal force F is used to pull a box placed on a floor. Variation in the force with position coordinate x measured along the floor is shown in the graph.



(a) Calculate the work done by the force in moving the box from x = 0 to x = 10 m.
(b) Calculate the work done by the force in moving the box from x = 10 m to x = 15 m.
(c) Calculate the work done by the force in moving the box from x = 0 to x = 15 m.

2. ENERGY

Energy is defined as the internal capacity to do work.
When we say that a body· has energy it means that it can do work.

2.1 Different forms of energy

Mechanical energy, electrical energy, optical (light) energy, acoustical (sound), molecular, atomic and nuclear energy etc, are various forms of energy.
These forms of energy can change from one form to the other.

2.2 Mass energy relation.

According to Einstein's mass-energy equivalence principle, mass and energy are inter convertible i.e. they can be changed into each other.
Equivalent energy corresponding to mass m is $E = mc^2$
where, m : mass of the particle
c : speed of light

- Energy is a scalar quantity· ..
- Dimensions : $[M1L^2T^{-2}]$
- SI Unit : joule

Other units: 1 erg.= 10^{-7} J 1kWh = 36×10^5 J
1 eV = 1.6×10^{-19} J 1 cal = 4.2 J

In mechanics we are only concerned with the mechanical energy, which is of two types.
(a) Kinetic energy (b) Potential energy

2.3 Kinetic Energy

Kinetic energy is the internal capacity of doing work of an object by virtue of its motion.

or

K.E. of a body can be calculated by the amount of work done in stopping the moving body or by the amount of work done in imparting the present velocity to the body from the state of rest.

If a particle of mass m is moving with velocity 'v' much less than the velocity of light, then the kinetic energy

K.E. is given by $\quad K.E. = \frac{1}{2}mv^2$

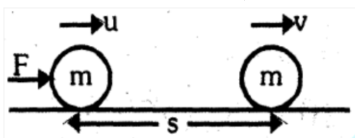
2.4 Work-Kinetic Energy Theorem

Work done by all the forces (conservative or non conservative, external or internal) acting on a particle or an object is equal to the change in its kinetic energy. So work done by all the forces = change in kinetic energy

$$W = \Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_1^2$$

Proof :

(i) For constant force :



$v^2 = u^2 + 2as$

$\Rightarrow \quad v^2 - u^2 = 2\left(\frac{F}{m}\right)s$

$\Rightarrow \quad s = \frac{(v^2 - u^2)m}{2F} = \frac{\Delta KE}{F}$

$\Rightarrow \quad Fs = \Delta KE$

$\Rightarrow \quad W = \Delta KE$

(ii) For variable force:

$$W = \int F dx = \int madx = \int m\left(\frac{vdv}{dx}\right)dx = \int_u^v mvdv = m\left[\frac{v^2}{2}\right]_u^v = \frac{m}{2}[v^2 - u^2]$$

$\Rightarrow \quad W = \Delta KE$

2.5 How to apply work-kinetic energy theorem

The work-kinetic energy theorem is deduced here for a single body moving relative to an inertial frame, therefore it is recommended at present, to use for a single body in inertial frame. The use work-kinetic energy theorem the following steps should be followed:

- Identify the initial and final positions as position 1 and 2 and write expressions for kinetic energies, whether known or unknown.
- Draw the free body diagram of the body at any intermediate stage between positions l and 2. The forces shown will help in deciding their work. Calculate work by each force and add them to obtain the total work done $W_{1\to2}$ by all the force.
- Use the work obtained in step 2 and kinetic energies obtained in step 1 in the equation $W_{1\to2} = K_2 - K_1$.

GOLDEN KEY POINTS

- If K.E. of the body decreases then work done is negative & vice-versa.

- In the above discussion, we have assumed that the work done by the force is effective only in changing the kinetic energy of the body. It should however be remembered that the work done on a body may also be stored in the system in the form of potential energy.
- As mass m and v^2 or $\vec{v}.\vec{v}$ are always positive so K.E. can never be negative .
- The kinetic energy depends on the frame of reference.
- The expression KE = $\frac{1}{2}mv^2$ holds even when the force applied varies in magnitude or in direction or in both .

Illustrations

Illustration 9.

A body of mass 0.8 kg has initial velocity $(3\hat{i}-4\hat{j})$ m/s and final velocity $(-6\hat{j}+2\hat{k})$ m/s. Find the change in kinetic energy of the body ? **[AIPMT (Mains) 2006]**

Solution:

Change in kinetic energy

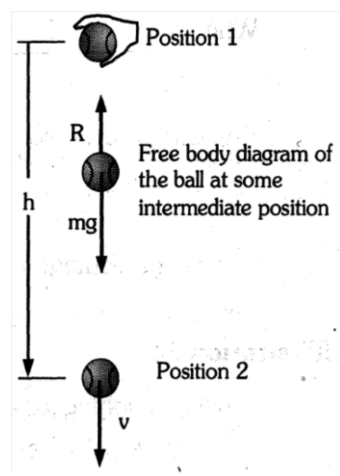
$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \text{ where } v_f = \sqrt{6^2+2^2} = \sqrt{40} \text{ and } v_i = \sqrt{3^2+4^2} = \sqrt{25}$$

$$= \frac{1}{2} \times 0.8\left(\left(\sqrt{40}\right)^2 - \left(\sqrt{25}\right)^2\right) = 0.4\,[40-25] = 0.4\,(15) = 6 \text{ joules.}$$

Illustration 10.

A 5 kg ball when falls through a height of 20 m acquires a speed of 10 m/s. Find the work done by air resistance.

Solution:



The ball start falling from position 1, where its speed is zero.
hence, kinetic energy is also zero

$K_1 = 0$ J ……(i)

During the downward motion of the ball, constant gravitational force mg acts downward and air resistance R of unknown magnitude acts upward as shown in the free body diagram. The ball reaches position 20 m below the position-1 with a speed v = 10 m/s, so the kinetic energy of the ball at position 2 is

$K_2 = \frac{1}{2}mv^2 = 250$ J …….(ii)

Work done by gravity

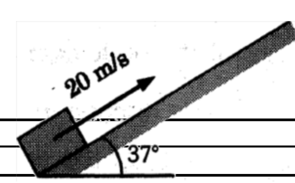
$W_{g,1\to2} = mgh = 1000$ J …….(iii)

Denoting the work done by the air resistance as $W_{R,1\to2}$ and making use of eq. (i), (ii) and (iii) work-kinetic energy theorem,

we have $W_{1\to2} = K_2 - K_1 \Rightarrow W_{g,1\to2} + W_{R,1\to2} = K_2 - K_1 \Rightarrow W_{R,1\to2} = -750$ J.

Illustration 11.

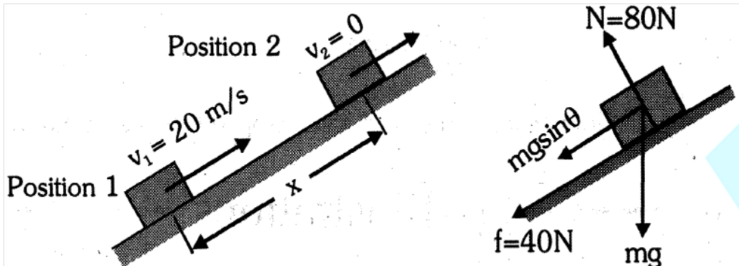
A box of mass m = 10 kg is projected up an inclined plane from its foot with a speed of 20 m/s as shown in the figure. The coefficient



of friction μ between the box and the plane is 0.5. Find the distance travelled by the box on the plane before it stops for the first time.

Solution:

The box start from position 1 with speed $v_1 = 20$ m/s and stops at position 2.



Kinetic energy at position 1: $K_1 = \frac{1}{2}mv_1^2 = 2000$ J

Kinetic energy at position 2: $K_2 = 0$

Work done by external forces as the box moves from position 1 to position 2 is,

$$W_{1\to2} = W_{g,1\to2} + W_{f,1\to2} = 60x - 40x = -100x \text{ J}$$

Applying work energy theorem for the motion of the box from position 1 to position 2, we have

$$W_{1\to2} = K_2 - K_1 \Rightarrow -100x = 0 - 2000 \Rightarrow x = 20 \text{ m}$$

Illustration 12.

Kinetic energy of a particle is increased by 300%. Find the percentage increase in its momentum.

Solution:

Kinetic energy $E = \frac{1}{2}mv^2$ momentum $p = mv$

When E is increased by 300%, $E' = E + 3E = 4E = 4(\frac{1}{2}mv^2) = 2mv^2$

If v' is the new velocity of the body, then $\frac{1}{2}mv^2 \Rightarrow v' = 2v$ So $p' = mv' = 2mv$

Hence, percentage change in momentum = $\frac{2mv - mv}{mv} \times 100 = 100$ %.

Illustration 13.

A bullet weighing 10 g is fired with a velocity 800 m/s. Its velocity decreases to 100 m/s after passing through a 1m thick mud wall. Find the average resistance offered by the mud wall.

Solution:

Work done by the average resistance (offered by the wall) = change in K.E. of the bullet.

$$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \Rightarrow F = \frac{m(v^2 - u^2)}{2s} = \frac{0.01(100^2 - 800^2)}{2\times1} = -3151 \text{ N}$$

$\Rightarrow$ Resistance offered = 3150 N.

Illustration 14.

In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with a speed of 200 m/s on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Solution:

Initial kinetic energy, $K_i = \frac{1}{2} \times \frac{50}{1000} \times 200 \times 200 = 1000$ J

Final kinetic energy, $K_f = \frac{10}{100} \times 1000 = 100$ J

If v_f emergent speed of the bullet, then

$$\frac{1}{2} \times \frac{50}{1000} \times v_f^2 = 100 \Rightarrow v_f^2 = 4000 \Rightarrow v_f = 63.2 \text{ m/s.}$$

Note that the speed is reduced by approximately 68% and not 90%.

Illustration 15.

A particle of mass m moves with velocity $v = a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from x = 0 to x = d.

Solution:

Work done by all forces = $W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Here $v_1 = a\sqrt{0} = 0$, $v_2 = a\sqrt{d}$, So $W = \frac{1}{2}ma^2d - 0 = \frac{1}{2}ma^2d$.

BEGINNER'S BOX - 2

1. If the linear momentum of a body is increased by 50%, then by what percentage will its kinetic energy increase?

2. The kinetic energy of a body is numerically equal to thrice the momentum of the body. Find the velocity of the body.

3. A body of mass 10 kg is released from the top of a tower which acquires a velocity of 10 m/s after falling through the distance of 20 m. Calculate the work done by the drag force of the air on the body? (take g = m/s^2) **[AIPMT (Mains) 2008]**

4. The displacement x of a body of mass 1 kg on smooth horizontal surface as a function of time t is given by $x = \frac{t^3}{3}$ (where x is in meters and t is in seconds). Find the work done by the external agent for the first on second.

3. CONSERVATIVE, NON-CONSERVATIVE & CENTRAL FORCE

3.1 Conservative force

- A force is said to be conservative if the work done by the force is independent of the path and depends only on initial and final positions.
- It does not depend on the nature of the path followed between the initial and the final positions.

Examples of Conservative force

All central forces are conservative like Gravitational, Electrostatic, Elastic force, Restoring force due to spring, Intermolecular force etc.

3.2 Central Force

A force whose line of action always passes through a fixed point (which is known as centre of force) and whose magnitude depends only on the distance from this point is known as **central force.**

$$\vec{F} = F(r)\hat{r}$$

All forces following inverse square law are called central forces.

$\vec{F} = \frac{k}{r^2}\hat{r}$ is a central force such as Gravitational force and Coulomb force.

- All central forces are conservative forces.
- Central forces are functions of position only.

3.3 Non Conservative Force

A force is said to be non-conservative if work done by the force in moving a body depends upon the path between the initial and final positions.

Work done in a closed path is not zero in a non-conservative force field.

Frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which the body is moved. It does not depend on the initial and final positions. The work done by frictional force in a round trip is not zero.

Example of non-conservative force

All velocity-dependent forces such as air resistance, viscous force are non-conservative forces.

3.4 Difference Between Conservative forces & Non-conservative Forces

Conservative Forces	Non-Conservative Forces
● Work done does not depend upon path.	● Work done depends upon path
● Work done in a round trip is zero e.g. gravitational force	● Work done in a round trip is not zero e.g. friction
● Central forces, spring forces etc, are conservative forces	● Forces which are velocity-dependent in nature e.g. dragging force, viscous force, etc
● when a conservative force acts within a system, the kinetic energy and potential energy can change. However, their sum, the mechanical energy of the system, does not change.	● Work done against a non-conservative force may be dissipated as heat energy.
● Work done is completely recoverable.	● Work done is not completely recoverable.

4. POTENTIAL ENERGY

4.1 Potential Energy

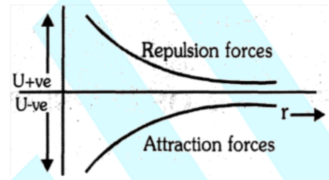
- The energy possessed by a body by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the external work done against the action of conservative force in order to shift it from a certain reference point (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the conservative force in moving the body from its present position to reference position.
- At a certain reference position, the potential energy of the body is assumed to be zero.

- Relationship between conservative force field and potential energy:

$$\vec{F} = \nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

- If force varies with only one dimension (say along x-axis) then $F = \frac{dU}{dx} \Rightarrow dU = -Fdx$

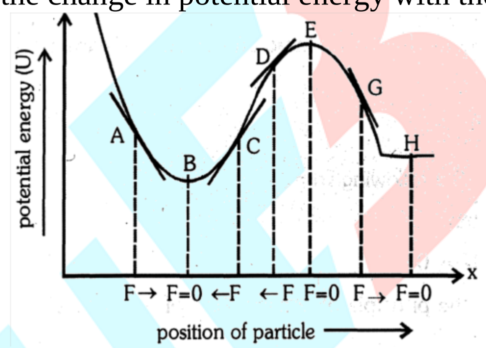
$$\Rightarrow \int_{U_1}^{U_2} dU = -\int_{x_1}^{x_2} Fdx \quad \Rightarrow \quad DU = -W_C$$



- Potential energy may be positive or negative or even zero
  (i) Potential energy is positive, if force field is repulsive in nature
  (ii) Potential energy is negative, if force field is attractive in nature
- If r ↑ (separation between body and force centre), U ↑, force field is attractive or vice-versa.
- If r ↑, U ↓, force field is repulsive in nature.

4.2 Potential energy curve and equilibrium

It is a curve which shows the change in potential energy with the position of a particle,



- **Stable Equilibrium :**
  After a particle is slightly displaced from its equilibrium position if it tends to come back towards equilibrium then it is said to be in stable equilibrium.

  At point **A :** slope $\frac{dU}{dx}$ is negative so F is positive

  At point **C :** slope $\frac{dU}{dx}$ is positive so F is negative At equilibrium $F = \frac{dU}{dx} = 0$

  At point **B :** it is the point of stable equilibrium.

  At point **B :** $U = U_{min}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = positive

- **Unstable equilibrium :**
  After a particle is slightly displaced from its equilibrium position, if it tends to move away from equilibrium position then it is said to be in unstable equilibrium.

  At point **D :** slope $\frac{dU}{dx}$ is positive so F is negative

  At point **G :** slope $\frac{dU}{dx}$ is negative so F is positive

  At point **E :** it is the point of unstable equilibrium

At point **E :** $U = U_{max'} \frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = negative

- **Neutral equilibrium :**

After a particle is slightly displaced from its equilibrium position if no force acts on it then the equilibrium is said to be neutral equilibrium.

Point **H** corresponds to neutral equilibrium. $\Rightarrow U$ = constant ; $\frac{dU}{dx} = 0,\ \frac{d^2U}{dx^2} = 0.$

5. LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor be destroyed, it only can be converted from one form to another.

5.1 Conservation of mechanical energy

The total potential energy of a system and the total kinetic energy of all the constituent bodies together is known as the mechanical energy of the system. If E, K and U respectively denote the total mechanical energy, total kinetic energy, and the total potential energy of a system in any configuration then we have

$$E = K + U$$

Consider a system on which no external force acts and all the internal forces are conservative. If we apply work-kinetic energy ($W_{1\to2} = K_2 - K_1$) theorem, the work $W_{1\to2}$ will be the work done by internal conservative forces, negative of which equals the change in potential energy. Rearranging the kinetic energy and potential energy terms, we have

$$E = K_1 + U_1 = K_2 + U_2$$

The above equation takes the following form

$$E = K + U = \text{constant}$$

$$\Delta E = 0 \Rightarrow AK + \Delta U = 0.$$

Above equations, express the principle of conservation of mechanical energy.

If there is no net work done by any external force or any internal non-conservative force, then the total mechanical energy of a system is conserved.

The principle of conservation of mechanical energy is developed from the work-kinetic energy principle for systems where change in configuration takes place under internal conservative forces only. Therefore, in physical situations, where external forces or non-conservative internal forces are involved, the use of work-kinetic energy principle should be preferred.

In systems, where external forces or internal non-conservative forces do work, the net work done by these forces becomes equal to the change in the mechanical energy of the system.

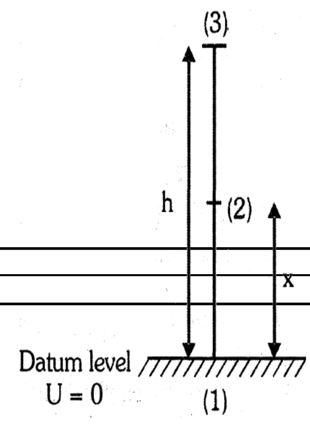
5.2 Application to gravitational field

Let a ball of mass m be dropped from position (3) (as shown in figure)

At point 1

PE = 0; KE = $\frac{1}{2}mv^2$ Θ $v = \sqrt{2gh}$ so KE = mgh

At point 2



$PE = mgx$; $KE = \frac{1}{2}mv^2$ $\because v = \sqrt{2g(h-x)}$ so $KE = mg(h-x)$

At point 3

$PE = mgh$; $KE = 0$

so during the motion of the ball at any position

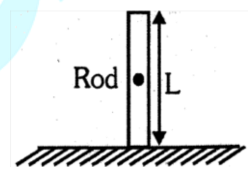
$\boxed{ME_{(1)} = ME_{(2)} = M_{(3)}}$ and $\boxed{PE = mgx}$; $\boxed{KE = mg(h-x)}$

GOLDEN KEY POINTS

- In presence of conservative forces mechanical energy remains conserved.
- Work done along a closed path is zero. i.e. $\oint \vec{F}.d\vec{r} = 0$
- Potential energy depends on the frame of reference but change in potential energy is independent of reference frame.
- Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.
- It is a function of position and does not depend on the path.
- Whenever work is done by the conservative forces the potential energy decreases and whenever work is done against the conservative forces, potential energy increases.
- For regularly shaped uniform bodies, the potential energy change can be calculated by considering their mass to be centered at the geometrical centre point.

  For Example :

  For a uniform vertical rod of length L

  $PE = mg\frac{L}{2}$

Illustrations

Illustration 16.

Force between the atoms of a diatomic molecule has its origin in the interaction between the electrons and the nuclei present in each atom. This force is conservative and associated potential energy U(r) is, to a good approximation, represented by the Lennard - Jones potential function.

$$U(r) = U_o\left\{\left(\frac{a}{r}\right)^{12} - \left(\frac{a}{r}\right)^{6}\right\}$$

Here r is the distance between the two atoms and U_o and a are positive constants. Develop expression for the associated force and find the equilibrium separation between the atoms.

Solution:

Using equation $F = -\frac{dU}{dr}$, we obtain the expression for the force

$$F = \frac{6U_0}{a}\left\{2\left(\frac{a}{r}\right)^{13} - \left(\frac{a}{r}\right)^{7}\right\}$$

At equilibrium, force must be zero. Therefore the equilibrium separation is $r_o = 2^{\frac{1}{6}}a$

Illustration 17.

The potential energy for a conservative force system is given by $U = ax^2 - bx$, where a and b are constants. Find out the (a) expression for force, (b) equilibrium position and (c) potential energy at equilibrium.

Solution:

(a) For conservative force $F = -\frac{dU}{dx} = -(2ax - b) = -2ax + b$

(b) At equilibrium $F = 0 \Rightarrow -2ax + b = 0 \Rightarrow x = \frac{b}{2a}$

(c) $U = a\,a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$.

Illustration 18.

The potential energy of a particle of mass 1 kg free to move along the x-axis is given by $U(x) = \left(\frac{x^2}{2} - x\right)$ joules.

For minimum U, $\frac{dU}{dx} = \frac{2x}{2} - 1 = 0$ and $\frac{d^2U}{dx^2} = 1 =$ positive

so at x = 1, U is minimum. Hence $U_{min} = -\frac{1}{2}$ J.

Total mechanical energy = Max KE + Min PE

$\Rightarrow$ Max KE $= \frac{1}{2}mv_{max}^2 = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2} \Rightarrow v_{max} = \sqrt{\frac{2}{1} \times \frac{5}{2}} = \sqrt{5}\,ms^{-1}$.

Illustration 19.

A body is dropped from height 8 m. After striking the surface it rises to 6 m, what is the fractional loss in kinetic energy during impact? Assuming the frictional resistance to be negligible:

(A) 1/2 (B) 1/4 (C) 1/6 (D) 1/8

Ans. (B)

Solution:

Fractional loss in kinetic energy $= \frac{\text{loss in kinetic energy}}{\text{initial kinetic energy}}$

$= \frac{\text{loss in potential energy}}{\text{initial potential energy}}$

$= \frac{mg(8-6)}{mg(8)} = \frac{2}{8} = \frac{1}{4}$

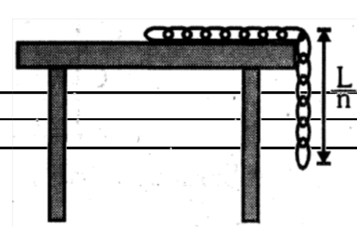
Illustration 20.

A chain of mass m and length L is held on a frictionless table in such a way that $\frac{1}{n}$th part is hanging below the edge of table. Calculate the work. done to pull the hanging part of the chain.

[AIPMT (Mains) 2008]

Solution:

Required work done = change in potential energy of chain



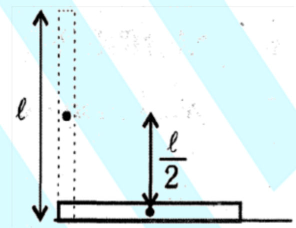
Now, let Potential energy (U) = 0 at table level
so potential energies of chain initially and finally are respectively

$$U_i = -mg\left(\frac{L}{2n}\right) = -\left(\frac{M}{L}\right)\frac{L}{n}g\left(\frac{L}{2n}\right) = -\frac{MgL}{2n^2},\ U_f = 0$$

$\therefore$ required work done = $U_f - U_i = \dfrac{MgL}{2n^2}$

Illustration 21.

A rod of mass 'm' and length 'λ' is placed in a horizontal position. Find the work done by the external force against gravity to alter its configuration from horizontal position to vertical position.



Solution:

Displacement between the positions of center of mass of rod is $\frac{l}{2}$

(in a direction parallel to the force of gravity)

$$W = -\text{change in PE of the rod} = mg\frac{l}{2}.$$

BEGINNER'S BOX - 3

1. **Match the following items -**

(A)	Conservative force	(p)	Mechanical energy = constant
(B)	Non conservative force	(q)	Kinetic energy = constant
		(r)	Work done in a closed path = 0
		(s)	$F = -\frac{dU}{dx}$
		(t)	Potential energy is not defined

2. The kinetic energy of a particle increases continuously with time. Then select correct alternative.

(A) the magnitude of its linear momentum is increasing continuously.

(B) its height above the ground must continuously decrease.

(C) the work done by all forces acting the particle must be positive.

(D) the resultant force on the particle must be parallel to the velocity at all times.

3. A particle moves in one dimensional field with total mechanical energy E. If its potential energy is U(x), then

(A) particle has zero speed where U(x) = 0

(B) particle has zero acceleration where U(x) = E

(C) particle has zero velocity where $\frac{dU(x)}{dx} = 0$

(D) particle has zero acceleration where $\frac{dU(x)}{dx} = 0$

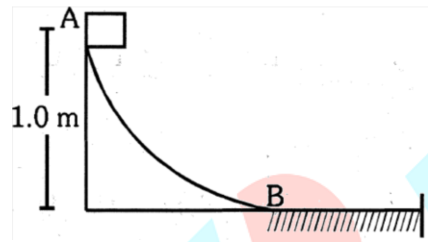
4. Match the following items-

Column-I		Coloumn-II	
A	Work done by all the forces	p	Change in mechanical

			energy
B	Work done by internal conservative forces	q	Change in kinetic energy
C	Work done by external forces	r	Negative of change in potential energy

5. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface. The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, then it would move a distance S on the rough surface.

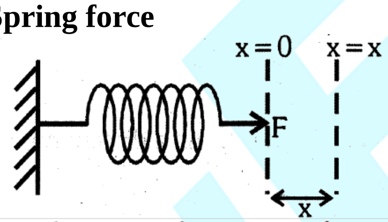
Calculate the value of S [g = 10 m/s^2]



6. SPRING POTENTIAL ENERGY AND SPRING - BLOCK SYSTEM

6.1 Work done by Spring force

$$W = \int_{x_1}^{x_2} F\cos\theta\,dx$$



- Work required to stretch a spring by a length x

  $x_1 = 0$ $\quad x_2 = x$ $\quad$ and $\quad \theta = 0°$

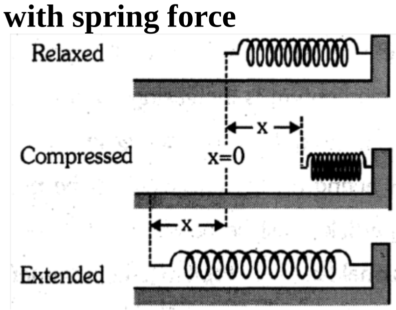
  $W = \int_0^x F\,dx$ $\quad \because F = kx$ $\quad W = \int_0^x kx\,dx$ $\quad \boxed{W = \frac{1}{2}kx^2}$

- Work required to stretch a spring from a length x_1 to x_2:

  Then $x_1 = x_1$ $\quad x_2 = x_2$ $\quad$ and $\quad \theta = 0°$

  $W = \int_{x_1}^{x_2} F\,dx$ $\quad \because F = kx$ $\quad W = \int_{x_1}^{x_2} kx\,dx$ $\quad \boxed{W = \frac{1}{2}k\left(x_2^2 - x_1^2\right)}$

6.2 Potential energy associated with spring force



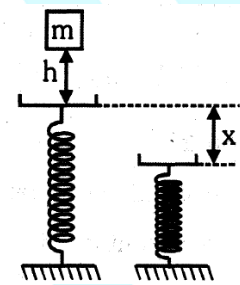
The potential energy of an ideal spring associated with a spring force when compressed or elongated by a distance x from its natural is given by the following expression $U = \frac{1}{2}kx^2$.

GOLDEN KEY POINTS

- A particle of mass m is freely released from a height h on a spring; if the spring gets compressed by x then :

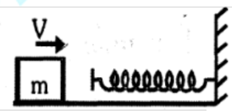
$$mgh + 0 \text{ (Elastic potential energy)} + 0 \text{ (Kinetic energy)} = -mgx + \frac{1}{2}kx^2 + 0 \text{ (Kinetic energy)}$$

$$\boxed{mg(h+x) = \frac{1}{2}kx^2}$$



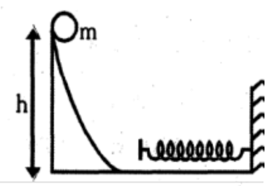
- If a body of mass m moving with speed v collides with a spring. The spring gets compressed by x_{max}, then

$$\frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2 \Rightarrow \boxed{\sqrt{\frac{m}{k}}v = x_{max}}$$



- If a body is released from a height h and collides with a spring.

$$mgh = \frac{1}{2}kx_{max}^2 \Rightarrow \sqrt{\frac{2mgh}{k}} = x_{max}$$



- If a block is attached to the lower end of a spring hanging from a fixed support
  - Equilibrium condition:

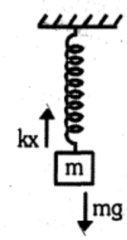
    $F_{net} = 0$

    $$mg - kx = 0 \Rightarrow \boxed{\frac{mg}{k} = x}$$

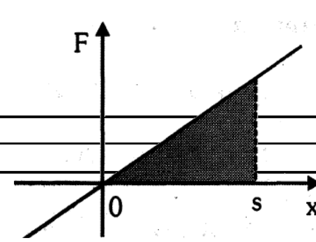
  - **Maximum extension:**

    Decrease in G.P.E. of the block = increase in P.E. of the spring

    $$mgx_{max} = \frac{1}{2}kx_{max}^2 \Rightarrow \boxed{\frac{2mg}{k} = x_{max}}$$



- When a spring is compressed or elongated from its natural length by the some amount x then the W.D. by spring force = $\frac{kx^2}{2}$.
- When spring is elongated or compressed from its natural length by the some amount x the W.D. by external force = $\frac{+kx^2}{2}$.
- The variation in F with extension x in the spring is linear therefore area under the force-extension graph can easily be calculated. This



area equals to the work done by the applied force. The graph showing variation in F with x is shown in the adjoining figure.

$$W_F = \int_0^s F dx = \text{Area of the shaded portion} = \frac{1}{2} k s^2$$

Illustration 22.

A spring of force constant 100 N/m is stretched by 5 cm. Find the work done by applied force.

Solution:

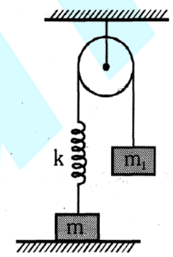
Spring is stretched by a length λ = 5 cm = 0.05 m

Work done for small displacement dx is dW = Fdx = kx dx

Total work done is $W = \int_0^l F dx = \int_0^l kx\, dx = \frac{1}{2} k l^2 = \frac{1}{2} 100\,(.05)^2 = 0.125$ J

Illustration 23.

For what minimum value of m_1 will the block of mass m just leave contact with the surface?



Solution:

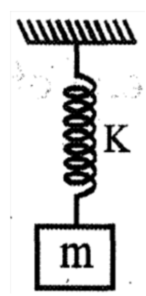
Let extension in the spring be x_0 due to m_1

Then $m_1 g x_0 = \frac{1}{2} k x_0^2 \quad \Rightarrow \quad k x_0 = 2 m_1 g$

but $k x_0 \geq mg$ so $2 m_1 g \geq mg \Rightarrow m_1 \geq \frac{m}{2}$; therefore minimum value of $m_1 = \frac{m}{2}$.

Illustration 24.

A block of mass m is lowered slowly from the natural length position of a massless spring by an external agent to equilibrium position. The extension produced in the spring was asked to two students



Student A : $\frac{1}{2} k x^2 = mgx \qquad \therefore\ x = \frac{2mg}{k}$

Student B : $mg = kx \qquad \therefore\ x = \frac{mg}{k}$

(A) Student A is incorrect, Student B is correct.
(B) Student A is correct, Student B is incorrect.
(C) Both are correct
(D) Both are incorrect

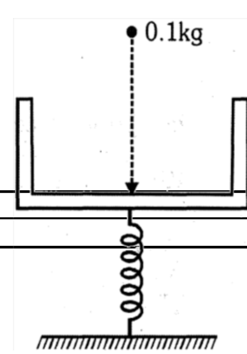
Ans. (A)

Solution:

In equilibrium position, $mg = kx \Rightarrow x = \frac{mg}{k}$ so student A is incorrect.

Illustration 25.

A massless platform is kept on a light elastic spring as shown in Fig. When a sand particle of 0.1 kg mass is dropped on the pan from a height of 0.24 m, the particle strikes the pan (in a perfectly inelastic manner)



and the spring compresses by 0.01 m. From what height should the particle be dropped so as to generate a compression of 0.04 m?

Solution:

$Mg(h + x) = \frac{1}{2}kx^2$ …..(1) & $mg(h' + x') = \frac{1}{2}kx'^2$ ….(2)

Divide (1) by (2)

$$\frac{h+x}{h'+x'} = \frac{x^2}{x'^2} \Rightarrow \frac{0.24+0.01}{h'+0.04} = \left(\frac{0.01}{0.04}\right)^2 \Rightarrow \frac{0.25}{h'+0.04}\frac{1}{16} \Rightarrow h' + 0.04 = 4 \Rightarrow h' = 4 - 0.04 = 3.96 \text{ m.}$$

Illustration 26.

A block strikes the free end of a horizontal spring with the other end fixed, placed on a smooth surface with a speed v. After compressing the spring by x, the speed of the block reduces to half Calculate the maximum compression of the spring.

Solution:

Let maximum compression be x_{max} then by COME

$$\frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}kx^2}{\frac{1}{2}kx_{max}^2} \Rightarrow \frac{3}{4} = \frac{x^2}{x_{max}^2} \Rightarrow x_{max} = \frac{2}{\sqrt{3}}x.$$

Illustration 27.

If in the above question KE of the block reduces to half, then find the out maximum compression of the spring.

Solution:

$$KE - \frac{KE}{2} = \frac{1}{2}kx^2 \Rightarrow \frac{KE}{2} = \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}kx_{max}^2 \Rightarrow kx^2 = \frac{1}{2}kx_{max}^2 \Rightarrow x_{max} = \sqrt{2}x.$$

Illustration 28.

A spring is initially compressed by x and then, it is further compressed by y. Find out the work done during the latter compression. (spring constant is k.)

Solution:

$$W_1 = \frac{1}{2}kx^2; \quad W_2 = \frac{1}{2}k(x+y)^2$$

$$W.D. = W_2 - W_1 = \frac{1}{2}k(x^2 + y^2 + 2xy) - \frac{1}{2}kx^2$$

$$\frac{1}{2}k(y^2 + 2xy) \Rightarrow \frac{1}{2}ky(y + 2x)$$

7. POWER

7.1 Average Power &. Instantaneous Power

- When we purchase a car or jeep we are interested in the horsepower of its engine. We know that, usually an engine with large horsepower is most effective in accelerating the automobile.

  In many cases, it is useful to know not just the total amount of work being done, but also how fast the work is being done. We define power as the rate at which work is being done.

$$\text{Average Power} = \frac{\text{Total Workdone}}{\text{Time taken to do work}} = \frac{\text{Total change in kinetic energy}}{\text{Total change in time}}$$

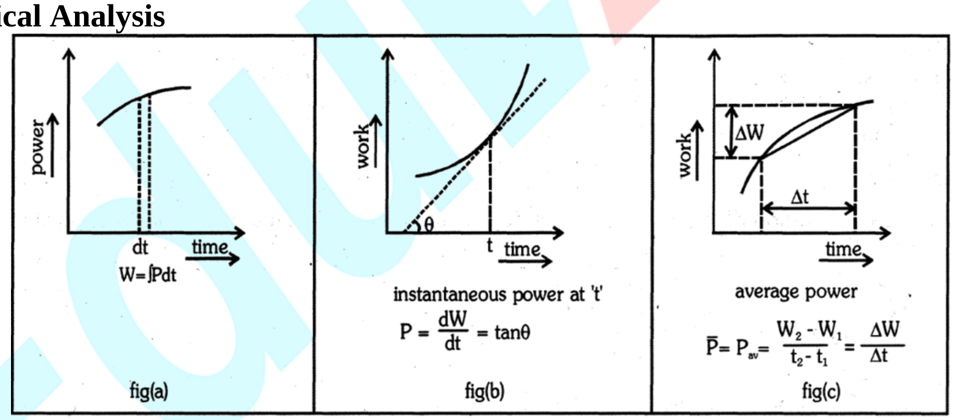
If ΔW is the amount of work done in a time interval Δt, then $P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$

- Instantaneous power is the scalar product of force and velocity at any instant.
  When work is measured in joules and t is in seconds, the unit for power is joules per second, which is called watt. For motors and engines, power is usually measured in horsepower, where 1 hp = 746 W. The definition of power is applicable to all types of energies also like mechanical, electrical, thermal.

  $$\text{Instantaneous power } P = \frac{\Delta W}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$$

  where v is the instantaneous velocity of the particle. Here dot product is used because only that component of force will contribute to power which is acting in the direction of instantaneous velocity.
- For a system of varying mass $F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$ If v = constant then $F = v\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$
- Power is a scalar quantity with dimensions $M^1L^2T^{-3}$
- SI unit of power is J/s or watt.
- l horsepower = 746 watts = 550 ft-lb/s.

7.2 Graphical Analysis



- Area under power-time graph gives the work done. $W = \int P dt$ (See Fig. a)
- The slope of tangent at a point on work-time graph,, gives instantaneous power (See Fig. b)
- The slope of a straight line joining two points on work-time graph gives average power between two points (See Fig. c)

7.3 Efficiency

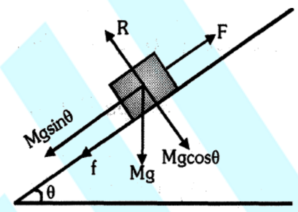
Machines are designed to convert energy into useful work, however because of frictional effects and other dissipative forces work performed by the machine is always less than the energy supplied to the machine.

The efficiency of a machine is given by $\eta = \frac{\text{work done}}{\text{energy input}}$

Illustration 29.

A truck pulls a mass of 1200 kg at a constant speed of 10 m/s on a levelled road. The tension in the coupling is 1000 N. What is the power spent on the mass? Fin the tension when the truck moves up a road of inclination 1 in 6.

Solution:



Force applied by truck f = 1000 N

Power spent in pulling the mass P = fv = 1000 × 10 = 10^4 W

Here sinθ = 1/6, the force required by the truck to move up is

$F = f + Mg\sin\theta \quad \Rightarrow \quad F = 1000\text{ N} + 1200 \times 9.8 \times \frac{1}{6} = 2960\text{ N}$

Illustration. 30

A body of mass m starting from rest from the origin moves along the x-axis with a constant power (P). Calculate the:

(i) relation between velocity and time.

(ii) relation between distance and time.

(iii) relation between velocity and distance.

Solution:

(i) $P = Fv = mav = m\frac{dv}{dt}v \Rightarrow \int_0^t \frac{P}{m}dt = \int_0^v v\,dv \Rightarrow \frac{P}{m}t = \frac{v^2}{2}$

$\Rightarrow \quad v = \sqrt{\frac{2P}{m}t} \quad \Rightarrow \quad \boxed{v \propto t^{1/2}}$ .....(1)

(ii) $\frac{dx}{dt} = \sqrt{\frac{2P}{m}}t^{1/2} \quad \Rightarrow \quad \int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$

$\Rightarrow \quad x = \sqrt{\frac{2P}{m}}\frac{2}{3}t^{3/2} \quad \Rightarrow \quad \boxed{x \propto t^{3/2}}$ ....(2)

(iii) From (1) & (2), $x \propto \left(t^{1/2}\right)^3 \quad \Rightarrow \quad \boxed{x \propto v^3}$ ....(3)

Illustration 31.

A pump is used to deliver water at a certain rate from a given pipe. By what amount should velocity of water, Force on the water and power of motor be increased to obtain n times water from the same pipe in the same time?

Solution:

Amount of water flowing per unit time $\frac{dm}{dt} = Av\rho$

v = velocity of flow, A is area of cross-section, ρ = density of liquid

To get n times water in the same time, $\left(\frac{dm}{dt}\right)' = n\frac{dm}{dt} \Rightarrow A v'\rho = nAv\rho \Rightarrow v' = nv$

$$F = \frac{vdm}{dt} \Rightarrow F' = v'\frac{dm'}{dt} = n^2 v\frac{dm}{dt} = n^2F$$

To gets n times water, force must be increased n^2 times.

$$\Theta\ P = v^2\frac{dm}{dt} \qquad \text{So } \frac{P'}{P} = \frac{v'^2(dm/dt)'}{v^2(dm/dt)} = \frac{n^2v^2 n\,dm/dt}{v^2\,dm/dt} = n^3 \quad \Rightarrow P' = n^3P$$

Thus to get n times water, the power must be increased n^3 times.

Illustration 32.

The force required to tow a boat at constant velocity is proportional to the speed. If a speed of 4.0 km/h requires 7.5 kW, how much power does a speed of 12 km/h require?

Solution:

Let the force be $F = \alpha v$, where v is speed and a is a constant of proportionality the power required is

$$P = Fv = \alpha v^2$$

Let P_1 be the power required for speed v_1 and P_2 be the power required for speed v_2.

$P_1 = 7.5$ kW and $v_2 = 3v_1$, P_2, $= \left(\frac{v_2}{v_1}\right)^2 P_1 \quad \Rightarrow \quad P_2 = (3)^2 \times 7.5$ kW $= 67.5$ kW.

Illustration 33.

A 30000 kg air-plane takes off at a speed of 50 m/s and 5 min. later it is at an elevation of 3 km with a speed of 100 m/s. What average power is required during this, 5 min. if 40 percent of the power is used in overcoming dissipative forces ?

Solution:

Energy supplied to the plane in 300 s (5 min.) is

$$(PE + KE)_{\text{at altitude}} - KE_{\text{ground}} = (30000 \times 9.8 \times 3000) + \frac{1}{2}(30000)\,(100)^2 - \frac{1}{2}(30000)\,(50)^2$$

$$= 882 \times 10^6 + 150 \times 10^6 - 37.5 \times 10^6 = 994.5 \text{ MJ}$$

For Average Power we have $0.6\,P = \frac{994.5\,\text{MJ}}{300\text{s}}$ so, P = 5.525 MW

Illustration 34.

In unloading grains from the hold of a ship, an elevator lifts the grains through a distance of 12m. Grains are discharged at the top of the elevator at a rate of 2.0 kg per second and the discharge speed of each grain particle is 3.0 m/s. Find the minimum horsepower of the motor that can elevate grains in this way.

Solution:

Work done by the motor each second, i.e. power $= mgh + \frac{1}{2}mv^2$

here the mass of grain discharged (and lifted) in one second m = 2.0 kg, v = 3.0 m/s and h = 12m

$\therefore$ power = 249 W = 0.33 H.P.

The motor must have an output of at least 0.33 H.P.

Illustration 35.

A pump can take out 7200 kg of water per hour from a 100m deep well. Calculate the power of the pump, assuming that its efficiency is 50%. ($g = 10\ m/s^2$)

Solution:

Output power = $\frac{mgh}{t} = \frac{7200 \times 10 \times 100}{3600}$ = 2000 W

Efficiency $\eta = \frac{\text{output power}}{\text{input power}}$

output power = $\frac{\text{output power}}{\eta} = \frac{2000 \times 100}{50}$ = 4 kW

Illustration 36.

A truck of mass 10,000 kg moves up an inclined plane rising 1 in 50 with a speed of 36 km/h. Find the power of the engine ($g = 10\ m/s^2$).

Solution:

Force against which work is done $F = mg \sin\theta = 10{,}000 \times 10 \times \frac{1}{50} = 2000$ N

speed $v = \frac{36 \times 5}{18} = 10$ m/s so $p = 2000 \times 10 = 20$ kW.

Illustration 37.

An engine pumps water of density p, through a hose pipe. Water leaves the hose pipe with a velocity v. Find the

(i) rate at which kinetic energy is imparted to water

(ii) power of the engine.

Solution:

(i) Rate of change of kinetic energy = $\frac{dE_k}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}v^2\frac{dm}{dt} = \frac{1}{2}v^2\frac{d}{dt}(\rho Ax)$

$= \frac{1}{2}\rho Av^2\frac{dx}{dt} = \frac{1}{2}\rho Av^3$

(ii) Power = $Fv = \left(v\frac{dm}{dt}\right)v = v^2\frac{dm}{dt} = v^2(\rho Av) = \rho Av^3$

BEGINNER'S BOX - 4

1. Two springs have their respective force constants K_1 and K_2. Both are stretched till their elastic potential energies are equal. If the stretching forces are F_1 and F_2 find $\frac{F_1}{F_2}$.

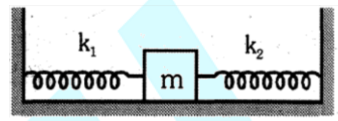
2. A long spring is stretched by 1 cm. If work done in this process is W, then find out the work done in further stretching it by 1 cm.

3. A 20 newtons stone falls accidentally from a height of 4 m on to a spring of stiffness constant 1960 N/m . Write down the equation to find out the maximum compression (x_m) of the spring.

4. A block of mass m moving with speed v compresses a spring through distance x before its speed is halved. What is the value of spring constant?

(A) $\frac{3mv^2}{4x^2}$ (B) $\frac{mv^2}{4x^2}$ (C) $\frac{mv^2}{2x^2}$ (D) $\frac{2mv^2}{x^2}$

5. A block of mass m is attached to two springs of spring constants k_1 and K_2 as shown in figure. The block is displaced by x towards right and released. The velocity of the block when it is at x/2 will be :



(A) $\sqrt{\frac{(k_1+k_2)x^2}{2m}}$ (B) $\sqrt{\frac{3}{4}\frac{(k_1+k_2)x^2}{m}}$ (C) $\sqrt{\frac{(k_1+k_2)x^2}{m}}$ (D) $\sqrt{\frac{(k_1+k_2)x^2}{4m}}$

6. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above. the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

7. An engine pumps up 100 kg of water through a height of 10 m in 5 s. If the efficiency of the engine is 60%, find out the power of the engine (g = 10 m/s^2).

ANSWERS

BEGINNER'S BOX - 1

1. 60º **2.** In second case work has to be done against gravity

3. $\frac{\mu Mgd\cos\theta}{\cos\theta+\mu\sin\theta}$ **4.** $x_2^2 - x_1^2$ **5.** 21 J **6.** 50 J

7. 100 J **8.** Ka^2 **9.** 833 units

10. (a) 75 J (b) –25 J (c) 50 J

BEGINNER'S BOX - 2

1. 125% **2.** 6 units **3.** –1500 J **4.** 0.5 J

BEGINNER'S BOX - 3

1. (A) –p, r, s, (B) –t **2.** (A, C) **3.** D

4. (A) –q, (B) –r, (C) –p **5.** 5 m

BEGINNER'S BOX - 4

1. $\sqrt{\frac{K_1}{K_2}}$ **2.** 3 W **3.** $49x_m^2 - x_m - 4 = 0$ **4.** A

5. B **6.** 43.6 kW **7.** 3.33 kW