

## TEMPERATURE & THERMAL EXPANSION

### 1. TEMPERATURE & TEMPERATURE SCALES

#### • Temperature

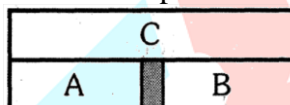
Temperature may be defined as the **degree of hotness or coldness** of a body. Heat energy flows from a body at higher temperature to that at lower temperature until their temperatures become equal. At this stage, the bodies are said to be in thermal equilibrium.

#### • Thermal equilibrium

Thermal equilibrium is a situation in which two objects would not exchange energy by heat or electromagnetic radiation if they were placed in thermal contact. Heat is the transfer of energy from one object to another object as a result of a difference in temperature between them.

#### • Zeroth law of thermodynamics

If objects A and B are separately in thermal equilibrium with a third object C (say thermometer), then objects A and B are in thermal equilibrium with each other. Zeroth law of thermodynamics introduces the concept of temperature. Two objects (or systems) are said to be in thermal equilibrium if their temperatures are same.



In measuring the temperature of a body, it is important that the thermometer should be in thermal equilibrium with the body whose temperature is to be measured.

#### • Measurement of Temperature

The branch of thermodynamics which deals with the measurement of temperature is called thermometry. A thermometer is a device used to measure the temperature of a body. The substances like liquids and gases which are used in the thermometer are called thermometric substances.

#### • Different Scales of Temperature

A thermometer can be graduated into following scales :

- (a) The Centigrade or Celsius scale ( $^{\circ}\text{C}$ )
- (b) The Fahrenheit scale ( $^{\circ}\text{F}$ )
- (c) Kelvin scale (K)

#### • Comparison between Different Temperature Scales

The general formula for the conversion between different temperature scales is:

$$\frac{K - 273}{100} = \frac{C}{100} = \frac{F - 32}{180} = \frac{X - \text{LFP}}{\text{UFP} - \text{LFP}}$$

Where  $X \rightarrow$  Reading in unknown temperature scale,  $\text{LFP} \rightarrow$  Lower Fixed Point,  $\text{UFP} \rightarrow$  Upper Fixed Point

$$\text{Change in temperature} \quad \frac{\Delta K}{100} = \frac{\Delta C}{100} = \frac{\Delta F}{180} = \frac{\Delta X}{\text{UFP} - \text{LFP}}$$

## GOLDEN KEY POINTS

- Although the temperature of a body can be raised without limit, it can not be lowered without limit and theoretically limiting low temperature is taken to be zero of the Kelvin scale (i.e. no negative temperature on Kelvin scale is possible).
- Though when universe was created  $10^{10}$  years ago, its temp. was about  $10^{39}$  K which at present is about 3K. The highest laboratory temperature is about  $10^8$  K (in fusion test reactor) while lowest  $10^{10}$  K (achieved in 1999 through nuclear spin cooling) Theory has established that zero Kelvin temperature can never be achieved practically.

## Illustrations

### Illustration 1.

Temperature of a patient is  $40^{\circ}\text{C}$ . find the temperature on Fahrenheit scale ?

**Solution:**

$$\frac{F-32}{180} = \frac{40-0}{100} \Rightarrow F = 104^{\circ}\text{F}$$

### Illustration 2.

At what temperature is the Fahrenheit scale reading equal to twice of Celsius ?

**Solution:**

$$\frac{F-32}{180} = \frac{C-0}{100} \Rightarrow \frac{2x-32}{180} = \frac{x-0}{100} \Rightarrow x = 160$$

### Illustration 3.

The lower and upper fixed points of a faulty thermometer are 5 W and 105 W. If the thermometer reads 25 W, what is the actual temperature in Celsius' scale?

**Solution:**

$$\frac{25-5}{100} = \frac{C-0}{100} \Rightarrow C = 20^{\circ}\text{C}$$

### Illustration 4.

A thermometer with an arbitrary scale has the ice point at  $-20^{\circ}$  and the steam point at  $180^{\circ}$ . When the thermometer reads  $5^{\circ}$ , a Centigrade thermometer will read

- (1)  $7.5^{\circ}\text{C}$       (2)  $12.5^{\circ}\text{C}$       (3)  $16.5^{\circ}\text{C}$       (4)  $-9.37^{\circ}\text{C}$

**Solution:**

$$\frac{C-0}{100-0} = \frac{t-(-20)}{180-(-20)} \quad (\text{Here } t = 5^{\circ})$$

### Illustration 5.

The temperature of an iron piece is raised from  $30^{\circ}\text{C}$  to  $90^{\circ}\text{C}$ . What is the change in its temperature on the Fahrenheit scale and on the Kelvin scale?

**Solution:**

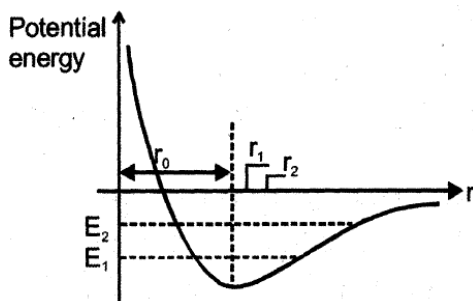
$$\Delta C = 90^{\circ} - 30^{\circ} = 60^{\circ}\text{C}$$

$$\text{Temperature difference on Fahrenheit Scale } \Delta F = \frac{9}{5} \Delta C = \frac{9}{5} (60^{\circ}\text{C}) = 108^{\circ}\text{F}$$

$$\text{Temperature difference on Kelvin Scale } \Delta K = \Delta C = 60\text{K}$$

## 2. THERMAL EXPANSION

When matter is heated without any change in its state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration increases and hence energy of atoms increases, hence the average distance between the atom increases. So the matter as a whole expands.



- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular forces are maximum in solids but minimum in gases.
- Solids can expand in one dimension (Linear expansion), two dimensions (Superficial expansion) and three dimensions (Volumetric expansion) while liquids and gases usually suffers change in volume only.

- **Linear expansion :**

$$\lambda = \lambda_0 (1 + \alpha \Delta\theta) \Rightarrow \Delta\lambda = \lambda_0 \alpha \Delta\theta$$

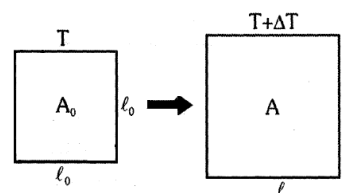


- **Superficial (areal) expansion :**

$$A = A_0 (1 + \beta \Delta\theta)$$

$$\text{Also } A_0 = l_0^2 \text{ and } A = \lambda^2$$

$$\text{So } \lambda^2 = l_0^2 (1 + \beta \Delta\theta) = [\lambda_0 (1 + \alpha \Delta\theta)]^2 \Rightarrow \beta = 2\alpha$$



- **Volumetric expansion :**

$$V = V_0 (1 + \gamma \Delta\theta) \text{ Also } V = \lambda^3 \text{ and } V_0 = l_0^3 \text{ so}$$

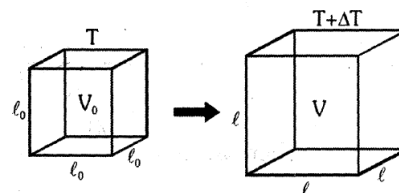
$$\gamma = 3\alpha$$

$$\lambda^3 = [\lambda_0 (1 + \alpha \Delta\theta)]^3 \Rightarrow 6\alpha = 3\beta = 2\gamma \text{ or } \alpha : \beta : \gamma = 1 : 2 : 3$$

$\alpha$  = coefficient of linear expansion

$\beta$  = coefficient of superficial expansion

$\gamma$  = coefficient of volumetric expansion



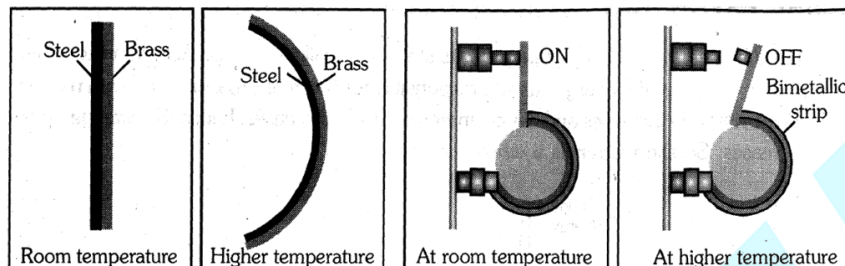
- **Contraction on heating :**

Some rubber like substances contract on heating because transverse vibration of atoms of substance dominate over longitudinal vibration which is responsible for expansion.

### Applications of thermal Expansion in Solids

- (a) **Bi-metallic strip :** When two strips of equal length but of different materials (different coefficient of linear expansion) are joined together, it is called "Bi-metallic strip" and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the

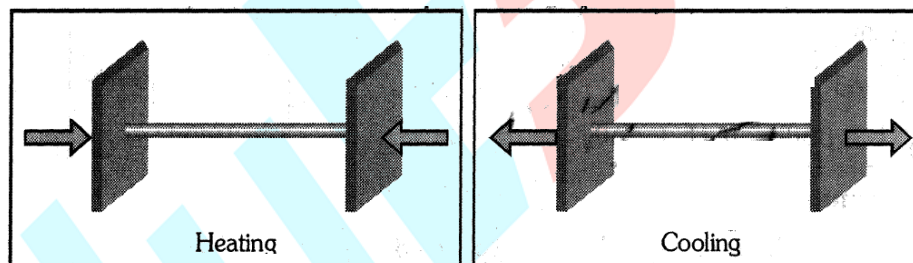
two metals. The strip will bend with metal of greater  $\alpha$  on outer side. Coefficient of expansion is more for brass than steel.



- (b) **Effect of temperature on the time period of a simple pendulum** : A pendulum clock keeps proper time at temperature  $\theta$ . If temperature is increased to  $\theta'$  ( $> \theta$ ) then due to linear expansion, length of pendulum increases and hence its time period will increase.

$$\text{Fractional change in time period } \frac{\Delta T}{T} = \frac{1}{2} \alpha \Delta \theta \quad (\because T \propto \sqrt{l} \therefore \frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta l}{l})$$

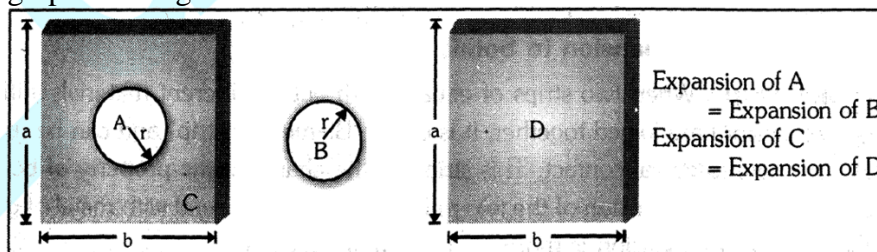
- Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time. Loss of time in a time period  $\Delta T = \frac{1}{2} \alpha \Delta \theta T$ .
- The clock will lose time i.e. will become slow if  $\theta' > \theta$  (in summer) and will gain time i.e. will become fast if  $\theta' < \theta$  (in winter).
- Since coefficient of linear expansion ( $\alpha$ ) is very small for invar, hence pendulums are made of invar to show the correct time in all seasons.



$$\text{Thermal strain} = \frac{\Delta L}{L} = \alpha \Delta \theta \quad \therefore \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta \theta} \quad \text{So thermal stress} = Y \alpha \Delta \theta \quad \therefore Y = \frac{\text{stress}}{\text{strain}}$$

So force on the supports  $F = Y A \alpha \Delta \theta$

- (d) **Expansion of cavity** : Thermal expansion of an isotropic object may be imagined as a photographic enlargement.



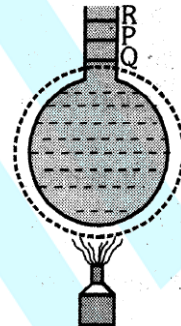
- (e) **Some other applications**

- When rails are laid down on the ground, space is left between the ends of two rails
- The transmission cable are not tightly fixed to the poles
- Test tubes, beakers and cubicles are made up of pyrex-glass or silica because they have very low value of coefficient of linear expansion

- The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel
- A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

### • Thermal Expansion in Liquids

- Liquids do not have linear and superficial expansion but these only have volumetric expansion.
- Since liquids are always to be heated along with a vessel which contains them so initially on heating the system (liquid + vessel), the level of liquid in vessel falls (as vessel expands more since it absorbs heat and liquid expands less) but later on, it starts rising due to faster expansion of the liquid.



QP → represents expansion of vessel

QR → represents the real expansion of liquid.

- The actual increase in the volume of the liquid  
= The apparent increase in the volume of liquid + the increase in the volume of the vessel.
- Liquids have two coefficients of volume expansion.

#### (i) Co-efficient of apparent expansion ( $\gamma_a$ )

It is due to apparent (that appears to be, but not in real) increase in the volume of liquid if expansion of vessel containing the liquid is not taken into account.

$$\gamma_a = \frac{\text{Apparent expansion in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

#### (ii) Co-efficient of real expansion ( $\gamma_r$ )

It is due to the actual increase in volume of liquid due to heating.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

- Also coefficient of expansion of flask  $\gamma_{\text{vessel}} = \frac{(\Delta V)_{\text{vessel}}}{V \times \Delta\theta}$

$$\gamma_{\text{Real}} = \gamma_{\text{Apparent}} + \gamma_{\text{Vessel}}$$

- Change (apparent change) in volume in liquid relative to vessel is

$$\Delta V_{\text{app}} = V(\gamma_{\text{Real}} - \gamma_{\text{Vessel}}) \Delta\theta = V(\gamma_r - 3\alpha)\Delta\theta$$

$\alpha$  = Coefficient of linear expansion of the vessel.

- **Different level of liquid in vessel**

$\gamma$	$\Delta V$	Level
$\gamma_{\text{Real}} > \gamma_{\text{Vessel}} (= 3\alpha) \Rightarrow \gamma_{\text{app}} > 0$	$\Delta V_{\text{app}}$ is positive	Level of liquid in vessel will rise on heating
$\gamma_{\text{Real}} < \gamma_{\text{Vessel}} (= 3\alpha) \Rightarrow \gamma_{\text{app}} < 0$	$\Delta V_{\text{app}}$ is negative	Level of liquid in vessel will fall on heating
$\gamma_{\text{Real}} = \gamma_{\text{Vessel}} (= 3\alpha) \Rightarrow \gamma_{\text{app}} = 0$	$\Delta V_{\text{app}} = 0$	Level of liquid in vessel will remain same

## GOLDEN KEY POINTS



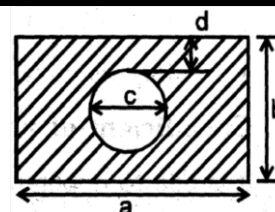
	K	C	F
Water boils	373.15	100	212
body temp.	310.2	37.0	98.6
Room temp.	300	27	80.6
Triple point of water	273.16	0.01	
Water freezes	273.15	0	32
Solid CO <sub>2</sub>	195	-78	-109
Hydrogen boils	20.7	-252.5	-422.5
Absolute zero	0	-273.15	-489.67

- Actually thermal expansion is always 3-D expansion. When other two dimensions of object are negligible with respect to one, then observations are significant only in one dimension and it is known as linear expansion.
- Relation between  $\alpha$ ,  $\beta$  and  $\gamma$ 
  - For isotropic solids:  $\alpha : \beta : \gamma = 1 : 2 : 3$  or  $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$
  - For non-isotropic solids  $\beta = \alpha_1 + \alpha_2$  and  $\gamma = \alpha_1 + \alpha_2 + \alpha_3$ . Here  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are coefficient of linear expansion in X, Y and Z direction.

### Illustrations

#### Illustration 6.

A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.

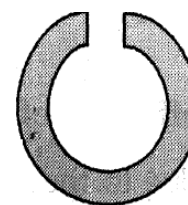


#### Solution:

Distance between any two point on an object increases with increase in temperature. So, all dimension a, b, c and d will increase

#### Illustration 7.

A small ring having small gap is shown in figure on heating what will happen to the size of gap.



#### Solution:

Gap will also increase. The reason is same as in above example.

#### Illustration 8.

What is the percentage change in length of 1 m iron rod if its temperature changes by 100°C.  $\alpha$  for iron is  $2 \times 10^{-5}/^\circ\text{C}$ .

#### Solution:

Percentage change in length due to temperature change

$$\% \Delta l = \frac{\Delta l}{l} \times 100 = \alpha \Delta \theta \times 100 = 2 \times 10^{-5} \times 100 \times 100 = 0.2\%$$

#### Illustration 9.

A concrete slab has a length of 10 m on a winter night when the temperature is  $0^{\circ}\text{C}$ . Find the length of the slab on a summer day when the temperature is  $35^{\circ}\text{C}$ . The coefficient of linear expansion of concrete is  $1.0 \times 10^{-5} / ^{\circ}\text{C}$ .

**Solution:**

$$\lambda_t = 10(1 + 1 \times 10^{-5} \times 35) = 10.0035 \text{ m}$$

**Illustration 10.**

A glass vessel of volume  $100 \text{ cm}^3$  is filled with mercury and is heated from  $25^{\circ}\text{C}$  to  $75^{\circ}\text{C}$ . What volume of mercury will overflow? Coefficient of linear expansion of glass =  $1.8 \times 10^{-6} / ^{\circ}\text{C}$  and coefficient of volume expansion of mercury is  $1.8 \times 10^{-4} / ^{\circ}\text{C}$ .

**Solution:**

$$\Delta V = v_0(\gamma_L - \gamma_L) \Delta T = 100 \times \{1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6}\} \times 50 \Rightarrow \Delta V = 0.87 \text{ cm}^3$$

**Illustration 11.**

There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if they are heated to the same temperature,

**Solution:**

As thermal expansion of isotropic solids is similar to true photographic enlargement, expansion of a cavity is same as if it had been a solid body of the same material

$$\text{i.e. } \Delta V = V \gamma \Delta \theta$$

As here  $V$ ,  $\gamma$  and  $\Delta \theta$  are same for both solid and hollow spheres treated (cavity); so the expansion of both will be equal.



**Illustration 12.**

A steel wire of cross-sectional area  $0.5 \text{ mm}^2$  is held between two fixed supports. If the wire is just taut at  $20^{\circ}\text{C}$ , determine the tension when the temperature falls to  $0^{\circ}\text{C}$ . Coefficient of linear expansion of steel is  $1.2 \times 10^{-5} / ^{\circ}\text{C}$  and its Young's modulus is  $2.0 \times 10^{11} \text{ N/m}^2$ .

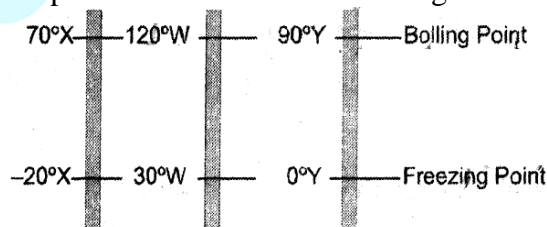
**Solution:**

Here final length is less than the original length so that strain is tensile and tensile force is given by

$$F = AY\alpha\Delta\theta = 0.5 \times 10^{-6} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 20 = 24 \text{ N}$$

### BEGINNER'S BOX - 1

- Write down the following temperatures in the increasing order  $50^{\circ}\text{F}$ ,  $50^{\circ}\text{C}$  and  $50 \text{ K}$ .
- The figure shows three temperature scales with the freezing and boiling points of water indicated.



- Rank the size of a degree on these scales, greatest first.
  - Rank the following temperatures, highest first :  $50^{\circ}\text{X}$ ,  $50^{\circ}\text{W}$  and  $50^{\circ}\text{Y}$ .
- What is the temperature at which we get the same reading on both the Centigrade and Fahrenheit scales?

4. A thin copper wire of length  $L$  increases in length by 1% when heated from temperature  $T_1$  to  $T_2$ . What is the percentage change in area when a thin copper plate having dimensions  $2L \times L$  is heated from  $T_1$  to  $T_2$  ?  
 (A) 1% (B) 2% (C) 3% (D) 4%
5. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at  $27.0^\circ\text{C}$ . What is the change in the diameter of the hole when the sheet is heated to  $227^\circ\text{C}$  ?  
 Coefficient of linear expansion of copper =  $1.70 \times 10^{-5}^\circ\text{C}^{-1}$
6. If a bimetallic strip is heated, it will  
 (A) bend towards the metal with lower thermal expansion coefficient.  
 (B) bend towards the metal with higher thermal expansion coefficient.  
 (C) twist itself into helix.  
 (D) have no bending.

## HEAT

When a hot body is placed in contact with a cold one, the former gets colder and the latter warmer. From this observation it is natural to conclude that a certain quantity of heat has passed from the hot body to the cold one. Heat is a form of energy.

Heat is felt by its effects. Some of the effects of heat are :

- (a) Change in the degree of hotness (b) Change in length, surface area and volume  
 (c) Change in state of a substance (d) Change in the resistance of a conductor  
 (e) Thermo e.m.f. effect

**SI Unit :** J (joule). Also measured in calorie.

- **Calorie**  
 It is defined as the amount of heat required to raise the temperature of 1 g water by  $1^\circ\text{C}$  or 1 K.
  - **International calorie**  
 International calorie is the amount of heat required to raise the temperature of 1 g water from  $14.5^\circ\text{C}$  to  $15.5^\circ\text{C}$  rise of temperature at pressure of 1 atm.
  - **kilo Calorie**  
 kilo calorie is defined as the amount of heat required to raise the temperature of 1 kg water from  $14.5^\circ\text{C}$  to  $15.5^\circ\text{C}$ . (1 kcal = 1000 calorie) at pressure of 1 atm.
  - **British thermal unit (B.T.U.)**  
 It is the amount of heat required to raise the temperature of one pound water by  $1^\circ\text{F}$ . (1 B.T.U. = 252 calorie).
- Mechanical equivalent of heat**  
 According to Joule, work may be converted into heat and vice-versa. The ratio of work done to the heat produced is always constant.  $\frac{W}{H} = \text{constant (J)} \Rightarrow W = J H$
- $W$  must be in joule, irrespective of nature of energy or work and  $H$  must be in calorie.  
 $J$  is called mechanical equivalent of heat. It is not a physical quantity but simply a conversion factor.  
 It converts unit of work into that of heat and vice-versa.  
 $J = 4.186 \text{ joule/cal}$  or  $4.186 \times 10^3 \text{ joule/kcal}$ . For rough calculations we take  $J = 4.2 \text{ joule/cal}$



**Specific heat (s or c)**

The amount of energy required to raise the temperature of unit mass of a substance by  $1^{\circ}\text{C}$  (or  $1\text{K}$ ) is called its specific heat. It is represented by  $s$  or  $c$ .

If the temperature of a substance of mass  $m$  changes from  $T$  to  $T + dT$  when it exchanges an amount of heat  $dQ$  with its surroundings then its specific heat is  $c = \frac{1}{m} \frac{dQ}{dT}$

The specific heat depends on the pressure, volume and temperature of the substance.

For liquids and solids, specific heat measurements are most often made at a constant pressure as functions of temperature, because constant pressure is quite easy to produce experimentally.

**Unit :** joule/kg-K, cal/ g  $^{\circ}\text{C}$

Specific heat of water :  $c_{\text{water}} = 1 \text{ cal/g-}^{\circ}\text{C} = 1 \text{ cal/g-K} = 1 \text{ kcal/kg-K} = 4200 \text{ joule/kg-K}$

When a substance does not undergo a change of state (i.e., liquid remains liquid or solid remains solid), then the amount of heat required to raise the temperature of mass  $m$  of the substance by an amount  $\Delta\theta$  is

$$Q = ms\Delta\theta$$

Variation in specific heat of water is less than 1% over the interval from  $0$  to  $100^{\circ}\text{C}$ . Such a small variation is typical for most solids and liquids, so their specific heats can generally be taken to be constant over fairly large temperature ranges.

- There are many possible processes to give heat to a gas.  
A specific heat can be associated to each such process which depends on the nature of process.
- Value of specific heats of gas can vary from zero (0) to infinity.
- Generally two types of specific heats are defined for a gas -  
(a) Specific heat at constant volume ( $C_v$ )      (b) Specific heat at constant pressure ( $C_p$ )
- These specific heats can be molar or gram.

**Molar heat capacity**

The amount of energy needed to raise the temperature of one mole of a substance by  $1^{\circ}\text{C}$  (or  $1\text{K}$ ) is called molar heat capacity. The molar heat capacity is the product of molecular weight and specific heat i.e.,

$$\text{Molar heat capacity } C = \text{Molecular weight (M)} \times \text{Specific heat(c)} \Rightarrow C = \frac{1}{\mu} \left( \frac{dQ}{dT} \right).$$

If the molecular mass of the substance is  $M$  and the mass of the substance is  $m$  then number of

$$\text{moles of the substance } \mu = \frac{m}{M} \Rightarrow C = \frac{m}{M} \left( \frac{dQ}{dT} \right)$$

**SI Unit :** J/mol-K

**Thermal capacity (Heat capacity)**

The quantity of heat required to raise the temperature of the whole substance through  $1^{\circ}\text{C}$  is called thermal capacity. The thermal capacity of mass  $m$  of the whole substance of specific heat ( $s$ ) =  $ms$

Thermal capacity = mass  $\times$  specific heat

Thermal capacity depends on property of material of the body and mass of the body.

**SI Unit :** cal/°C or cal/K,

**Dimensions:**  $ML^2T^{-2}K^{-1}$ .

### Water equivalent of a body

As the specific heat of water is unity so the thermal capacity of a body (ms) represents its water equivalent also.

- Mass of water having the same thermal capacity as the body is called the water equivalent of the body
- The water equivalent of a body is the amount of water that absorbs or gives out the same amount of heat as is done by the body when heated or cooled through 1°C.  
Water equivalent = mass of body  $\times$  specific heat of material  $\Rightarrow$  (w = ms).

### Latent heat or Hidden heat

When phase of a body changes, change of phase takes place at constant temperature [melting point or boiling point] and heat released or absorbed is  $Q = mL$  where L is latent heat. Heat is absorbed if solid converts into liquid (at melting point) or liquid convert into vapours (at boiling point) and heat is released if liquid converts into solid or vapours convert into liquid,

- **Latent heat of fusion**  
It is the quantity of heat (in kilocalories) required to change 1 kg mass of a substance from solid to liquid state at its melting point. Latent heat of fusion for ice : 80 kcal/kg = 80 cal /g.
- **Latent heat of vaporization**  
The quantity of heat required to change its 1 kg mass from liquid to vapour state at its boiling point.  
Latent heat of vaporisation for water : 536 kcal/kg = 536 cal/g  $\approx$  540 cal/g

### Change of State

- **Melting**  
Conversion of solid into liquid state at constant temperature is known as melting.
- **Boiling**  
Evaporation within the whole mass of the liquid is called boiling. Boiling takes place at a constant temperature known as boiling point. A liquid boils when the saturated vapour pressure on its surface is equal to atmospheric pressure. Boiling point reduces on decreasing pressure.
- **Evaporation**  
Conversion of liquid into vapours at all temperatures is called evaporation. It is a surface phenomenon. Greater the temperature, faster is the evaporation. Smaller the boiling point of liquid, more rapid is the evaporation. Smaller the humidity, more is the evaporation. Evaporation increases on decreasing pressure that is why evaporation is faster in vacuum.

### Heat of evaporation

Heat required to change unit mass of a liquid into vapour at a given temperature is called heat of evaporation at that temperature.

- **Sublimation**

Direct conversion of solid into vapour state is called sublimation.

- **Heat of sublimation**

Heat required to change unit mass of solid directly into vapours at a given temperature is called, heat of sublimation at that temperature.

- Camphor and ammonium chloride sublimes on heating in normal conditions.

- A block of ice sublimes into vapours on the surface of moon because of very-very low pressure on its surface

- **Condensation**

The process of conversion from gaseous or vapour state to liquid state is known as condensation .

These materials again get converted to vapour or gaseous state on heating.

- **Hoar frost**

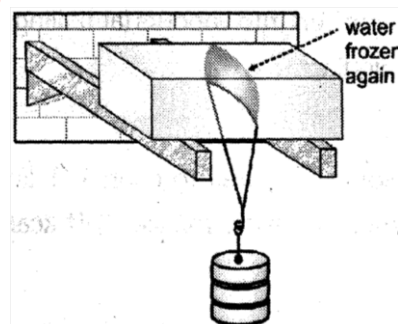
Direct conversion of vapours into solid is called frost, This process is just reverse of the process of sublimation.

**Ex. :** Formation of snow by freezing of clouds.

- **Regelation**

Regelation is the melting of ice caused by pressure and its resolidification when the pressure is removed. Ice shrinks when it melts, and if pressure is applied, deliberately promoting shrinkage, it is found that melting is thereby assisted. In other words, melting of cold ice is ordinarily effected by using the temperature, but if pressure is present to help with the shrinkage the temperature need not be raised so much.

Ice heals up after being cut through by the wire. Melting takes place under the wire because pressure lowers the melting temperature Refreezing (regelation) occurs above the wire when the water escapes to normal pressure again.



Increase of pressure lowers the melting (or freezing) point of water. Conversely, if a substance expands on melting, the melting point is raised by pressure.

### **Phase of a substance**

The phase of a substance is defined as its form which is homogeneous, physically distinct and mechanically separable from the other forms of that substance.

### **Phase diagram**

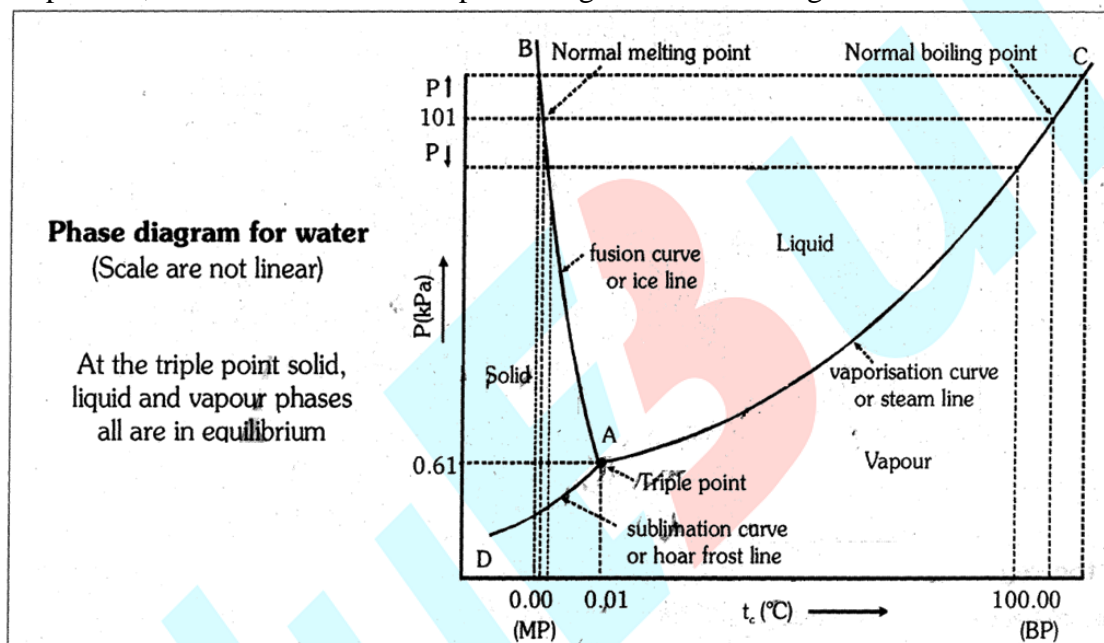
- A phase diagram is a graph in which pressure (P) is represented along the y-axis and temperature (T) is represented along the x-axis.

- **Characteristics of Phase diagram**

- (i) Different phases of a substance can be shown on a phase diagram.
- (ii) A region on the phase diagram represents a single phase of the substance, a curve represents equilibrium between two phases and a common point represents equilibrium between three phases.
- (iii) A phase diagram helps to determine the condition under which the different phases are in equilibrium.
- (iv) A phase diagram is useful for finding a convenient way in which a desired change of phase can be produced.

### Phase diagram for water

The phase diagram for water consists of three curves AB, AC and AD meeting each other at the point A, these curves divide the phase diagram into three regions.



Region to the left of the curve AB and above the curve AD represents the solid phase of water (ice). The region to the right of the curve AB and above the curve AC represents the liquid phase of water. The region below the curves AC and AD represents the gaseous phase of water (i.e. water vapour). A curve on the phase diagram represents the boundary between two phases of the substance.

### Along any curve the two phases can coexist in equilibrium

- Along curve AB, ice and water can remain in equilibrium. This curve is called fusion curve or ice line. This curve shows that the melting point of ice decreases with increase in pressure.
- Along the curve AC, water and water vapour can remain in equilibrium. The curve is called vaporisation curve or steam line. The curve shows that the boiling point of water increases with increase in pressure.
- Along the curve AD, ice and water vapour can remain in equilibrium. This curve is called sublimation curve or hoar frost line.

### Triple point of water

The three curves in the phase diagram of water meet at a single point A, which is called the triple point of water. The triple point of water represents the co-existence of all the three phases of water ice water and water vapour in equilibrium. The pressure corresponding to triple point of water is  $6.03 \times 10^{-3}$  atmosphere or 4.58 mm of Hg and temperature corresponding to it is 273.16K.

- **Significance of triple point of water**

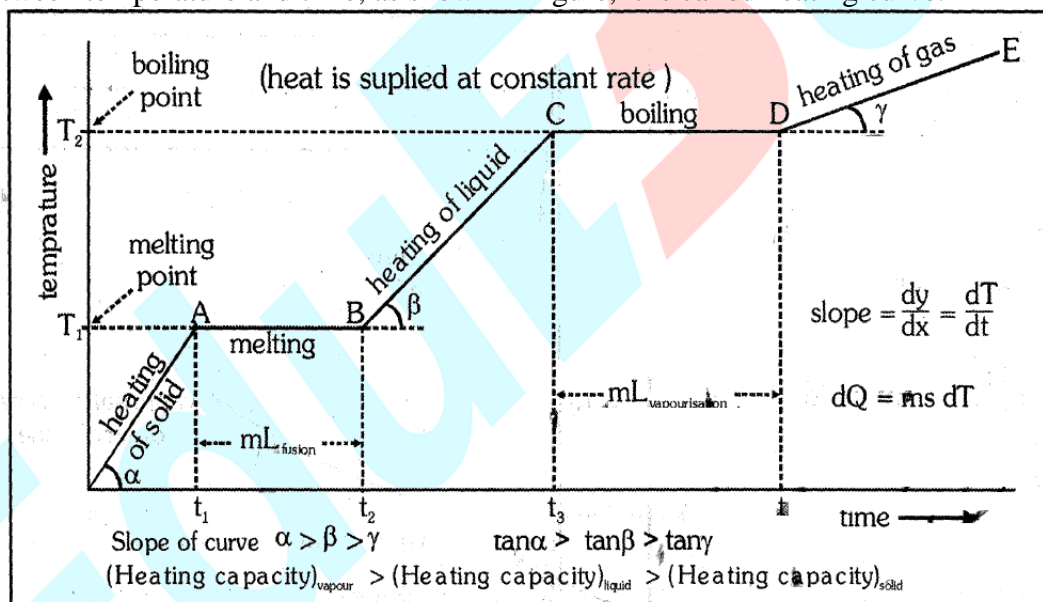
Triple point of water represents a unique condition and it is used to define the absolute temperature. While making Kelvin's absolute scale, upper fixed point is 273.16 K and lower fixed point is 0 K. One kelvin of temperature is  $\frac{1}{273.16}$  part of the temperature of triple point of water.

**Effect of change in pressure on M.P. and B.P. for water**

- If  $P \uparrow \xrightarrow{\text{then result}} \text{M.P.} \downarrow \text{ \& B.P.} \uparrow$
- If  $P \uparrow \xrightarrow{\text{then result}} \text{M.P.} \uparrow \text{ \& B.P.} \downarrow$

**Example :**

If to a given mass (m) of a solid, heat is supplied at constant rate and a graph is plotted between temperature and time, as shown in figure, it is called heating curve.



- **In the region OA**

Rate of heat supply P is constant and temperature of solid is changing with time.

$$\text{So, } Q = mc_s \Delta T \Rightarrow P \Delta t = mc_s \Delta T \quad [\Theta Q = P \Delta t]$$

$$\Theta \frac{\Delta T}{\Delta t} = \text{The slope of temperature-time curve so specific heat of solid } c_s \propto \frac{1}{\text{slope of line OA}}$$

specific heat (or thermal capacity) is inversely proportional to the slope of temperature-time curve.

- **In the region AB**

Temperature is constant, so it represents change of state, i.e., melting of solid at melting point  $T_1$ . At point A melting starts and at point B all solid is converted into liquid. So



between A and B substance is partly solid and partly liquid. If  $L_F$  is the latent heat of fusion then

$$Q = mL_F \Rightarrow L_F = \frac{P(t_2 - t_1)}{m} \quad [\text{as } Q = P(t_2 - t_1)] \Rightarrow L_F \propto \text{length of line AB}$$

i.e., Latent heat of fusion is proportional to the length, of line of zero slope.

$$[\text{In this region specific heat} \propto \frac{1}{\tan 0^\circ} = \infty]$$

- **In the region BC**

Temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC,  $c_L \propto \frac{1}{\text{slope of line BC}}$

- **In the region CD**

Temperature is constant, so it represents change of state, i.e., liquid is boiling at boiling point  $T_2$ . At C all substance is in liquid state while at D is vapour state and between C and D partly liquid and partly gas. The length of line CD is proportional to latent heat of vaporisation, i.e.,  $L_V \propto \text{Length of line CD}$ .

$$[\text{In this region specific heat} \propto \frac{1}{\tan 0^\circ} = \infty]$$

- **In the region DE**

The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

### **Law of Mixtures : Principle of calorimetry**

- When two bodies at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

**Heat lost = Heat gained** (Principle of calorimetry)

Principle of calorimetry represents the law of conservation of heat energy.

- Temperature of mixture (T) is always  $\geq$  lower temperature ( $T_L$ ) and  $\leq$  higher temperature ( $T_H$ ),  $T_L \leq T \leq T_H$

The temperature of mixture can never be lesser than lower temperature (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Further more usually, rise in temperature of one body may not be equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

## Illustrations

### **Illustration 13.**

5 g ice at  $0^\circ\text{C}$  is mixed with 5 g of steam at  $100^\circ\text{C}$ . What is the final temperature?

### **Solution:**

Heat required by ice to raise its temperature to  $100^\circ\text{C}$ ,

$$Q_1 = m_1 L_1 + m_1 c_1 \Delta \theta_1 = 5 \times 80 + 5 \times 1 \times 100 = 400 + 500 = 900 \text{ cal}$$

$$\text{Heat given by steam when condensed } Q_2 = m_2 L_2 = 5 \times 536 = 2680 \text{ cal}$$

As  $Q_2 > Q_1$ . This means that whole steam is not even condensed.

Hence temperature of mixture will remain at  $100^\circ\text{C}$ .

#### Illustration 14.

A calorimeter of heat capacity  $100 \text{ J/K}$  is at room temperature of  $30^\circ\text{C}$ .  $100 \text{ g}$  of water  $40^\circ\text{C}$  of specific heat  $4200 \text{ J/kg-K}$  is poured into the calorimeter. What is the temperature of water in calorimeter ?

#### Solution:

Let the temperature of water in calorimeter is  $t$ . Then heat lost by water = heat gained by calorimeter

$$(0.1) \times 4200 \times (40 - t) = 100 (t - 30) \Rightarrow 420 \times 40 - 420 t = 100 t - 3000 \Rightarrow t = 38.07^\circ\text{C}$$

#### Illustration 15.

Find the quantity of heat required to convert  $40 \text{ g}$  of ice at  $-20^\circ\text{C}$  into water at  $20^\circ\text{C}$ .

Given  $L_f = 0.336 \times 10^6 \text{ J/kg}$ . Specific heat of ice =  $2100 \text{ J/kg-K}$ , specific heat of water =  $4200 \text{ J/kg-K}$

#### Solution:

Heat required to raise the temperature of ice from  $-20^\circ\text{C}$  to  $0^\circ\text{C} = 0.04 \times 2100 \times 20 = 1680 \text{ J}$

Heat required to convert the ice into water at  $0^\circ\text{C} = mL = 0.04 \times 0.336 \times 10^6 = 13440 \text{ J}$

Heat required to heat water from  $0^\circ\text{C}$  to  $20^\circ\text{C} = 0.04 \times 4200 \times 20 = 3360 \text{ J}$

Total heat required =  $1680 + 13440 + 3360 = 18480 \text{ J}$

#### Illustration 16.

Steam at  $100^\circ\text{C}$  is passed into  $1.1 \text{ kg}$  of water contained in a calorimeter of water equivalent  $0.02 \text{ kg}$  at  $15^\circ\text{C}$  till the temperature of the calorimeter and its contents rises to  $80^\circ\text{C}$ . What is the mass of steam condensed?

Latent heat of steam =  $536 \text{ cal/g}$ .

#### Solution:

Heat required by (calorimeter + water)

$$Q = (m_1 c_1 + m_2 c_2) \Delta \theta = (0.02 + 1.1 \times 1) (80 - 15) = 72.8 \text{ kcal}$$

If  $m$  is mass of steam condensed, then heat given by steam

$$Q = mL + mc \Delta \theta = m \times 536 + m \times 1 \times (100 - 80) = 556 m \quad \therefore 556 m = 72.8$$

$$\therefore \text{Mass of steam condensed } m = \frac{72.8}{556} = 0.130 \text{ kg}$$

#### Illustration 17.

An iron block of mass  $2 \text{ kg}$ , fall from a height  $10 \text{ m}$ . After colliding with the ground it loses  $25\%$  energy to surroundings. Then find the temperature rise of the block.

(Take specific heat of iron.  $470 \text{ J/kg } ^\circ\text{C}$ )

#### Solution:

$$mS\Delta\theta = \frac{3}{4} mgh \quad \Rightarrow \quad \Delta\theta = \frac{3 \times 10 \times 10}{4 \times 470} = 159^\circ\text{C}$$

**Illustration 18.**

The temperature of equal masses of three different liquids A, B, and C are  $10^{\circ}\text{C}$ ,  $15^{\circ}\text{C}$  and  $20^{\circ}\text{C}$  respectively. The temperature when A and B are mixed is  $13^{\circ}\text{C}$  and when B and C are mixed, it is  $16^{\circ}\text{C}$ . What will be the temperature when A and C are mixed?

**Solution:**

when A and B are mixed

$$mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13) \quad \dots\dots(1)$$

$$3S_1 = 2S_2$$

when B and C are mixed

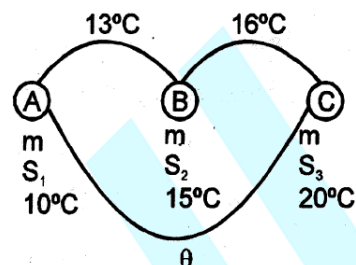
$$S_2 \times 1 = S_3 \times 4 \quad \dots\dots(2)$$

when C and A are mixed

$$S_1(\theta - 10) = S_3 \times (20 - \theta) \quad \dots\dots(3)$$

by using equation (1), (2) and (3)

$$\text{we get } \theta = \frac{140}{11}^{\circ}\text{C}$$

**Illustration 19.**

5 kg of steam at  $100^{\circ}\text{C}$  is mixed with 10 kg of ice at  $0^{\circ}\text{C}$ . Choose incorrect alternative

(Give  $s_{\text{water}} = 1 \text{ cal/g}^{\circ}\text{C}$ ,  $L_F = 80 \text{ cal/g}$ ,  $L_V = 540 \text{ cal/g}$ )

(A) Equilibrium temperature of mixture is  $160^{\circ}\text{C}$

(B) Equilibrium temperature of mixture is  $100^{\circ}\text{C}$

(C) At equilibrium, mixture contains  $13\frac{1}{3}$  kg of water

(D) at equilibrium, mixture contains  $1\frac{2}{3}$  kg of steam

**Ans. (A)**

**Solution:**

Required heat

10 kg ice ( $0^{\circ}\text{C}$ )

↓ 800 kcal

10 kg water ( $0^{\circ}\text{C}$ )

↓ 1000 kcal

10 kg water ( $100^{\circ}\text{C}$ )

Available heat

5 kg steam ( $100^{\circ}\text{C}$ )

↓ 2700 Kcal

5 kg water ( $100^{\circ}\text{C}$ )

So available heat is more than required heat therefore final temperature will be  $100^{\circ}\text{C}$ .

$$\text{Mass of heat condensed} = \frac{800 + 1000}{540} = \frac{10}{3} \text{ kg. Total mass of water} = 10 + \frac{10}{3} = \frac{40}{3} = 13\frac{1}{3} \text{ kg}$$

$$\text{Total mass of steam} = 5 - \frac{10}{3} = \frac{5}{3} = 1\frac{2}{3} \text{ kg}$$

**BEGINNER'S BOX - 2**

1. A bullet of mass 10 g is moving with speed 400 m/s. Find its kinetic energy in calories ?
2. Calculate amount of heat required to convert 1 kg steam from  $100^{\circ}\text{C}$  to  $200^{\circ}\text{C}$  steam ?

3. Calculate heat required to raise the temperature of 1 g of water by  $1^{\circ}\text{C}$  ?
4. 420 J of energy supplied to 10 g of water will raise its temperature by ?
5. The ratio of the densities of the two bodies is 3 : 4 and the ratio of specific heats is 4 : 3. Find the ratio of their thermal capacities for unit volume ?
6. Heat releases by 1 kg steam at  $150^{\circ}\text{C}$  if it is converted into 1 kg water at  $50^{\circ}\text{C}$ .
7. 200 g water is filled in a calorimetry of negligible heat capacity. It is heated till its temperature is increase by  $20^{\circ}\text{C}$ . Find the heat supplied to the water.
8. A bullet of mass 5 gm is moving with speed 400 m/s strike a target. Then calculate rise of temperature of bullet. Assuming all the lose in kinetic energy is converted into heat energy of bullet is its specific heat is  $500 \text{ J/kg}^{\circ}\text{C}$ .
9. 1 kg ice at  $-10^{\circ}\text{C}$  is mixed with 1 kg water at  $100^{\circ}\text{C}$ . Then find equilibrium temperature and mixture content.
10. 540 g of ice at  $0^{\circ}\text{C}$  is mixed with 540 g of water at  $80^{\circ}\text{C}$ . The final temperature of the mixture is (Given latent heat of fusion of ice =  $80 \text{ cal/g}$  and specific heat capacity of water =  $1 \text{ cal/g}^{\circ}\text{C}$ )  
 (A)  $0^{\circ}\text{C}$                       (B)  $40^{\circ}\text{C}$                       (C)  $80^{\circ}\text{C}$                       (D) less than  $0^{\circ}\text{C}$

## MODES OF HEAT TRANSFER

Heat is a form of energy which transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any one of the following modes :

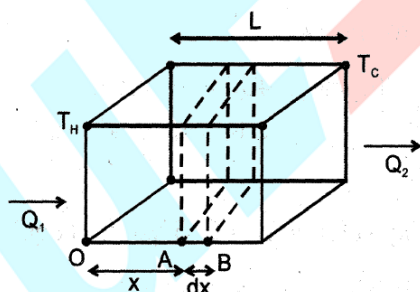
- **Conduction**  
The process in which the material takes an active part by molecular action and energy is passed from one particle to another is called conduction. It is predominant in solids.
- **Convection**  
The transfer of energy by actual motion of particle of medium from one place to another is called convection. It is predominant in fluids (liquids and gases).
- **Radiation**  
Quickest way of transmission of heat is known as radiation. In this mode of energy transmission, heat is transferred from one place to another without effecting the inter-venning medium.

Conduction	Convection	Radiation
Heat Transfer due to Temperature difference	Heat transfer due to density difference	Heat transfer without any medium
Due to free electron or vibration motion of molecules	Actual motion of particles	Electromagnetic radiation
Heat transfer in solids and Hg	Heat transfer in fluids (Liquid + gas)	No medium required
Slow process	Slow process	Fast process ( $3 \times 10^8$ m/sec)
Irregular path	Irregular path	Straight line (like light)

### 3.1 Thermal conduction

The process by which heat is transferred from hot part to cold part of body through the transfer of energy from one particle to another particle of the body without the actual movement of the particles from their equilibrium position is called conduction. The process of conduction takes place only in solid body (except Hg). Heat transfer by conduction from one part of body to another continues till their temperatures become equal.

For example if you hold an iron rod with one of its end on a fire for some time the handle will become hot. The heat is transferred from the fire to the handle by conduction along the length of iron rod. The vibrational amplitude of atoms and electrons of the iron rod at the hot end takes relatively higher value due to the higher temperature of their environment. These increased vibrational amplitude are transferred along the rod, from atom to atom during collision between adjacent atoms. In this way a region of rising temperature extends itself along the rod to your hand.



Consider a slab of face area  $A$ , Lateral thickness  $L$ , whose faces have temperatures  $T_H$  and  $T_C$  ( $T_H > T_C$ ). Now consider two cross sections in the slab at positions  $A$  and  $B$  separated by a lateral distance of  $dx$ .

Let temperature of face  $A$  be  $T$  and that of face  $B$  be  $T + \Delta T$ . Then experiments show that  $Q$ , the amount of heat crossing the area  $A$  of the slab at position  $x$  in time  $t$  is given by

$$\frac{Q}{t} = -Ka \frac{dT}{dx} \quad \text{or} \quad \frac{Q}{t} = \frac{KA(T_H - T_C)}{L}$$

$K \rightarrow$  Thermal conductivity

$\rightarrow$  It is the measure of the ability of material to conduct the heat.

Here  $K$  is a constant depending on nature of the material of the slab and is named thermal conductivity of the material, and the quantity  $\left(\frac{dT}{dx}\right)$  is called temperature gradient. The  $(-)$  sign shows heat flows from high temperature to low temperature ( $\Delta T$  is a  $-ve$  quantity).

#### Steady State



If the temperature of a cross-section at any position  $x$  in the above slab remains constant with time (remember, it does vary with position  $x$ ), the slab is said to be in steady state and temperature of rod is not same.

Remember steady state is distinct from thermal equilibrium for which temperature at any position ( $x$ ) in the slab must be same.

For a conductor in steady state there is no absorption or emission of heat at any cross-section. (as temperature at each point remains constant with time). The left and right faces are maintained at constant temperatures  $T_H$  and  $T_C$  respectively and all other faces must be covered with adiabatic walls so that no heat escapes through them and same amount of heat flows through each cross-section in a given interval of time.

Hence  $Q_1 = Q = Q_2$ . Consequently the temperature gradient is constant throughout the slab.

Hence, 
$$\frac{dT}{dx} = \frac{\Delta T}{L} = \frac{T_f - T_i}{L} = \frac{T_C - T_H}{L}$$

and 
$$\frac{Q}{t} = -KA \frac{\Delta T}{L} \Rightarrow \frac{Q}{t} = KA \left( \frac{T_H - T_C}{L} \right)$$

Here  $Q$  is the amount of heat flowing through a cross-section of slab at any position in a time interval  $t$ .

### Thermal conductivity (K) :

- It depends on nature of material.
- Order of thermal conductivity  $\boxed{Ag > Cu > Au > Al}$ 
  - For Ag maximum is (410 W/mK)
  - For Freon minimum is 12 (0.008 W/mK)
- **SI Unit :**  $J s^{-1} m^{-1} K^{-1}$       **Dimensions :**  $M^1 L^{-3} \theta^{-1}$
- For an ideal or perfect conductor of heat the value of  $K = \infty$
- For an ideal or perfect bad conductor or insulator the value of  $K = 0$
- For cooking the food, low specific heat and high conductivity utensils are most suitable.

### Application of Thermal Conduction

- In winter, the iron chairs appear to be colder than the wooden chairs.
- Cooking utensils are made of aluminium and brass whereas their handles are made of wood.
- Ice is covered in gunny bags to prevent melting of ice .
- We feel warm in woollen clothes and fur coat.
- Two thin blankets are warmer than a single blanket of double the thickness.
- Birds often swell their feathers in winter .
- A new quilt is warmer than old one .

### Thermal Resistance to conduction

If you are interested in insulating your house from cold weather or for that matter keeping the meal hot in your tiffin-box, you are more interested in poor heat conductors, rather than good conductors. For this reason, the concept of thermal resistance  $R$  has been introduced.

For a slab of cross-section A, Lateral thickness L and thermal conductivity K,  $R = \frac{L}{KA}$

In terms of R, the amount of heat flowing through a slab in steady-state (in time t) =  $\frac{Q}{t} = \frac{(T_H - T_L)}{R}$

If we name  $\frac{Q}{t}$  as thermal current  $i_T$  then,  $i_T = \frac{T_H - T_L}{R}$

This is mathematically equivalent to OHM's law, with temperature doing the role of electric potential. Hence results derived from OHM's law are also valid for thermal conduction.

More over, for a slab in steady state we have seen earlier that the thermal current  $i_L$  remains same at each cross-section. This is analogous to Kirchhoff's current law in electricity, which can be very conveniently applied to thermal conduction.

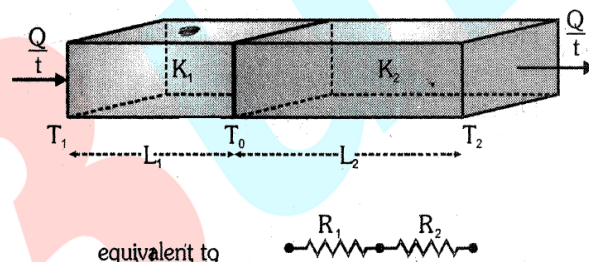
- Equivalent conductivity for Heat flow through slabs in series**

$$R_{eq} = R_1 + R_2$$

$$\frac{L_1 + L_2}{K_{eq} A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}$$

Equivalent thermal conductivity of the system is

$$K_{eq} = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}} = \frac{\sum L_i}{\sum \frac{L_i}{K_i}}$$



- Equivalent thermal conductivity for Heat flow through slabs in parallel**

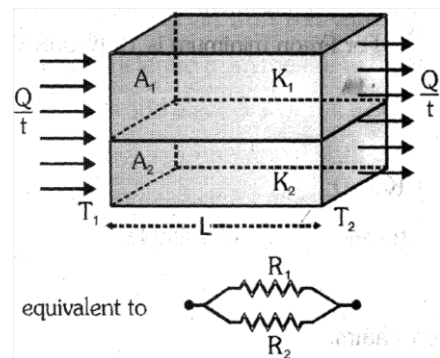
$$R = \frac{L}{KA}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K_{eq}}{L} (A_1 + A_2) = \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L}$$

Equivalent thermal conductivity

$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2} = \frac{\sum K_i A_i}{\sum A_i}$$



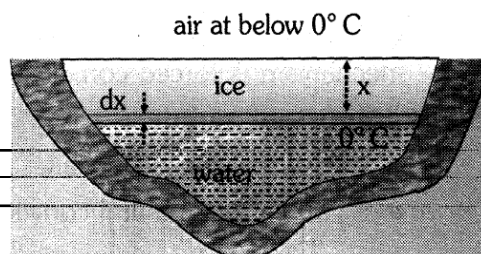
### Growth of Ice on Lakes

In winter atmospheric temperature falls below 0°C and water in the lake start freezing. Let at time t thickness of ice on the surface of the lake = x and air temperature = -θ°C

The temperature of water in contact with the lower surface of ice = 0°C

Let area of the lake = A

Heat escaping through ice in time dt is



$$dQ = KA \frac{[0 - (-\theta)]}{x} dt$$

Due to escape of this heat increasing extra thickness of ice =  $dx$

Mass of this extra thickness of ice is  $m = \rho V = \rho A \cdot dx$

$$dQ = mL = (\rho A \cdot dx) L$$

$$\therefore KA \frac{\theta}{x} = (\rho A \cdot dx) L \Rightarrow dt = \frac{\rho L}{K\theta} x \, dx$$

So time taken by ice to grow a thickness  $x$  is  $t = \frac{\rho L}{K\theta} \int_0^x x \, dx = \frac{1}{2} \frac{\rho L}{K\theta} x^2$

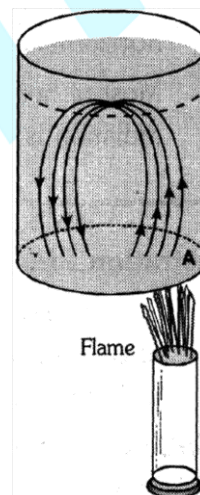
So time taken by ice to grow from thickness  $x_1$  to thickness  $x_2$  is

$$t = t_2 - t_1 = \frac{1}{2} \frac{\rho L}{K\theta} (x_2^2 - x_1^2) \quad \text{and} \quad t \propto (x_2^2 - x_1^2)$$

Time taken to double and triple the thickness ratio  $t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2$  So  $t_1 : t_2 : t_3 :: 1 : 4 : 9$

### 3.2 Convection

Convection requires a medium and is the process in which heat is transferred from one place to other by actual movement of heated substance (usually fluid). The type of convection which results from difference in densities is called natural convection (for example, a fluid in a container heated through its bottom). However, if a heated fluid is forced to move by a blower, fan or pump, the process is called forced convection.



#### Phenomena Based on convection :

##### (i) Land and sea breezes :

The heat from the Sun is absorbed more rapidly by land than by sea-water. Moreover, the specific heat of land is low as compared to that of sea-water. Consequently, the rise in temperature of land is higher as compared to that of sea-water. To sum-up, land is hotter than the sea during day time. As a result of this the colder air over the sea blows towards the land. This is called sea-breeze.

At night, air blows from land towards sea. This is called land breeze.

##### (ii) Formation of trade winds :

The surface of Earth near the equator gets heated strongly. So, the air in contact with the surface of Earth at the equator expands and rises upwards. As a result of this a low pressure is created at the equator.

At the poles, the air in the upper atmosphere gets cooled and comes down. So, a high pressure is created at the poles. Due to difference of pressures at the poles and equator, the air at the poles moves towards the equator, rises up, moves towards poles and so on. In this way, a wind is formed in the atmosphere. The rotation of the Earth also affects the motion of the wind. Due to anti-clockwise rotation of Earth the warm wind blowing from equator to north drifts towards east. The steady wind blowing from north-East to equator, near the surface of Earth, is called trade wind.

##### (iii) Monsoons:

In summer, the peninsular mass of central Asia becomes more strongly heated than the water of the Indian Ocean. This is due to the fact that the specific heat of water is much higher than that of the soil and rocks. Hot air from the heated land mass rises up and moves towards the

Indian ocean. Air filled with moisture flows over the Indian ocean on the south towards heated land mass. When obstructed by mountains, the moist air rushes upwards to great height. In the process, it gets cooled. Consequently, the moisture condenses and falls as rain.

**(iv) Ventillation :**

Ventillators or exhaust fans in a room help to remove impure and warm air from a room. The fresh air from outside blows into the room. This is all due to the forced convection current set up in the room.

**(v) To regulate temperature in the human body :**

Heat transfer in the human body involves a combination of mechanisms. These together maintain a remarkably uniform temperature in the human body in spite of large changes in environmental conditions. The chief internal mechanism is forced convection. The heart serves as the pump and the blood as the circulating fluid.

**Some important points :**

- Natural convection takes place from bottom to top due to gravity while forced convection in any direction.
- In case of natural convection, convection currents move warm air upwards and cool air downwards. This is why heating is done from base, while cooling from the top.
- Natural convection is not possible in a gravity free region such as a freely falling lift or an orbiting satellite.
- Natural convection plays an important role in ventilation, in changing climate and weather and in forming land and sea breezes and trade winds.
- The forced convection of blood in our body by a pump (heart) helps in keeping the temperature of body constant.
- Convection takes place in fluids (gases and liquid)

**GOLDEN KEY POINTS**

- For heat propagation via natural convection, temperature gradient exists in vertical direction and not in horizontal direction.
- Most of heat transfer that is taking place on Earth is by convection, the contribution due to conduction and radiation is very small.

**Illustrations**

**Illustration 20.**

One face of an aluminium cube of edge 2 meter is maintained at  $100^{\circ}\text{C}$  and the other end is maintained at  $0^{\circ}\text{C}$ . All other surfaces are covered by adiabatic walls. Find the amount of heat flowing through the cube in 5 seconds. (thermal conductivity of aluminium is  $209 \text{ W/m}^{\circ}\text{C}$ )

**Solution:**

Heat will flow from the end at  $100^{\circ}\text{C}$  to the end at  $0^{\circ}\text{C}$ .

Area of cross-section perpendicular to direction of heat flow,  $A = 4\text{m}^2$  then  $\frac{Q}{t} = KA \frac{(T_H - T_C)}{L}$

$$Q = \frac{(209 \text{ W/m}^{\circ}\text{C})(4\text{m}^2)(100^{\circ}\text{C} - 0^{\circ}\text{C})(5\text{sec})}{2\text{m}} = 209 \text{ kJ}$$

**Illustration 21.**

Three identical rods of length 1m each, having cross-section area of 1 cm<sup>2</sup> each and made of Aluminium, copper and steel respectively are maintained at temperatures of 12°C, 4°C and 50°C respectively at their separate ends. Find the temperature of their common junction.

[  $K_{Cu} = 400 \text{ W/m-K}$ ,  $K_{Al} = 200 \text{ W/m-K}$ ,  $K_{steel} = 50 \text{ W/m-K}$  ]

**Solution:**

$$R_{Al} = \frac{L}{KA} = \frac{1}{200 \times 10^{-4}} = \frac{10^4}{200}$$

$$\text{Similarly } R_{steel} = \frac{10^4}{50} \quad \text{and} \quad R_{copper} = \frac{10^4}{400}$$

Let temperature of common junction =  $T$

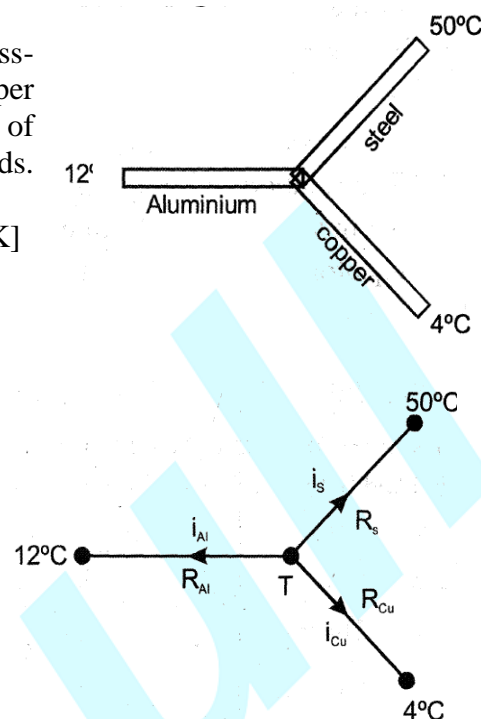
then from Kirchhoff's current law,  $i_{Al} + i_{steel} + i_{Cu} = 0$

$$\Rightarrow \frac{T-12}{R_{Al}} + \frac{T-50}{R_{steel}} - \frac{T-4}{R_{Cu}} = 0$$

$$\Rightarrow (T-12) 200 + (T-50) 50 + (T-4) 400 = 0$$

$$\Rightarrow 4(T-12) + (T-50) + 8(T-4) = 0$$

$$\Rightarrow 13T = 48 + 50 + 32 = 130 \quad \Rightarrow \quad T = 10^\circ\text{C}$$

**Illustration 22.**

The thermal conductivity of brick is  $1.7 \text{ W m}^{-1} \text{ K}^{-1}$  and that of cement is  $2.9 \text{ W m}^{-1} \text{ K}^{-1}$ . What thickness of cement will have same insulation as the brick of thickness 20 cm? Assuming their area to be same.

**Solution:**

Since  $Q = \frac{KA(T_1 - T_2)t}{L}$ . For same insulation by the brick and cement ;  $Q$ ,  $A$  ( $T_1 - T_2$ ) and  $t$  do not change.

Hence,  $\frac{K}{L}$  remain constant. If  $K_1$  and  $K_2$  be the thermal conductivities of brick and cement respectively and  $L_1$  and  $L_2$  be the required thickness then  $\frac{K_1}{L_1} = \frac{K_2}{L_2}$  or  $\frac{1.7}{20} = \frac{2.9}{L_2}$

$$\therefore L_2 = \frac{2.9}{1.7} \times 20 = 34.12 \text{ cm.}$$

**Illustration 23.**

Two vessels of different materials are identical in size and wall-thickness. They are filled with equal quantities of ice at 0°C. If the ice melts completely, in 10 and 25 minutes respectively then compare the coefficients of thermal conductivity of the materials of the vessels.

**Solution:**

Let  $K_1$  and  $K_2$  be the coefficients of thermal conductivity of the materials, and  $t_1$  and  $t_2$  be the time in which ice melts in the two vessels. Since both the vessels are identical, so  $A$  and  $L$  in both the cases is same.

$$\text{Now, } Q = \frac{K_1 A (\theta_1 - \theta_2) t_1}{L} = \frac{K_2 A (\theta_1 - \theta_2) t_2}{L} \quad \Rightarrow \quad \frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{25 \text{ min}}{10 \text{ min}} = \frac{5}{2}$$



**Illustration 24.**

Two plates of equal areas are placed in contact with each other. Their thickness are 2.0 cm and 5.0 cm respectively. The temperature of the external surface of the first plate is  $-20^{\circ}\text{C}$  and that of the external surface of the second plate is  $20^{\circ}\text{C}$ . What will be the temperature of the contact surface if the plate (i) are of the same material, (ii) have thermal conductivities in the ratio 2 : 5.

**Solution:**

Rate of flow of heat I the plates is  $\frac{Q}{t} = \frac{K_1 A (\theta_1 - \theta)}{L_1} = \frac{K_2 A (\theta - \theta_2)}{L_2} \dots (1)$

(i) Here  $\theta_1 = -20^{\circ}\text{C}$ ,  $\theta_2 = 20^{\circ}\text{C}$

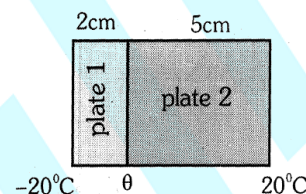
$L_1 = 2 \text{ cm} = 0.02 \text{ m}$ ,  $L_2 = 5 \text{ cm} = 0.05 \text{ m}$  and  $K_1 = K_2 = K$

$\therefore$  equation (i) becomes  $\frac{KA(-20 - \theta)}{0.02} = \frac{KA(\theta - 20)}{0.05}$

$\therefore 5(-20 - \theta) = 2(\theta - 20) \Rightarrow -100 - 5\theta = 2\theta - 40 \Rightarrow 7\theta = -60 \Rightarrow \theta = -8.6^{\circ}\text{C}$

(ii)  $\frac{K_1}{K_2} = \frac{2}{5}$  or  $K_1 = \frac{2}{5} K_2$

$\therefore$  from equation (i)  $\frac{2/5 K_2 A (-20 - \theta)}{0.02} = \frac{K_2 A (\theta - 20)}{0.05} \Rightarrow -20 - \theta = \theta - 20 \therefore \theta = 0^{\circ}\text{C}$

**Illustration 25.**

Two thin concentric shells made of copper with radius  $r_1$  and  $r_2$  ( $r_2 > r_1$ ) have a material of thermal conductivity  $K$  filled between them. The inner and outer spheres are maintained at temperatures  $T_H$  and  $T_C$  respectively by keeping a heater of power  $P$  at the centre of the two spheres. Find the value of  $P$ .

**Solution :**

Heat flowing per second through each cross-section of the sphere =  $P = i$ .

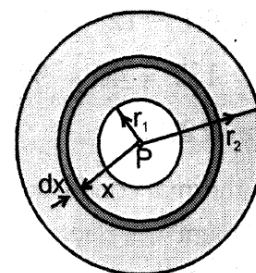
Thermal resistance of the spherical shell of radius  $x$  and thickness  $dx$ ,

$$dR = \frac{dx}{K \cdot 4\pi x^2}$$

$$\Rightarrow R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2 \cdot K} = \frac{1}{4\pi K} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$

Thermal current

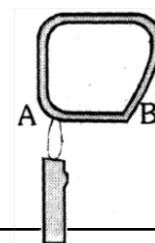
$$i = P = \frac{T_H - T_C}{R} = \frac{4\pi K (T_H - T_C) r_1 r_2}{(r_2 - r_1)}$$

**Illustration 26.**

Water in a closed tube is heated with one arm vertically placed above the lamp. In what direction water will begin to circulate along the tube ?

**Solution:**

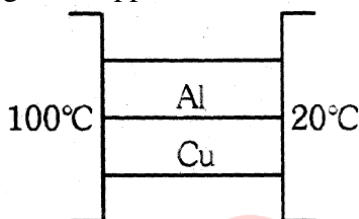
On heating the liquid at A will become lighter and will rise up. This will push the liquid in the tube upwards and so the liquid in the tube will move clockwise i.e. from B to A.



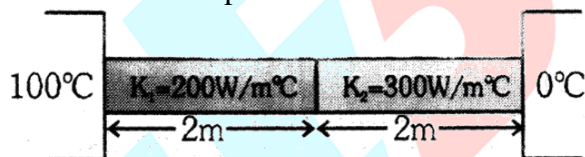
## BEGINNER'S BOX - 3

1. Explain why :
  - (a) A brass tumbler feels much colder than a wooden tray on a chilly day
  - (b) Two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double thickness ?
  - (c) Mud-houses are colder in summer and warmer in winter ?
  - (d) In winter birds sit with their wings spread out ?
  - (e) Woollen clothes are warmer than cotton clothes ?

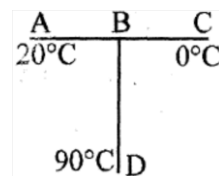
2. Two metal cubes with 3 cm-edges of copper and aluminium are arranged as shown in figure. Find



- (a) The total thermal current from one reservoir to the other.
  - (b) The ratio of the thermal current carried by the copper cube to that carried by the aluminium cube. Thermal conductivity of copper is  $60 \text{ W/m-K}$  and that of aluminium is  $40 \text{ W/m-K}$ .
3. For shown situation, calculate the temperature of the common interface.



4. The temperature at the ends of a uniform rod of length 100 cm are respectively  $95^\circ\text{C}$  and  $5^\circ\text{C}$ . What will be the temperature at a point 30 cm far from the hotter end ? Also calculate the temperature gradient.
5. Three conducting rods of same material and cross-section are shown in figure. Temperature of A, D and C are maintained at  $20^\circ\text{C}$ ,  $90^\circ\text{C}$  and  $0^\circ\text{C}$ . Find the ratio of length BD and BC if there is no heat flow in AB.



### 3.3 Thermal Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. When a body is heated and placed in vacuum, it loses heat even when there is no medium surrounding it. The heat can not go out from the body by the process of conduction or convection since both of these process require the presence of a material medium between source and surrounding objects.

The process by which heat is lost in this case is called radiation. This does not require the presence of any material medium.

It is radiation by which heat from the Sun reaches the Earth. Radiation has the following properties:

- (a) Radiant energy travels in straight lines and when some object is placed in the path, its shadow is formed at the detector.

- (b) It is reflected and refracted or can be made to interfere. The reflection or refraction are exactly as in case of light.
- (c) It can travel through vacuum.
- (d) Intensity of radiation follows the law of inverse square.
- (e) Thermal radiation can be polarised in the same way as light by transmission through a nicol prism.
- (f) Radiation takes place in solid, liquid & gases.

All these and many other properties establish that heat radiation has nearly all the properties possessed by light and these are also electromagnetic waves with the only difference of wavelength or frequency. The wavelength of thermal radiation is larger than that of visible light.

- When radiation passes through any medium then radiations slightly absorbed medium according to its absorptive power so temperature of medium slightly increases.
- In order to obtain a spectrum of radiation, a special prism is used like KCl Rock salt prism Fluorspar prism. Normal glass prism or Quartz prism can not be used (because it absorbs some radiation).
- Radiation intensity is measured with a specific device named as Bolometer.
- Heat radiation are always obtained in infra-red region of electromagnetic wave spectrum so they are called Infra-red rays.
- Thermal radiation when incident on a surface, then exert pressure on the surface which is known as Radiation Pressure.

#### Basic Fundamental definitions

- **Absorptive power or absorptive coefficient (a) :** The ratio of amount of radiation absorbed by a surface ( $Q_a$ ) to the amount of radiation incident (Q) upon it, is defined as the coefficient of absorption  $a = \frac{Q_a}{Q}$ . It is unitless and dimensionless.
- **Spectral absorptive power ( $a_\lambda$ )**  $a_\lambda = \frac{Q_{a\lambda}}{Q_\lambda}$  : also called monochromatic absorptive coefficient

At a given wavelength  $a = \int_0^\infty a_\lambda d\lambda$ . For ideal black body  $a_\lambda$  and  $a = 1$ ,  $a$  and  $a_\lambda$  are unitless

- **Emissive power (e) :** The amount of heat radiation emitted by unit surface area in unit second at a particular temperature.  
**SI UNIT:**  $\text{J/m}^2\text{-s}$  or  $\text{watt/m}^2$
- **Spectral Emissive power ( $e_\lambda$ ) :** The amount of heat radiation emitted by unit area of the body in one second in unit spectral region at a given wavelength.

Emissive power or total emissive power  $e = \int_0^\infty e_\lambda d\lambda$

**SI UNIT :**  $\text{W/m}^2\text{-}\text{\AA}$

#### Emissivity (e)

- **Absolute emissivity or emissivity :** Radiation energy given out by a unit surface area of a body in unit time corresponding to unit temperature difference w.r.t. the surroundings is called Emissivity.  
**SI unit :**  $\text{W/m}^2 \text{ K}$

- **Relative emissivity ( $e_r$ ) :**  $e_r = \frac{Q_{GB}}{Q_{IBB}} = \frac{e_{GB}}{E_{IBB}} = \frac{\text{emitted radiation by gray body}}{\text{emitted radiation by ideal black body}}$

GB = gray or general body, IBB = Ideal black body

(i) No unit

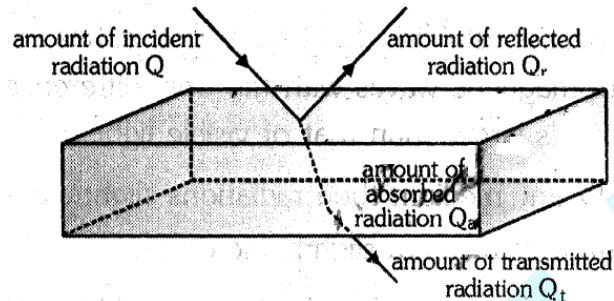
(ii) For ideal black body  $e_r = 1$

(iii) range  $0 < e_r < 1$

### Spectral, emissive, absorptive and transmittive power of a given body surface

Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts. (a) Reflection (b) Absorption (c) Transmission

- **From energy conservation**



$$Q = Q_r + Q_a + Q_t \Rightarrow \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = 1 \Rightarrow r + a + t = 1$$

$$\text{Reflective Coefficient } r = \frac{Q_r}{Q}, \text{ Absorptive Coefficient } a = \frac{Q_a}{Q},$$

$$\text{Transmittive Coefficient } t = \frac{Q_t}{Q}$$

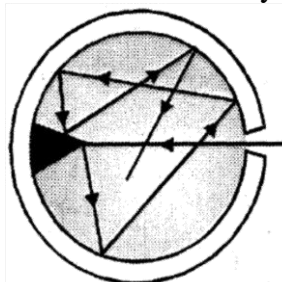
$r = 1$ and	$a = 0,$	$t = 0$	$\Rightarrow$	Perfect reflector
$a = 1$ and	$r = 0,$	$t = 0$	$\Rightarrow$	Ideal absorber (ideal black body)
$t = 1$ and	$a = 0,$	$r = 0$	$\Rightarrow$	Perfect transmitter (diathermanous)

$$\text{Reflection power (r)} = \left[ \frac{Q_r}{Q} \times 100 \right] \%, \text{ Absorption power (a)} = \left[ \frac{Q_a}{Q} \times 100 \right] \%$$

$$\text{Transmission power (t)} = \left[ \frac{Q_t}{Q} \times 100 \right] \%$$

### Ideal Black Body

- A body surface which absorbs all incident thermal radiations at low temperature, irrespective of their wave length and emits out all these absorbed radiations at high temperature is assumed to be an ideal black body surface.
- The identical parameters of. an ideal black body is given by  $a = a_\lambda = 1$  and  $r = 0 = t, e_r = 1$



Ferry's ideal black body

- The nature of emitted radiations from surface of ideal black body only depends on its temperature

- The radiations emitted from surface of ideal black body are called as either full or white radiations.
- At any temperature the spectral energy distribution curve for surface of an ideal black body is always continuous and according to this concept if the spectrum of a heat source obtained is continuous then it must be ideal black body like kerosene lamp; oil lamp, heating filament etc.
- There are two experimentally ideal black body  
(a) Ferry's ideal black body (b) Wien's ideal black body.
- At low temperature, surface of ideal black body is a perfect absorber and at a high temperature it proves to be a perfect emitter.
- An ideal black body need not be of black colour (eg. Sun).

### Prevost's theory of heat energy exchange

According to Prevost, at every possible temperature (except zero kelvin temperature) there is a continuous heat energy exchange between body and its surrounding and this exchange carry on for infinite time. The relation between temperature difference of body with its surrounding decides whether the body experience cooling effect or heating effect.

**When a cold body is placed in the hot surrounding :** The body radiates less energy and absorbs more energy from the surrounding, therefore the temperature of body increases. (Heating effect).

**When a hot body placed in cooler surrounding:** The body radiates more energy and absorbs less energy from the surroundings. Therefore temperature of body decrease (cooling effect)

### When the temperature of a body is equal to the temperature of the surrounding

The energy radiated per unit time by the body is equal to the energy absorbed per unit time by the body, therefore its temperature remains constant and the body is in thermal equilibrium with surrounding. Hence no heating and cooling effects are seen.

### KIRCHHOFF'S LAW

At a given temperature for all bodies the ratio of their spectral emissive power ( $e_\lambda$ ) to spectral absorptive power ( $a_\lambda$ ) is constant and this constant is equal to spectral emissive power ( $E_\lambda$ ) of the ideal black body at same temperature

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant} \quad \left[ \frac{e_\lambda}{a_\lambda} \right]_1 = \left[ \frac{e_\lambda}{a_\lambda} \right]_2 = \text{constant} \quad \text{Hence } e_\lambda \propto a_\lambda$$

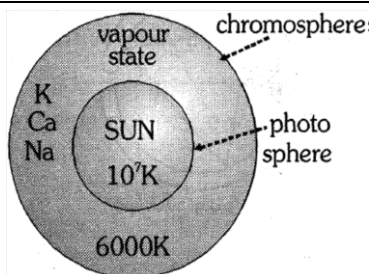
Good absorbers are good emitters and bad absorbers are bad emitters

### Applications of Kirchoff Law

#### • Fraunhofer's lines

Fraunhofer lines are dark lines in the spectrum of the Sun. When white light emitted from the central core of the Sun (Photosphere) passes through its atmosphere (chromosphere) some of the radiations are absorbed by the gases present, resulting in dark lines in the spectrum of Sun. At the time of total solar eclipse direct light rays emitted from photosphere cannot reach on the Earth and only rays from chromosphere are able to reach on the Earth surface. At that time we observe bright fraunhofer lines.





- **In deserts days are hot and nights cold**

Sand is rough and black, so it is a good absorber and hence in deserts, days (when radiation from Sun is incident on sand) will be very hot. Now in accordance with Kirchhoff's Law, good absorber is a good emitter.

So nights (when sand emits radiation) will be cold.

### Stefan's Law

The amount of radiation emitted per second per unit area by ideal black body is directly proportional to the fourth power of its absolute temperature.

Amount radiation emitted  $E \propto T^4$  where  $T$  = temperature of ideal black body (in K)  
 $E = \sigma T^4$  (This law is true for only ideal black body)

**SI Unit :**  $E = \text{watt/m}^2$   $\sigma \rightarrow$  Stefan's constant  $= 5.67 \times 10^{-8} \text{ watt /m}^2 \text{ K}^4$  (universal constant)

**Dimensions of  $\sigma$  :**  $M^1 L^0 T^{-3} \theta^{-4}$

Total radiation energy emitted out by surface of area  $A$  in time  $t$  :

Ideal black body  $Q_{\text{IBB}} = \sigma A T^4 t$  and for any other body  $Q_{\text{GB}} = e \sigma A T^4 t$

### Rate of emission of radiation

When temperature of surrounding  $T_0$  (Let  $T_0 < T$ )

Rate of emission of radiation from per unit area of ideal black body surface  $E_1 = \sigma T^4$

Rate of emission or absorption of radiation (per unit area) from surrounding  $E_2 = \sigma T_0^4$

Net rate of loss of radiation per unit area from ideal black body surface is

$$E = E_1 = \sigma T^4 - \sigma T_0^4 = \sigma (T^4 - T_0^4)$$

Net loss of radiation energy from entire surface area in time  $t$  is  $Q_{\text{IBB}} = \sigma A (T^4 - T_0^4) t$

For any other body  $Q_{\text{GB}} = e_r A \sigma (T^4 - T_0^4) t$

If in time  $dt$  the net heat energy loss for ideal black body is  $dQ$  and because of this its temperature falls by  $dT$

Rate of loss of heat (IBB)  $R_H = \frac{dQ}{dt} = \sigma A (T^4 - T_0^4) \text{ J/s}$

It is also equal to emitted power or radiation emitted per second

Rate of fall in temperature (Rate of cooling)  $R_F = \frac{dT}{dt} = \frac{\sigma A}{ms} (T^4 - T_0^4) \quad \Theta \quad \frac{dQ}{dt} = ms \frac{dT}{dt}$

Body	$R_H$	$R_F$
(i) Two solid sphere (same material) (same $T, T_0, s, \rho$ ) (different radius $r_1, r_2$ )	$R_H \propto r^2$	$R_F \propto \frac{1}{r}$
(ii) Two solid sphere (different material) (same $T, T_0$ )	$R_H \propto r^2$	$R_F \propto \frac{1}{rps}$
(iii) Different shape bodies (Cube, sphere, cylinder flat surface) (const. $T, T_0, V$ , same materials) surface	$R_H \propto A$ • maximum for flat surface • minimum for spherical	$R_F \propto \frac{A}{V}$ • maximum for flat surface • minimum for spherical surface
(iv) Two sphere (one solid and another hollow) ( $T, T_0, s, A$ are same)	• $R_H$ is same for both	$R_F$ • maximum for hollow sphere • minimum for solid sphere
(v) Different bodies made of different material ( $T, T_0, M, A$ ) (s-different)	• $R_H$ is same for all bodies	$R_F \propto \frac{1}{s}$

Where  $T$  and  $T_0$  absolute temperature of body and surrounding,  $M$  = mass of body,  
 $s$  = Specific heat,  $\rho$  = density,  $V$  = volume

• **When a body cools by radiation, its cooling depends on :**

- Nature of radiating surface : greater the emissivity ( $e_r$ ), faster will be the cooling.
- Area of radiating surface : greater the area of radiating surface faster will be the cooling.
- Mass of radiating body : greater the mass of radiating body slower will be the cooling.
- Specific heat of radiating body : greater the specific heat of radiating body slower will be the cooling.
- Temperature of radiating body : greater the temperature of radiating body faster will be the cooling.

## NEWTON'S LAW OF COOLING

Rate of cooling  $\left(\frac{dT}{dt}\right)$  is directly proportional to excess of temperature of the body over that of surrounding

$$[(\text{when } (T - T_0) \neq 35^\circ\text{C})] \quad -\frac{dT}{dt} \propto (T - T_0)$$

$T$  = temperature of body [all temperatures in  $^\circ\text{C}$ ]

$T_0$  = temperature of surrounding,

$T - T_0$  = excess of temperature ( $T > T_0$ )

If the temperature of body is decreased by  $dT$  in time  $dt$  then rate of fall of temperature

$$-\frac{dT}{dt} = K (T - T_0)$$

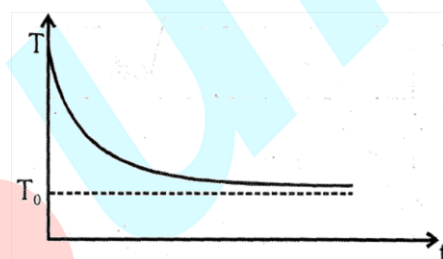
Where negative sign indicates that the rate of cooling is decreasing with time.

### For Numerical Problems, Newton's Law of cooling

If the temperature of body decreases from  $T_1$  to  $T_2$  and temperature of surroundings is  $T_0$  then

$$\text{Average excess of temperature} = \left[ \frac{T_1 + T_2}{2} - T_0 \right]$$

$$\Rightarrow \left[ \frac{T_1 - T_2}{t} \right] = +K \left[ \frac{T_1 + T_2}{2} - T_0 \right]$$



### Limitations of Newton's Law of Cooling

- Temperature difference should not exceed  $35^\circ\text{C}$ ,  $(T - T_0) \neq 35^\circ\text{C}$
- Loss of heat should only be by radiation.
- This law is an extended form of Stefan-Boltzman's law.

### Derivation of Newton's law from Stefan's Boltzman law

$$\frac{dT}{dt} = \frac{\sigma A}{ms} (T^4 - T_0^4) \quad \begin{cases} T - T_0 = \Delta T \\ T = T_0 + \Delta T \end{cases}$$

$$\frac{dT}{dt} = \frac{\sigma A}{ms} [(T_0 + \Delta T)^4 - T_0^4] \quad \text{if } x \ll 1 \text{ then } (1 + x)^n = 1 + nx \text{ (Binomial theorem)}$$

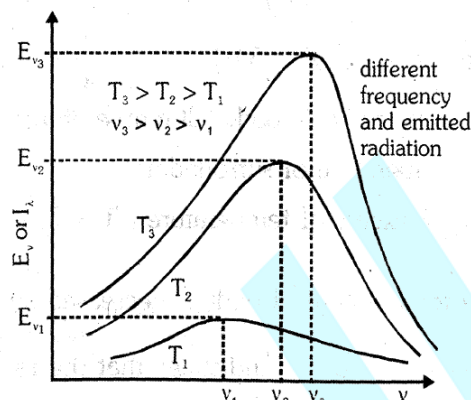
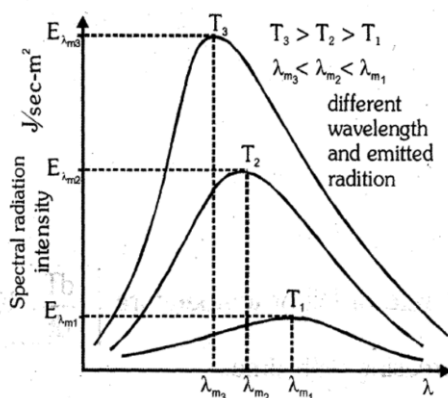
$$\frac{dT}{dt} = \frac{\sigma A}{ms} \left[ T_0^4 \left( 1 + \frac{\Delta T}{T_0} \right)^4 - T_0^4 \right] = \frac{\sigma A}{ms} T_0^4 \left[ \left( 1 + \frac{\Delta T}{T_0} \right)^4 - 1 \right] = \frac{\sigma A}{ms} T_0^4 \left[ 1 + 4 \frac{\Delta T}{T_0} - 1 \right]$$

$$\frac{dT}{dt} = \left[ 4 \frac{\sigma A}{ms} T_0^3 \right] \Delta T \quad \Rightarrow \quad \frac{dT}{dt} = K \Delta T \quad \text{here constant } K = \frac{4\sigma A T_0^3}{ms}$$

**Newton's law of cooling**  $\frac{dT}{dt} \propto \Delta T$  (for small temperature difference)

### Spectral Energy distribution curve of Black Body radiations

Practically given by : Lumers and Pringshem      Mathematically given by : Plank



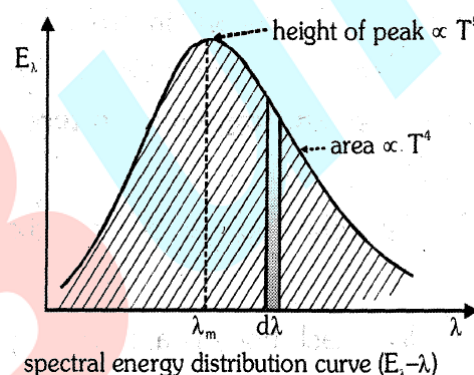
### Results from these Graphs

(i)  $\lambda_m \propto \frac{1}{T}$       Wien's displacement law

$$\lambda_m T = b$$

$$\lambda_m T_1 = \lambda_m T_2$$

(ii)  $E_{\lambda_m} \propto T^5$



(iii) i.e. Area  $\int_0^{\infty} E_{\lambda} d\lambda = E \sigma T^4$  Area between curve and  $\lambda$  axis gives the emissive power of body

$$\text{Hence } \frac{A_1}{A_2} = \left[ \frac{T_1}{T_2} \right]^4$$

### Wein's Displacement Law

The wavelength corresponding to maximum emission of radiation decrease with increasing temperature  $\left[ \lambda_m \propto \frac{1}{T} \right]$ . This is known as Wein's displacement law.

$$\lambda_m T = b \quad \text{where } b \text{ is Wein's constant} = 2.89 \times 10^{-3} \text{ mK}$$

**Dimensions of b :**  $M^0 L^1 T^0 \theta^1$

**Relation between frequency and temperature**  $\nu_m = \frac{c}{b} T$  [ $c = \lambda \times \nu$ ]

### Solar constant 'S'

The Sun emits radiant energy continuously in space of which an in significant part reaches the Earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant. The solar constant S is taken to be  $1340 \text{ watt/m}^2$  or  $1.937 \text{ Cal/cm}^2\text{-minute}$

### • Temperature of the Sun

Let  $R$  be the radius of the Sun and ' $d$ ' be the radius of Earth's orbit around the Sun. Let  $E$  be the energy emitted by the Sun per second per unit area. The total energy emitted by the Sun in one second =  $E.A = E \times 4\pi R^2$ . (This energy is falling on a sphere of radius equal to the radius of the Earth's orbit around the Sun i.e., on a sphere of surface area  $4\pi d^2$ )

So, The energy falling per unit time per unit area of Earth

$$= \frac{4\pi R^2 \times E}{4\pi d^2} = \frac{ER^2}{d^2}$$

$$R = 7 \times 10^8 \text{ m}, \quad d = 1.5 \times 10^{11} \text{ m}, \quad \sigma = 5.7 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

$$\text{Solar constant} \quad S = \frac{ER^2}{d^2}$$

$$\text{By Stefan's Law} \quad E = \sigma T^4$$

$$S = \frac{\sigma T^4 R^2}{d^2} \Rightarrow T = \left[ \frac{S \times d^2}{\sigma \times R^2} \right]^{\frac{1}{4}} = \left[ \frac{1340 \times (1.5 \times 10^{11})^2}{5.7 \times 10^{-8} \times (7 \times 10^8)^2} \right]^{\frac{1}{4}} = 5732 \text{ K}$$

### GOLDEN KEY POINT

- At absolute zero temperature (zero kelvin) all atoms of a given substance remains in ground state, so, at this temperature emission and absorption of radiation from any substance is impossible, so Prevost's heat energy exchange theory does not applied at this temperature, so it is called **limiting temperature** of prevost's theory.
- With the help of Prevost's theory rate of cooling of any body w.r.t. its surroundings can be worked out (applied to Stefan Boltzman law, Newton's law of cooling.)
- For a constant temperature the spectral emmisseive power of an ideal black body is a constant parameter
- The practical confirmation of Kirchhoff's law carried out by Rishi apparatus and the main base of this apparatus is a Lessilie container.
- The main conclusion predicted from Kirchhof's law can be expressed as  

Good absorber	$f$	Good emitter
Bad absorber	$f$	Bad emitter
(at Low temperature)		(at high temperature)
- If all of  $T, T_0, m, s, V, \rho$ , are same for different shape body then  $R_F$  and  $R_H$  will be maximum for the flat surface.
- If a solid and hollow sphere are taken with all the parameters same then hollow will cool down at fast rate.
- Rate of temperature fall,  $R_F \propto \frac{1}{s} \propto \frac{dT}{dt}$   
 so  $dt \propto s$ . If condition of specific heat is  $s_1 > s_2 > s_3$   
 and if all cooled same temperature i.e. temperature fall is also identical for all then required time  $t \propto s$   
 $\therefore t_1 > t_2 > t_3$



- Spectral energy distribution curves are continuous. At any temperature in all possible wavelength radiation between  $(0 - \infty)$  are emitted but quantity of radiation are different for different wavelength.
- As the wave length increases the amount of radiation emitted first increase, becomes maximum and then decreases.
- At a particular temperature the area enclosed between the spectral energy curve and wavelength axis shows the emissive power of the body.

$$\text{Area} = \int_0^{\infty} E_{\lambda} d\lambda = E = \sigma T^4$$

## Illustrations

### Illustration 27.

If  $\lambda_m$  for the moon is 14.5 micron, then find its temperature.

#### Solution:

Wien's displacement law  $\lambda_m T = b$

$$\therefore T = \frac{b}{\lambda_m} = \frac{2.89 \times 10^{-3}}{14.5 \times 10^{-6}} = 199.3K$$

### Illustration 28.

Total radiation incident on body is 400 J, If 20% of incident radiation reflected back and 120 J is absorbed by body. Then find out transmittive power in percentage.

#### Solution:

$$Q = Q_t + Q_r + Q_a \Rightarrow 400 = 80 + 120 + Q_t \Rightarrow Q_t = 200$$

$$\text{So transmittive power is } \frac{Q_t}{Q} \times 100\% = 50\%$$

### Illustration 29.

The operating temperature of a tungsten filament in an incandescent lamp is 2000 K and its emissivity is 0.3. Find the surface area of the filament of a 25 watt lamp. Stefan's constant  $s = 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$

#### Solution:

⊖ Rate of emission = wattage of the lamp

$$\therefore W = Ae\sigma T^4 \Rightarrow A = \frac{W}{e\sigma T^4} = \frac{25}{0.3 \times 5.67 \times 10^{-8} \times (2000)^4} = 0.918 \times 10^{-4} \text{ m}^2$$

### Illustration 30.

Draw a graph between  $\log E$  and  $\log T$

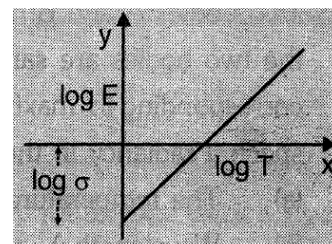
#### Solution:

$$E = \sigma T^4 \quad (\text{taking log})$$

$$\log E = \log (\sigma T^4)$$

$$\log E = 4\log T + \log \sigma \quad \text{This is equivalent to } y = mx - C$$

( $\sigma < 1$  so its log is negative quantity)



**Illustration 31.**

If temperature of ideal black body is increased by 50%, what will be percentage increase in quantity of radiations emitted from its surface.

**Solution:**

$$E \propto T^4 \quad \text{and} \quad \therefore E' \propto (1.5)^4 T^4 \propto \left[\frac{15}{10}\right]^4 T^4 \propto \left[\frac{3}{2}\right]^4 T^4 \propto \frac{81}{16} T^4$$

$$\frac{E' - E}{E} \times 100\% = \left[ \frac{\frac{81}{16} T^4 - T^4}{T^4} \right] \times 100\% = 406\% \approx 400\%$$

**Illustration 32.**

If temperature of ideal black body is decreased from  $T$  to  $\frac{T}{2}$  then find out percentage loss in emissive rate

**Solution:**

$$E \propto T^4, \quad E' \propto \left[\frac{T}{2}\right]^4 \quad \frac{T^4}{16}$$

$$\left[ \frac{E - E'}{E} \right] \times 100\% = \left[ 1 - \frac{1}{16} \right] \times 100\% = \frac{15}{16} \times 100\% \approx 94\% \quad \text{Remaining is 6\% (Approx.)}$$

**Illustration 33.**

Calculate the temperature at which a perfect black body radiates at the rate of  $5.67 \text{ W cm}^{-2}$ . Stefan's constant is  $5.67 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}$ .

**Solution:**

$$\text{Given } E = 5.67 \text{ W cm}^{-2} = 5.67 \times 10^{+4} \text{ W m}^{-2}, \quad \sigma = 5.67 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}$$

$$\text{Using, } E = \sigma T^4; \quad T^4 = \frac{E}{\sigma} \quad \text{or } T = \left[ \frac{E}{\sigma} \right]^{\frac{1}{4}} = \left[ \frac{5.67 \times 10^{+4}}{5.67 \times 10^{-8}} \right]^{\frac{1}{4}} = (10^{12})^{1/4} = 1000 \text{ K}$$

**Illustration 34.**

The temperature of furnace is  $2000^\circ\text{C}$ , in its spectrum the maximum intensity is obtained at about  $4000\text{\AA}$ . If the maximum intensity is at  $2000\text{\AA}$  calculate the temperature of the furnace in  $^\circ\text{C}$ .

**Solution:**

$$\text{by using } \lambda_m T = b, \quad 4000 (2000 + 273) = 2000(T) \Rightarrow T = 4546\text{K.}$$

$$\text{The temperature of furnace} = 4546 - 273 = 4273^\circ\text{C}$$

**Illustration 35.**

Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same the two bodies emit total radiant power at the same rate. The wavelength  $\lambda_B$  corresponding to maximum spectral radiance of B is shifted from the wavelength corresponding to maximum spectral radiance in the radiation of A by  $1.0 \mu\text{m}$ . If the temperature of A is  $5802\text{K}$ , Calculate :-

- The temperature of B
- Wavelength  $\lambda_B$

**Solution:**

- As both bodies A and B having same radiant power

$$\therefore P_A = P_B \Rightarrow e_A \sigma A_A T_A^4 = e_B \sigma A_B T_B^4 \Rightarrow (0.01) \sigma A T_A^4 = (0.81) \sigma A T_B^4$$

$$T_B = \left( \frac{0.01}{0.81} \right)^{1/4} T_A = \frac{T_A}{3} = \frac{5802}{3} = 1934 \text{ K}$$

(b) According to wein's displacement law

$$\lambda_A T_A = \lambda_B T_B \Rightarrow \lambda_B = \left( \frac{5802}{1934} \right) \lambda_A = 3 \lambda_A$$

$$\text{As } \lambda_B - \lambda_A = 1 \mu\text{m} \Rightarrow \lambda_B - \frac{\lambda_B}{3} = 1 \mu\text{m} \Rightarrow \frac{2\lambda_B}{3} = 1 \mu\text{m} \Rightarrow \lambda_B = 1.5 \mu\text{m}.$$

### Illustration 36.

When a metallic body is heated in a furnace, then what colour will appear as temperature increases.

**Solution:**

$$T \propto \frac{1}{\lambda_m}$$

As Temperature increases colour of body will appear from red, yellow, green, blue and the white.

### Illustration 37.

Define (i) Steady state and (ii) Temperature gradient in conduction of heat through a conducting rod.

**Solution:**

- (i) When one end of a rod is heated, the temperature of various points of the rod changes continuously but after some time a state is reached, when the temperature of each cross-section becomes steady which is called steady state. In this state the heat received by any section will be totally transferred to the next section so no heat is absorbed by any cross section .
- (ii) Temperature gradient is defined as the rate of change of temperature with distance in the direction of flow of heat.

### Illustration 38.

Assuming Newton's law of cooling to be valid. The temperature of body changes from  $60^\circ\text{C}$  to  $40^\circ\text{C}$  in 7 minutes. Temperature of surroundings being  $10^\circ\text{C}$ , find its temperature after next 7 minutes.

**Solution:**

$$\text{According to Newton's law of cooling } \frac{T_1 - t_2}{t} = K \left( \frac{T_1 + T_2}{2} - T_0 \right)$$

Since the temperature decreases from  $60^\circ\text{C}$  to  $40^\circ\text{C}$  in 7 minutes

$$\frac{60 - 40}{7} = K \left( \frac{60 + 40}{2} - 10 \right) \Rightarrow K (50 - 10) \Rightarrow K = \frac{1}{14}$$

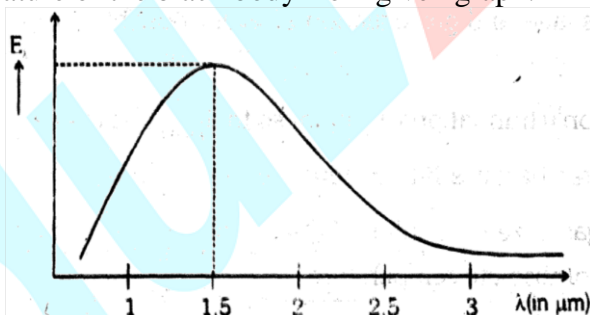
$$\text{If the temperature of object becomes } T_2 \text{ in next 7 minutes then } \frac{40 - T_2}{7} = \frac{1}{14} \left( \frac{40 + T_2}{2} - 10 \right)$$

$$\Rightarrow 40 - T_2 = \frac{1}{4} (40 + T_2 - 20) \Rightarrow 160 - 4T_2 = 20 + T_2$$

$$\Rightarrow 5T_2 = 140 \Rightarrow T_2 = 28^\circ\text{C}$$

## BEGINNER'S BOX - 4

1. Explain why :
  - (a) a body with large reflectivity is a poor emitter.
  - (b) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace
  - (c) the earth without its atmosphere would be inhospitably cold
  - (d) Heat is generated continuously in an electric heater but its temperature remains constant after some time.
  - (e) The bottom of a cooking vessel is made black ?
  - (f) On winter night you feel warmer when clouds cover the sky than when the sky is clear.
  - (g) A thermos or vacuum flask can keep hot things hot and cold things cold for a long time, how?
2. Two spherical ideal black bodies of radii  $r_1$  and  $r_2$  are having surface temperature  $T_1$  and  $T_2$  respectively, If both radiate the same power. Then calculate the ratio of  $\frac{T_1}{T_2}$ .
3. If a liquid takes 30 sec. in cooling from  $80^\circ\text{C}$  to  $70^\circ\text{C}$  and 70 sec in cooling from  $60^\circ\text{C}$  to  $50^\circ\text{C}$ , then find the room temperature.
4. A body cools in 10 minutes from  $60^\circ\text{C}$  to  $40^\circ\text{C}$ . What will be its temperature after next 10 minutes? The temperature of the surrounding is  $10^\circ\text{C}$ .
5. Calculate the temperature of the black body from given graph.



## KINETIC THEORY OF GASES

The properties of the gases are entirely different from those of solid and liquid. In case of gases, thermal expansion is very large as compared to solids and liquids. To state the conditions of a gas, its volume, pressure and temperature must be specified.

**Intermolecular force**                      Solid > liquid > real gas > ideal gas (zero)

**Potential energy**                              Solid < liquid < real gas < ideal gas (zero)

**At a given temperature for solid, liquid and gas:**

- (i) Internal kinetic energy                      : Same for all
- (ii) Internal potential Energy                      : Maximum for deal gas ( $PE = 0$ ) and minimum for solids ( $PE = -ve$ )
- (iii) Internal Energy                              : Maximum for Ideal gas and Minimum for solid

**Notation :**

$\mu$ = Molar amount of gas = $M/M_w = N/N_0$	$m$ = mass of a gas molecule
$N_0$ = Avogadro constant = $6.023 \times 10^{23}$ molecules/mole	$M$ = mass of gas
$R$ = Universal gas constant = $8.31 \text{ J/mole}^\circ\text{K}$ $\approx 2 \text{ cal/mole}^\circ\text{K}$ , [Dimension = $\text{ML}^2\text{T}^{-2}\mu^{-1}\theta^{-1}$ ]	$M_w$ = molecular weight
$r$ = Specific gas constant ( $r = R/M_w$ )	$N$ = No. of gas molecules
$k$ = Boltzmann constant = ( $k = R/N_0$ ) = $1.38 \times 10^{-23} \text{ J/kelvin}$	$n$ = Molecular density ( $n = N/V$ )
	$\rho$ = gas density ( $\rho = M/V = mn$ )

N.T.P. (Normal temperature and pressure)		S.T.P. (Standard Temperature and Pressure)
Temperature	$0^\circ\text{C} = 273.15 \text{ K}$	$0.01^\circ\text{C} = 273.16 \text{ K}$
Pressure	$1 \text{ atm} = 1.01325 \times 10^5 \text{ N/m}^2$ $= 1.01325 \times 10^5 \text{ pascal}$	$1 \text{ atm}$
Volume	$22.4 \text{ litre}$	$22.4 \text{ litre}$

$$1 \text{ lit} = 10^{-3} \text{ m}^3 = 10^3 \text{ cm}^3 = 10^3 \text{ cc} = 10^3 \text{ ml}$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3 = 1 \text{ g/cc}, \rho_{\text{ice}} = 900 \text{ kg/m}^3 = 0.9 \text{ g/cc}$$

### Ideal Gas Concept

- Volume of gas molecules is negligible as compared to volume container.  
So volume of gas = volume of container (Except 0 K)
- No intermolecular forces act between gas molecules.

### Properties of Ideal Gas

- A gas which follows all gas laws and gas equation at every possible temperature and pressure is known as ideal or perfect gas.
- Ideal gas molecules can do only translational motion, so their kinetic energy is only translational kinetic energy
- Ideal gas can not be liquified because IMF is zero
- Potential energy of ideal gas is zero so internal energy of ideal gas is perfectly translational K.E. of gas.

It is directly proportional to absolute temperature.

So, internal energy depends only and only on its temperature.

$$E_{\text{trans}} \propto T$$

$$\text{For a substance } U = U_{\text{KE}} + U_{\text{PE}}$$

$U_{\text{KE}}$  : depends only on T,  $U_{\text{PE}}$  : depends upon intermolecular force (Always negative)

- Specific heat of ideal gas is constant quantity and it does not change with temperature
- All real gases behaves as an ideal gas at high temperature and low pressure and low density.
- Gas molecules have point mass and negligible volume and velocity is very high ( $10^7 \text{ cm/s}$ ). That's why there is no effect of gravity on them.

### Equation of state for Ideal gas

$$PV = \mu RT \quad \text{or} \quad PV = \left[ \frac{mN}{mN_0} \right] RT = \left[ \frac{R}{N_0} \right] N T = NkT$$



where  $\mu$  = number of moles of gas

$k$  = Boltzmann constant

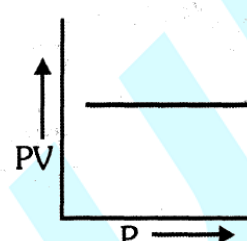
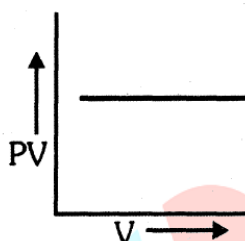
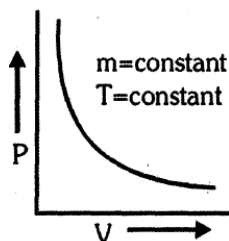
$$\Rightarrow \frac{P}{\rho} = \frac{RT}{M_w} = \frac{kT}{m}$$

## Gas Laws

### • Boyle's Law

According to it for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, i.e.,  $V \propto \frac{1}{P}$

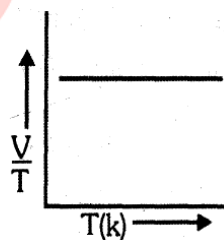
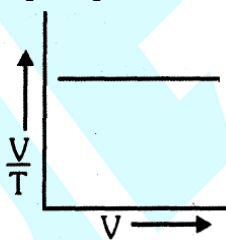
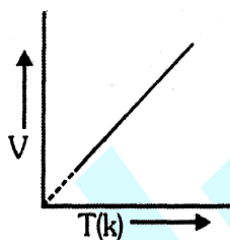
if  $m$  and  $T$  = Constant  $\Rightarrow P_1 V_1 = P_2 V_2$



### • Charle's Law

According to it for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature, i.e.  $V \propto T$

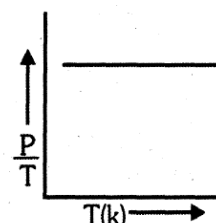
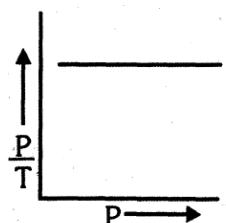
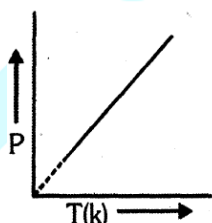
if  $m$  and  $P$  = Constant  $\Rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2}$



### • Gay-Lussac's Law

According to it, for a given mass of an ideal gas at constant volume, pressure of a gas is directly proportional to its absolute temperature, i.e.,  $P \propto T$

if  $m$  and  $V$  = constant  $\Rightarrow \frac{P_1}{P_2} = \frac{T_1}{T_2}$



### Avogadro's Law

According to it, at same temperature and pressure, equal volume of all gases contain equal number of molecules, i.e.,  $N_1 = N_2$  if  $P$ ,  $V$  and  $T$  are same.

### Dalton's Partial Pressure Mixture Law :

According to it, the pressure exerted by mixture of non-reactive gases is equal to the sum of partial pressure of each component gases present in the mixture, i.e.,  $P = P_1 + P_2 + \dots$

### Illustrations

#### Illustration 39.

By increasing temperature of gas by  $5^\circ\text{C}$  its pressure increases by 0.5% from its initial value at constant volume then what is initial temperature of gas ?

**Solution:**

$$\Theta \text{ At constant volume } T \propto P \quad \therefore \frac{\Delta T}{T} \times 100 = \frac{\Delta P}{P} \times 100 = 0.5 \Rightarrow T = \frac{5 \times 100}{0.5} = 1000\text{K}$$

#### Illustration 40.

Calculate the value of universal gas constant at STP.

**Solution:**

$$\text{Universal gas constant is given by } R = \frac{PV}{T}$$

One mole of all gases at S.T.P. occupy volume  $V = 22.4 \text{ litre} = 22.4 \times 10^{-3} \text{ m}^3$

$P = 760 \text{ mm of Hg} = 760 \times 10^{-3} \times 13.6 \times 10^3 \times 9.80 \text{ N m}^{-2}$ ,  $T = 273.16 \text{ K}$

$$\therefore R = \frac{760 \times 10^{-3} \times 13.6 \times 10^3 \times 9.80 \times 22.4 \times 10^{-3}}{273.16} = 8.314 \text{ L mol}^{-1} \text{ K}^{-1}$$

#### Illustration 41.

A closed container of volume  $0.02 \text{ m}^3$  contains a mixture of neon and argon gases at a temperature of  $27^\circ\text{C}$  and pressure of  $1 \times 10^5 \text{ N/m}^2$ . The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the mass the individual gases in the container, assuming them to be ideal. Given :  $R = 8.314 \text{ J/mol-K}$ .

**Solution:**

Let  $m$  gram be the mass of neon. Then, the mass of argon is  $(28 - m)\text{g}$ .

$$\text{Total number of moles of the mixture, } \mu = \frac{m}{20} + \frac{28 - m}{40} = \frac{28 + m}{40} \quad \dots(i)$$

$$\text{Now, } \mu = \frac{PV}{RT} = \frac{1 \times 10^5 \times 0.02}{8.314 \times 300} = 0.8 \quad \dots(ii)$$

$$\text{By (i) and (ii), } \frac{28 + m}{40} = 0.8 \Rightarrow 28 + m = 32 \Rightarrow m = 4 \text{ gram}$$

$$\text{or mass of argon} = (28 - 4)\text{g} = 24 \text{ g}$$

#### Illustration 42.

Calculate the temperature of the Sun if density is  $1.4 \text{ g cm}^{-3}$ , pressure is  $1.4 \times 10^9$  atmosphere and average molecular weight of gases in the Sun is 2 g/mole. [Given  $R = 8.4 \text{ J mol}^{-1} \text{ K}^{-1}$ ]

**Solution:**

$$PV = \mu RT \Rightarrow T = \frac{PV}{\mu R} \quad \dots(i) \quad \text{But } \mu = \frac{M}{M_w} \text{ and } \rho = \frac{M}{V} \quad \therefore \mu = \frac{\rho V}{M_w}$$

$$\text{From equation (i)} \quad T = \frac{PVM_w}{\rho VR} = \frac{PM_w}{\rho R} = \frac{1.4 \times 10^9 \times 1.01 \times 10^5 \times 2 \times 10^{-3}}{1.4 \times 1000 \times 8.4} = 2.4 \times 10^7 \text{ K}$$

#### Illustration 43.

At the top of a mountain a thermometer reads  $7^{\circ}\text{C}$  and barometer reads 70 cm of Hg. At the bottom of the mountain they read  $27^{\circ}\text{C}$  and 76 cm of Hg respectively. Compare the density of the air at the top with that at the bottom.

**Solution:**

$$\text{By gas equation } PV = \frac{M}{M_w} RT \Rightarrow \frac{P}{\rho T} = \frac{R}{M_w} \left[ Q\mu = \frac{M}{M_w} \text{ and } \frac{M}{V} = \rho \right]$$

$$\text{Now as } M_w \text{ and } R \text{ are same for top and bottom } \left[ \frac{P}{\rho T} \right]_T = \left[ \frac{P}{\rho T} \right]_B$$

$$\text{So } \frac{\rho_T}{\rho_B} = \frac{P_T}{P_B} \times \frac{T_B}{T_T} = \frac{70}{76} \times \frac{300}{280} = \frac{75}{76} = 0.9868$$

**Illustration 44.**

A sample of oxygen of volume of 500 cc at a pressure of 2 atm is compressed to a volume of 400 cc. What pressure is needed to do this if the temperature is kept constant ?

**Solution:**

$$\text{Temperature is constant, so } P_1 V_1 = P_2 V_2 \therefore P_2 = P_1 \frac{V_1}{V_2} = 2 \left[ \frac{500}{400} \right] = 2.5 \text{ atm}$$

**Illustration 45.**

A vessel of volume  $8.0 \times 10^{-3} \text{ m}^3$  contains an ideal gas at 300 K and 200 k Pa. The gas is allowed to leak till the pressure falls to 125 kPa. Calculate the amount of the gas leaked assuming that the temperature remains constant.

**Solution:**

As the gas leaks out the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel is given by  $\mu = \frac{PV}{RT}$ .

The number of moles in the vessel before the leakage is  $\mu = \frac{P_1 V}{RT}$  and that after the leakage is

$$\mu_2 = \frac{P_2 V}{RT}$$

$$\text{The amount leaked is } \mu_1 - \mu_2 = \frac{(200 - 125) \times 10^3 \times 8.0 \times 10^{-3}}{8.3 \times 300} = 0.24 \text{ mole}$$

**Illustration 46.**

1500 ml of a gas at a room temperature of  $23^{\circ}\text{C}$  is inhaled by a person whose body temperature is  $37^{\circ}\text{C}$ , if the pressure and mass stay constant, what will be the volume of the gas in the lungs of the person ?

**Solution:**

$T_1 = 273 + 23 = 296 \text{ K}$ ;  $T_2 = 273 + 37 = 310 \text{ K}$ . Pressure and amount of the gas are kept constant,

$$\text{So } \frac{V_1}{T_1} = \frac{V_2}{T_2} \therefore V_2 = V_1 \times \frac{T_2}{T_1} = 1500 \times \frac{310}{296} = 1570.95 \text{ ml}$$

**Illustration 47.**

A sample of  $O_2$  is at a pressure of 1 atm when the volume is 100 ml and its temperature is  $27^\circ C$ . What will be the temperature of the gas if the pressure becomes 2 atm and volume remains 100 ml.

**Solution:**

$$T_1 = 273 + 27 = 300 \text{ K}$$

$$\text{For constant volume } \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow T_2 = T_1 \times \frac{P_2}{P_1} = 300 \times \frac{2}{1} = 600 \text{ K} = 600 - 273 = 327^\circ C$$

**BEGINNER'S BOX - 5**

1. A vessel of volume  $8.3 \times 10^{-3} \text{ m}^3$  contains an ideal gas at temperature  $27^\circ C$  and pressure 200 kPa. The gas is allowed to leak till the pressure falls to 100 kPa and temperature increases to  $327^\circ C$ . What is the amount of gas in moles will be leaked out ?
2. Two closed vessels of equal volume contain air at 105 kPa, 300 K and are connected through a narrow tube. If one of the vessels is now maintained at 300 K and other at 400 K, What will be the pressure in the vessels ?
3. The volume of a gas is 1 litre at the pressure  $1.2 \times 10^7 \text{ Nm}^{-2}$  and temperature 400 K. Calculate the number of molecules in the gas.
4. A vessel contains two non-reactive gases : neon (monatomic) and oxygen (diatomic) The ratio of their partial pressures is 3 : 2. Estimate the ratio of
  - (a) number of molecules and
  - (b) mass density of neon and oxygen in the vessel.
 Atomic mass of Ne = 20.2 u. molecular mass of  $O_2 = 32.0 \text{ u}$ .

**The kinetic theory of gases**

- Rudolph Clausius (1822–88) and James Clark Maxwell (1831–75) developed the kinetic theory of gases in order to explain gas laws in terms of the motion of the gas molecules. The theory is based on following assumptions regarding the motion of molecules and the nature of the gases.

**Basic postulates of Kinetic theory of gases.**

- Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical but are different than those of another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparison to inter molecular distance ( $10^{-9} \text{ m}$ )

**Assumptions regarding motion :**

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity (very high speed).
- The number of molecules moving with most probable speed is maximum.

**Assumptions regarding collision:**

- The gas molecules keep colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (ie., the total energy before collision = total energy after the collision.)

#### Assumptions regarding force:

- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.

#### Assumptions regarding pressure:

- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

#### Assumptions regarding density:

- The density of gas is constant at all points of the container.

#### Properties/ Assumptions of Ideal gas

- The molecules of a gas are in a state of continuous random motion. They move with all possible velocities in all possible directions. They obey Newton's law of motion.
- Mean momentum = 0; Mean velocity  $\langle \vec{v} \rangle = 0$ ;  $\langle v^2 \rangle \neq 0$  (Non zero);  $\langle v^3 \rangle = \langle v^5 \rangle = 0$
- The average distance travelled by a molecule between two successive collisions is called as mean free path ( $\lambda_m$ ) of the molecule.
- The time during which a collision takes place is negligible as compared to time taken by the molecule to cover the mean free path. At NTP ratio of time of collision to time of motion to cover mean free path is  $10^{-8} : 1$ .
- When a gas is taken into a vessel it is uniformly distributed in entire volume of vessel such that its mass density, molecular density, motion of molecules etc. all are identical for all direction, therefore root mean velocity.  
 $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$  are equal. Pressure exerted by the gas in all direction  $P_x = P_y = P_z = P \rightarrow$  equal
- All those assumptions can be justified, if number of gas molecules are taken very large i.e.,  $10^{23}$  molecules/cm<sup>3</sup>.

#### Different speeds of gas molecules

##### Average velocity

Because molecules are in random motion in all possible direction with all possible velocity. Therefore, the average velocity of the gas molecules in container is zero.

$$\langle \vec{v} \rangle = \frac{\vec{v}_1 + \vec{v}_2 + \dots + \vec{v}_n}{N} = 0$$

$$\text{RMS speed of molecules} \quad v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}} = 1.73 \sqrt{\frac{kT}{m}}$$

##### Average speed or Mean speed of molecules :

By maxwell's velocity distribution law  $v_M$  or  $\langle |\vec{v}| \rangle \equiv v_{\text{mean}}$

$$\langle |\vec{v}| \rangle \equiv v_{\text{mean}} = \frac{|\vec{v}_1| + |\vec{v}_2| + \dots + |\vec{v}_n|}{n} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8kT}{\pi m}} = 1.59 \sqrt{\frac{kT}{m}}$$

#### Most probable speed of molecules ( $v_{\text{mp}}$ )



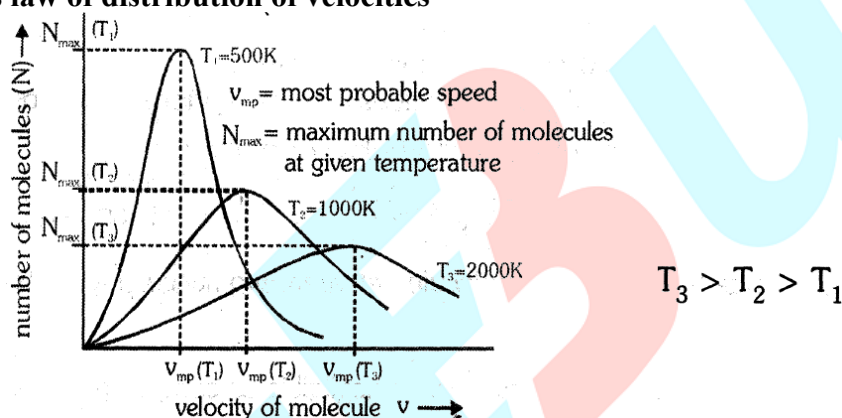
At a given temperature the speed to which maximum number of molecules belongs is called as most probable speed ( $v_{mp}$ )

$$v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2kT}{m}} = 1.41 \sqrt{\frac{kT}{m}}$$

**Velocity of sound in gas medium ( $v_s$ )**  $v_{sound} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M_w}} = \sqrt{\frac{\gamma kT}{m}}$

- At any temperature  $v_{rms} > v_{Mean} > v_{MP} > v_{sound}$  (always)
- For a gas at any temperature (T)  $\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}}$ ,  $\frac{v_{rms}}{v_{MP}} = \sqrt{\frac{3}{2}}$
- A temperature is not possible at which above order can be changed  
 $v_{rms} \neq v_{Mean} \neq v_{MP} \neq v_{sound}$  (always)

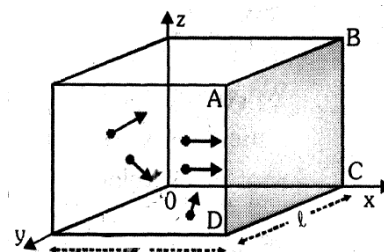
### Maxwell's law of distribution of velocities



### Expression for Pressure of An Ideal gas

Consider an ideal gas enclosed in a cubical vessel of length  $\lambda$ .

Suppose there are 'N' molecules in a gas which are moving with velocities  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_N$ .



If we consider any single molecule then its instantaneous  $\vec{v}$  can be expressed as  $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

Due to random motion of the molecule

$$v_x = v_y = v_z; |\vec{v}| = v_x \sqrt{3} = v_y \sqrt{3} = v_z \sqrt{3} = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Suppose a molecule of mass  $m$  is moving with a velocity  $v_x$  towards the face ABCD, It strikes the face of the cubical vessel and returns back to strike the opposite face.

Change in momentum of the molecule per collision  $\Delta p = -mv_x - mv_x = -2mv_x$

Momentum transferred to the wall of the vessel per molecule per collision  $\Delta p = 2mv_x$

The distance travelled by the molecule in going to face ABCD and coming back is  $2\lambda$ .

So, the time between two successive collision is  $\Delta t = \frac{2\lambda}{v_x}$

Number of collision per sec per molecule is  $f_c = \frac{v_x}{2l} = \frac{\text{molecule velocity}}{\text{mean free path}}$ ;  $f_c = \frac{v_{rms}}{\lambda_m}$  or  $f_c = \frac{v_m}{\lambda_m}$

Hence momentum transferred to the wall per second by the molecule is equal to force applied on the wall

$$\text{Therefore, force } F = (2 m v_x) \frac{v_x}{2l} = \frac{m v_x^2}{l} = \frac{m v^2}{3l} \quad [\text{As } V = \sqrt{3} V_x]$$

$$\text{Pressure exerted by gas molecule } P = \frac{F}{A} = \frac{1}{3} \frac{m v^2}{l \times A} \Rightarrow P = \frac{1}{3} \frac{m v^2}{V} \quad [\ominus A \times l = V]$$

$$\text{Pressure exerted by gas } P = \sum \frac{1}{3} \frac{m v^2}{V} = \sum \frac{1}{3} \frac{m v^2}{V} \times \frac{N}{N} = \frac{1}{3} \frac{m N}{V} \frac{\sum v^2}{N} = \frac{1}{3} \frac{m N}{V} v_{rms}^2$$

$$v_{rms}^2 = \frac{3PV}{M} = \frac{3\mu RT}{\mu M_w} \Rightarrow v_{rms} = \sqrt{\frac{3RT}{M_w}}, P = \frac{1}{3} \frac{M}{V} v_{rms}^2 = \frac{1}{3} \rho v_{rms}^2$$

- Average number of molecules for each wall =  $\frac{N}{6}$ .

$$\text{No. of molecules along each axis} = \frac{N}{3} \quad (N_x = N_y = N_z)$$

- $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \frac{v_{rms}^2}{3}$  Root mean square velocity along any axis for gas molecule is

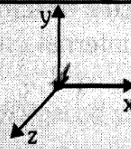
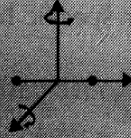
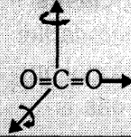
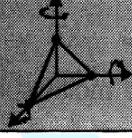
$$(v_{rms})_x = (v_{rms})_y = (v_{rms})_z = \frac{v_{rms}}{\sqrt{3}}$$

All gas laws and gas equation can be obtained by expression of pressure of gas (except Joule's law)

### Degree of freedom (f)

- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called its degrees of freedom.
- The number of independent coordinates required to specify the dynamic state of a system is called its degrees of freedom.
- The degrees of freedom are of three types :
  - (a) Translational Degree of freedom :** There are maximum three degree of freedom corresponding to translational motion.
  - (b) Rotational Degree of freedom :** The number of degrees of freedom in the case depends on the structure of the molecule.
  - (c) Vibrational Degree of freedom :** It is exhibited at high temperature.

**Degree of freedom for different gases according to atomicity of gas at low temperature**

Atomicity of gas	Translational	Rotational	Total	
Monoatomic Ex. Ar, Ne, Ideal gas etc	3	0	3	
Diatomic Ex. O <sub>2</sub> , Cl <sub>2</sub> , N <sub>2</sub> etc.	3	2	5	
Triatomic (linear) Ex. CO <sub>2</sub> , C <sub>2</sub> H <sub>2</sub>	3	2	5	
Triatomic (Non-linear) or Polyatomic Ex. H <sub>2</sub> O, NH <sub>3</sub> , CH <sub>4</sub>	3	3	6	

At high temperature a diatomic molecule has 7 degrees of freedom.  
(3 translational, 2 rotational and 2 vibrational)

### Maxwell's law of equipartition of energy

The total kinetic energy of a gas molecules is equally distributed among its all degree of freedom and the energy associated with each degree of freedom at absolute temperature T is  $\frac{1}{2} kT$

### For on molecule of gas

Energy related with each degree of freedom =  $\frac{1}{2} kT$

Energy related with all degree of freedom =  $\frac{f}{2} kT$     Q  $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \frac{v_{rms}^2}{3} \Rightarrow \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$

So energy related with one degree of freedom =  $\frac{1}{2} m \frac{v_{rms}^2}{3} = \frac{3}{2} \frac{kT}{3} = \frac{1}{2} kT$

### Different K.E. of gas (Internal Energy)

- **Translational kinetic energy ( $E_T$ )**  $E_T = \frac{1}{2} M v_{rms}^2 = \frac{3}{2} PV$

Kinetic energy of volume V is  $\frac{1}{2} M v_{rms}^2$

[Note : Total internal energy of ideal gas is total kinetic energy]

- **Energy per unit volume or energy density ( $E_V$ )**

$$E_V = \frac{\text{Total energy}}{\text{Volume}} = \frac{E}{V}; E_V = \frac{1}{2} \left[ \frac{M}{V} \right] v_{rms}^2 = \frac{1}{2} \rho v_{rms}^2$$

$$\Theta P = \frac{3}{2} \left[ \frac{1}{2} \rho v_{rms}^2 \right] \therefore E_V = \frac{3}{2} P$$

- **Molar K.E. or Mean Molar K.E. (E)**

$$E = \frac{1}{2} M_w v_{rms}^2 \text{ for } N_0 \text{ molecules or } M_w \text{ (gram)}$$

$$E = \frac{3}{2} RT = \frac{3}{2} N_0 kT$$

- Molecular kinetic energy or mean molecular K.E. ( $\bar{E}$ )**

$$E = \frac{1}{2} M_w v_{rms}^2, \quad \bar{E} = \frac{E}{N_0} = \frac{3}{2} \frac{RT}{N_0} = \frac{3}{2} kT$$

- Kinetic energy of 1 g mass ( $E_m$ )**  $= \frac{3}{2} \frac{RT}{M_w} = \frac{3}{2} \frac{N_0 \times kT}{N_0 m} = \frac{3}{2} \frac{kT}{m}$

## SPECIFIC HEAT CAPACITY

### Monoatomic Gases

The molecule of a monoatomic gas has only three translational degrees of freedom. Thus, the average energy of a molecule at temperature T is  $(3/2)kT$ .

Total internal energy of a mole of such a gas is

$$U = \frac{3}{2} kT \times N_A = \frac{3}{2} RT$$

The molar specific heat at constant volume,  $C_v$ , is

$$C_v \text{ (monoatomic gas)} = \frac{dU}{dT} = \frac{3}{2} R$$

### Diatomic Gases

As explained earlier, a diatomic molecule treated as a rigid rotator like a dumbbell has 5 degrees of freedom : 3 translational and 2 rotational. Using the law of equipartition of energy, the total internal energy of a mole of such a gas is

$$U = \frac{5}{2} kT \times N_A = \frac{5}{2} RT$$

The molar specific heats are then given by

$$C_v \text{ (rigid diatomic)} = \frac{5}{2} R$$

## MEAN FREE PATH

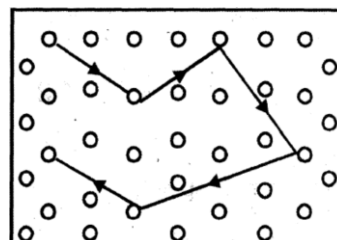
A molecule in its path undergoes a number of collisions so the path traversed by it is not a straight line but somewhat zigzag. Between two successive collisions a molecule travels in a straight line with uniform velocity. As motion is random thus the distance travelled by molecule between two successive collisions is not always equal.

The average distance travelled by a molecule between two successive collisions is called as mean free path ( $\lambda_m$ ) of the molecule.

$$\lambda_m = \frac{1}{\sqrt{2} \pi d^2 n}$$

Here  $n = \frac{N}{V}$  = No. of molecules per unit volume.

d = diameter of a molecule



## GOLDEN KEY POINTS



- At a given temperature graph drawn between molecular velocity and number of molecules is known as velocity distribution curve.
- The velocities of molecules of a gas are in between zero and infinity ( $0 - \infty$ )
- With the increase in the temperature, the most probable velocity and maximum molecule velocity both increases.
- The number of molecules within certain velocity range is constant although the velocity of molecule changes continuously at particular temperature.
- The area enclosed between the ( $N - v$ ) curve and the velocity axis represents the total number of molecules.

On the basis of velocity distribution Maxwell established the law of equipartition of energy for gases at any temperature.

- Except 0 K, at any temperature  $T$ ,  $E > E_m > \bar{E}$
- At a common temperature, for all ideal gas  
 $E$  and  $\bar{E}$  are same while  $E_m$  is different and depends upon nature of gas ( $M_w$  or  $m$ )

- For thermal equilibrium of gases, temperature of each gas is same and this temperature called as temperature of mixture ( $T_m$ ) which can be find out on basis of conservation of energy (All gases are of same atomicity).

$$T_m = \frac{\sum NT}{\sum N} = \frac{N_1 T_1 + N_2 T_2 + \dots + N_n T_n}{N_1 + N_2 + \dots + N_n}$$

- **1 mole gas :** Mean translational kinetic energy =  $\frac{3}{2} RT$  ; Total kinetic energy =  $\frac{f}{2} RT$

**1 molecule of gases :** Mean translational kinetic energy =  $\frac{3}{2} kT$  ; Total kinetic energy =  $\frac{f}{2} kT$ ;

$f \rightarrow$  Degree of freedom

- Specific heats of all substances approach zero as  $T \rightarrow 0$ . This is related to the fact that degrees of freedom get frozen and ineffective at low temperatures .

## Illustrations

### Illustration 48.

The velocities of ten particles in  $\text{ms}^{-1}$  are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate

- (i) average speed and                      (ii) rms speed                      (iii) most probable speed.

**Solution:**

(i) average speed,  $v_{av} = \frac{0 + 2 + 3 + 4 + 4 + 4 + 5 + 5 + 6 + 9}{10}$   
 $= \frac{42}{10} = 4.2 \text{ ms}^{-1}$

(ii) rms speed,  $v_{rms} = \left[ \frac{(0)^2 + (2)^2 + (3)^2 + (4)^2 + (4)^2 + (4)^2 + (5)^2 + (5)^2 + (6)^2 + (9)^2}{10} \right]^{1/2}$   
 $= \left[ \frac{228}{10} \right]^{1/2} = 4.77 \text{ ms}^{-1}$

(iii) most probable speed  $v_{mp} = 4 \text{ m/s}$



**Illustration 49.**

At what temperature root mean square velocity of hydrogen becomes double of its value at S.T.P., pressure remaining constant ?

**Solution:**

Let  $v_1$  be the r.m.s. velocity at S.T.P. and  $v_2$  be the r.m.s. velocity at unknown temperature  $T_2$ .

$$\therefore \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2}$$

$$\text{or } T_2 = T_1 \left[ \frac{v_2}{v_1} \right]^2 = 273 \times (2)^2 = 273 \times 4 = 1092 \text{ K} = (1092 - 273) = 819^\circ\text{C}$$

**Illustration 50.**

Calculate rms velocity of oxygen molecule at  $27^\circ\text{C}$

**Solution:**

Temperature,  $T = 27^\circ\text{C}$

$$\Rightarrow 273 + 27 = 300 \text{ K.}$$

Molecular weight of oxygen =  $32 \times 10^{-3} \text{ kg}$  and  $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

$$\text{rms velocity is } v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{32 \times 10^{-3}}} = 483.5 \text{ ms}^{-1}$$

**Illustration 51.**

Calculate the kinetic energy of a gram molecule of argon at  $127^\circ\text{C}$ .

**Solution:**

Temperature,  $T = 127^\circ\text{C} = 273 + 127 = 400 \text{ K}$ ,  $R = 8.31 \text{ J/mol K}$

$$\text{K.E. per gram molecule of argon} = \frac{3}{2} R T = \frac{3}{2} \times 8.31 \times 400 = 4986 \text{ J}$$

**Illustration 52.**

The mass of a hydrogen molecule is  $3.32 \times 10^{-27} \text{ kg}$ . If  $10^{23}$  molecules are colliding per second on a stationary wall of area  $2 \text{ cm}^2$  at an angle of  $45^\circ$  to the normal to the wall and reflected elastically with a speed  $10^3 \text{ m/s}$ . Find the pressure exerted on the wall. (in  $\text{N/m}^2$ )

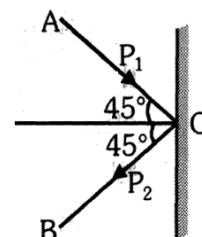
**Solution:**

As the impact is elastic  $\therefore |\vec{p}_1| = |\vec{p}_2| = p = mv = 3.32 \times 10^{-24} \text{ kg m/s}$

The change in momentum along the normal  $\Delta p = |\vec{p}_1 - \vec{p}_2| = 2p \cos 45^\circ = \sqrt{2}p$

If  $f$  is the collision frequency then force applied on the wall  $F = \frac{\Delta p}{\Delta t} = \Delta p \times f = \sqrt{2}pf$

$$\therefore \text{Pressure } P = \frac{F}{A} = \frac{\sqrt{2}pf}{A} = \frac{\sqrt{2} \times 3.32 \times 10^{-24} \times 10^{23}}{2 \times 10^{-4}} = 2.347 \times 10^3 \text{ N/m}^2$$

**BEGINNER'S BOX - 6**

- Given : Avogadro number  $N = 6.02 \times 10^{23}$  molecules/mole and Boltzmann's constant  $k = 1.38 \times 10^{-23} \text{ J/(molecule-K)}$ . Calculate.:

- (a) The average kinetic energy of translation of an oxygen molecule at  $27^{\circ}\text{C}$
  - (b) The total kinetic energy of an oxygen molecule at  $27^{\circ}\text{C}$
  - (c) The total kinetic energy of 1 mole of oxygen gas at  $27^{\circ}\text{C}$ .
2. The temperature of gas is  $-73^{\circ}\text{C}$  To what temperature should it be heated so that
    - (a) average kinetic energy of 1 molecule is doubled ?
    - (b) the root mean square velocity of the molecules is doubled?
  3. For a gas  $\frac{R}{C_p} = 0.4$ . For this gas calculate the following -
    - (a) atomicity and degree of freedom
    - (b) value of  $C_v$  and  $\gamma$
    - (c) mean gram – molecular kinetic energy at 300 K temperature
  4. If three molecules are having speeds  $v_1$ ,  $v_2$  and  $v_3$  respectively, then what will be their average speed and root mean square speed ?
  5. Four molecules of a gas are having speeds of 1, 4, 8 and  $16 \text{ ms}^{-1}$ . Find the root mean square velocity of the gas molecules ?
  6. A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is  $27^{\circ}\text{C}$  Obtain the ratio of
    - (a) Average translational kinetic energy per molecule, and
    - (b) root mean square speed ( $v_{\text{rms}}$ ) of the molecules of the two gases.
 Atomic mass of argon = 39.9 u :  
 Molecular mass of chlorine = 70.9 u.

## THERMODYNAMICS

Branch of physics which deals with the inter-conversion between heat energy and any other form of energy is known as thermodynamics. In this branch of physics we deal with the processes involving heat, work and internal energy. In this branch of science the conversion of heat into mechanical work and vice versa is studied.

- **Thermodynamic System**  
 The system which can be represented in terms of pressure (P), volume (V) and temperature (T), is known as thermodynamic system. A specified portion of matter consisting of one or more substances on which the effects of thermodynamic variables such as temperature, volume and pressure are to be studied, is called a system.  
 e.g. A gas enclosed in a cylinder fitted with a piston is a system.
- **Surrounding**  
 Anything outside the system, which exchanges energy with the system and which tends to change the properties of the system is called its surrounding.
- **Heterogeneous System**  
 A system which is not uniform throughout is said to be heterogeneous. e.g. A system consisting of two or more immiscible liquids.
- **Homogeneous System**

A system is said to be homogeneous if it is completely uniform throughout. e.g. Pure solid or liquid.

- **Isolated System**

A system in which no exchange of matter and energy with the surrounding take place, is said to be an isolated system.

- **Universe**

The system and Its surrounding are together known as the universe.

- **Thermodynamic variables of the system**

(i) Composition ( $\mu$ )	(ii) Temperature (T)	(iii) Volume (V)
(iv) Pressure (P)	(v) Mass	

- **Thermodynamic state**

The state of a system can be described completely by composition temperature volume and pressure. If a system is homogeneous and has definite mass and composition, then state of the system can be described by the remaining three variables namely temperature, pressure and volume. These variables are inter related by equation  $PV = \mu RT$ . The thermodynamic state of the system is its condition as identified by two independent thermodynamic variables (P, V or P, T or V, T).

- **Internal Energy**

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration. The energy due to molecular motion is called internal kinetic energy ( $U_k$ ) and that due to molecular configuration is called internal potential energy ( $U_p$ ).

$$dU = dU_k + dU_p$$

If there no intermolecular forces, then  $dU_p = 0$  and  $dU = dU_k = m c_v dT$  [ideal gas]

$c_v$  = Specific heat at constant volume and  $dT$  = Infinitesimal change in temperature

$m$  = Mass of system

$M$  = Molecular weight

Molar heat capacity  $C_v = Mc_v$  For  $\mu$ -moles of ideal gas  $dU = \mu C_v dT = \frac{m}{M} C_v dT$

Internal energy in the absence of inter-molecular forces is simply the function of temperature and state only,

it is independent of path followed.

$$\Delta U = U_f - U_i$$

$U_i$  = Internal energies in initial state and  $U_f$  = Internal energies in final state

- **Internal Energy**

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration. The energy due to molecular motion is called internal kinetic energy ( $U_k$ ) and that due to molecular configuration is called internal potential energy ( $U_p$ )

$$dU = dU_k + dU_p$$

If there no intermolecular forces, then  $dU_p = 0$  and  $dU = dU_k = m c_v dT$  [ideal gas] i

$c_v$  = Specific heat at constant volume and  $dT$  = Infinitesimal change in temperature

$m$  = Mass of system

$M$  = Molecular weight

molar heat capacity  $C_v = Mc_v$  For  $\mu$ -moles of ideal gas  $dU = \mu C_v dT = \frac{m}{M} C_v dT$

Internal energy in the absence of inter-molecular forces is simply the function of temperature and state only,

it is independent of path followed.

$$\Delta U = U_f - U_i$$

$U_i$  = Internal energies in initial state and  $U_f$  = Internal energies in final state

- **Thermodynamic Processes**

In the thermodynamic process pressure, volume, temperature and entropy of the system change with time. Thermodynamic process is said to take place if change occurs in the state of a thermodynamic system.

- **Sign convention used for the study of thermodynamic processes**

Heat gained by a system Positive

Positive

Heat lost by a system

Negative

Work done by a system

Positive

Work done on the system

Negative

Increase in the internal energy of system

Positive

Decrease in the internal energy of system

Negative

- **Indicator Diagram or P–V Diagram**

In the equation of state of a gas  $PV = \mu RT$  two thermodynamic variables are sufficient to describe the behavior of a thermodynamic system.

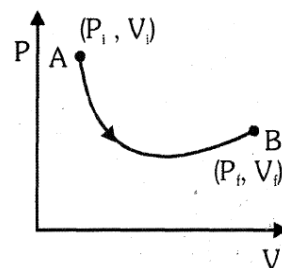
If any two of the three variables  $P$ ,  $V$  and  $T$  are known then the third can be calculated.

P–V diagram is a graph between the volume  $V$  and the pressure  $P$  of the system.

The volume is plotted against X-axis while the pressure is plotted against Y-axis.

The point A represents the initial stage of the system. Initial pressure of the system is  $P_i$  and initial volume of the system  $V_i$ .

The point B represents the final state of the system.  $P_f$  and  $V_f$  are the final pressure and final volume respectively of the system. The points between A and B represent the intermediate states of the system. With the help of the indicator diagram we calculate the amount of work done by the gas or on the gas during expansion or compression.



- **Cyclic process**

Cyclic process is a thermodynamic process in which the system returns to its initial stage after undergoing a series of changes.

- **Non-cyclic process**

Non-cyclic process is a process in which the system does not return to its initial stage.

- **Quasi-static or equilibrium process**

Quasi-static is a thermodynamic process which proceeds extremely slowly such that at every instant of time, the temperature and pressure are the same in all parts of the system.

- Reversible and Irreversible processes**

A reversible process is one in which the changes in heat and work of direct process from initial to a final state are exactly retraced in opposite sense in the reverse process and the system and surroundings are left in their initial states. The reversibility is an ideal concept and can not realized in practice.

The process which is not reversible is the irreversible process. In nature the processes are irreversible.

**Work done by thermodynamic system**

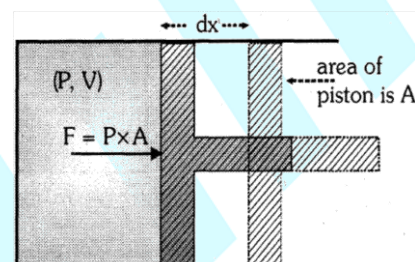
One of the simple example of a thermodynamic system is gas in a cylinder with a movable piston.

- If the gas expands against the piston**

Gas exerts a force on the piston and displace it through a distance and does work on the piston.

- If the piston compresses the gas**

When piston moved inward, work is done on the gas.



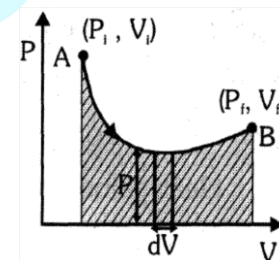
- The work associated with volume changes**

If pressure of gas on the piston =  $P$ .

Then the force on the piston due to gas is  $F = PA$

When the piston is pushed outward an infinitesimal distance  $dx$ , the work done by the gas is  $dW = F \times dx = PA \, dx$

The change in volume of the gas is  $dV = A \, dx$ ,  $\therefore dW = P \, dV$



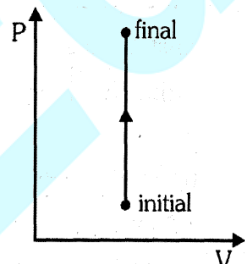
For a finite change in volume from  $V_i$  to  $V_f$ , this equation is then integrated between

$$V_i \text{ to } V_f \text{ to find the net work done } W = \int_{V_i}^{V_f} dW = \int_{V_i}^{V_f} P \, dV$$

Hence the work done by a gas is equal to the area under  $P$ - $V$  graph.

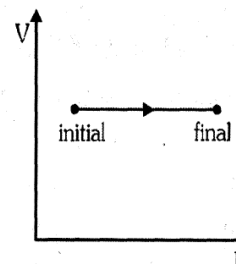
Following different cases are possible.

**(i) Volume is constant**



$V = \text{constant}$

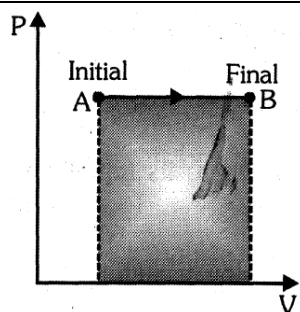
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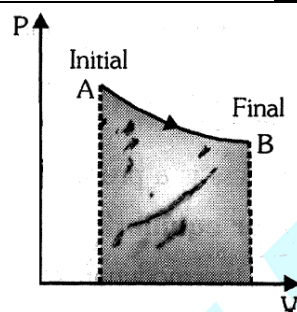
$W = 0$

**(ii) Volume is increasing**

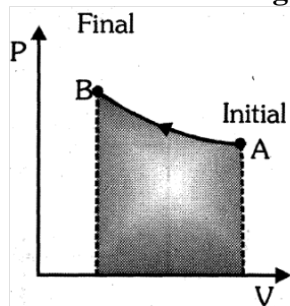




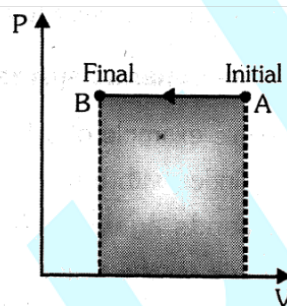
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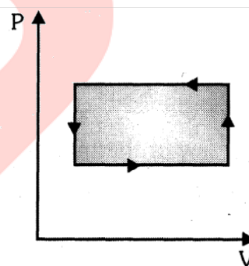
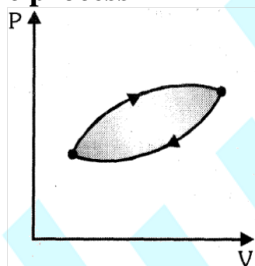
## (iii) Volume is decreasing



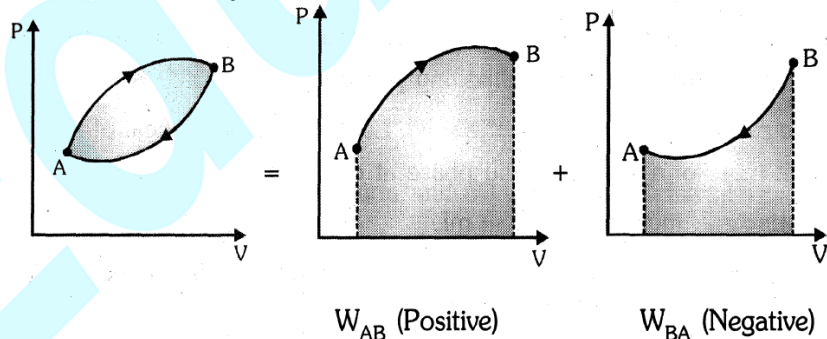
or



## (iv) Cyclic process



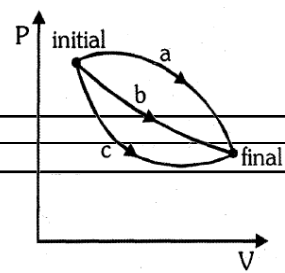
## Work Done in Clockwise Cycle



$$W_{\text{cyclic}} = W_{AB} \text{ (positive)} + W_{AB} \text{ (negative)} =$$



## First law of thermodynamics



Let a gas in a cylinder with a movable piston changes from an initial equilibrium state to a final equilibrium state.

System change its state through path 'a' :

The heat absorbed by the system in this process =  $\delta Q_a$

The work done by the system =  $\delta W_a$

**Again for path 'b' :**

Heat absorbed by the system =  $\delta Q_b$ , Work done by the system =  $\delta W_b$ .

It is experimental fact that the  $\delta Q_a - \delta W_a = \delta Q_b - \delta W_b$

Both  $\delta Q$  and  $\delta W$  depend on the thermodynamic path taken between two equilibrium states, but difference

$(\delta Q - \delta W)$  does not depend on path in between two definite states of the system.

So, there is a function (internal energy) of the thermodynamic coordinates (P, V and T) whose final value ( $U_f$ ) minus its initial value ( $U_i$ ) equals the change  $\delta Q - \delta W$  in the process.

**$dU = \delta Q - \delta W$ .** This is the first law of thermodynamics.

Heat supplied to the system and work done by the system are path dependent so they are denoted by  $\delta Q$  and  $\delta W$  respectively. Change in internal energy  $\Delta U = U_f - U_i$  does not depend on path it depends only on initial and final positions of the system. So, it is denoted by  $dU$  (or  $\Delta U$ )

### First Law of Thermodynamics

If some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system and the external work done by the system.  $\delta Q = dU + \delta W$  or  $Q = W + \Delta U$

- This law is applicable to every process in nature
- The first law of thermodynamics introduces the concept of internal energy.
- The first law of thermodynamics is based on the law of conservation of energy.
- $\delta Q$ ,  $dU$  and  $\delta W$  must be expressed in the same units (either in units of work or in units of heat).
- This law is applicable to all the three phases of matter, i.e., solid, liquid and gas.
- $dU$  is a characteristic of the state of a system, it may be any type of internal energy- translational kinetic energy, vibrational, rotational kinetic energy, binding energy etc.

### Application of First Law of Thermodynamics

#### • Melting Process :

When a substance melts, the change in volume ( $dV$ ) is very small and can, therefore, be neglected. The temperature of a substance remains unchanged during melting process.

Let us consider the melting of a mass  $m$  of the solid. Let  $L$  be the latent heat of fusion i.e., the heat required  $L$  to change a unit mass of a solid to liquid phase at constant temperature.

Heat absorbed during melting process,  $Q = mL$

By the first law of thermodynamics,  $Q = \Delta U + W = \Delta U + P\Delta V$ , where  $\Delta V$  is volume change during phase change.

#### • Boiling Process :

When a liquid is heated, it changes into vapour at constant temperature (called boiling point) and pressure. When water is heated at normal atmospheric pressure, it boils at  $100^\circ\text{C}$ . The temperature remains unchanged during the boiling process. Let us consider the vaporisation of liquid of mass  $m$ . Let  $V_\lambda$  and  $V_v$  be the volumes of the liquid and vapours respectively.

The work done in expanding at constant temperature and pressure  $P$ ,  $\delta W = P\Delta V = P(V_v - V_\lambda)$

Let the latent heat of vaporisation =  $L$   $\therefore$  Heat absorbed during boiling process,  $\delta Q = mL$

Let  $U_\lambda$  and  $U_v$  be the internal energies of the liquid and vapours respectively then  $\Delta U = U_v - U_\lambda$

According to the first law of thermodynamics,  $\delta Q = \Delta U + \delta W$   $\therefore mL = (U_v - U_\lambda) + P(V_v - V_\lambda)$

## Illustrations

### Illustration 53.

The pressure of one mole monoatomic gas increases linearly from  $4 \times 10^5 \text{ Nm}^{-2}$  to  $8 \times 10^5 \text{ Nm}^{-2}$  when its volume increases from  $0.2 \text{ m}^3$  to  $0.5 \text{ m}^3$ . Calculate.

- (i) Work done by the gas, (ii) Increase in the internal energy,  
(iii) Amount of heat supplied, (iv) Molar heat capacity of the gas  
 $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

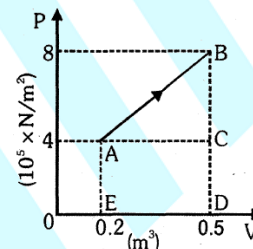
### Solution:

$$P_1 = 4 \times 10^5 \text{ Nm}^{-2} \quad P_2 = 8 \times 10^5 \text{ Nm}^{-2}, \quad V_1 = 0.2 \text{ m}^3, \quad V_2 = 0.5 \text{ m}^3$$

- (i) Work done by the gas = Area under P-V graph (Area ABCDEA)

$$= \frac{1}{2} (AE + BD) \times AC = \frac{1}{2} (4 \times 10^5 + 8 \times 10^5) \times (0.5 - 0.2)$$

$$= \frac{1}{2} \times 12 \times 10^5 \times 0.3 = 1.8 \times 10^5 \text{ J}$$



- (ii) Increase in internal energy  $\Delta U = C_v (T_2 - T_1) = \frac{C_v}{R} (P_2 V_2 - P_1 V_1)$

$$\text{For monoatomic gas } C_v = \frac{3}{2} R$$

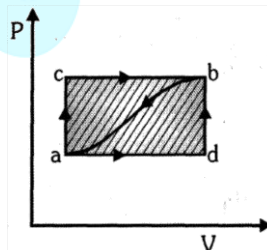
$$\therefore \Delta U = \frac{3}{2} [(8 \times 10^5 \times 0.5) - (4 \times 10^5 \times 0.2)] = \frac{3}{2} [4 \times 10^5 - 0.8 \times 10^5] = 4.8 \times 10^5 \text{ J}$$

- (iii)  $Q = \Delta U + W = 4.8 \times 10^5 + 1.8 \times 10^5 = 6.6 \times 10^5 \text{ J}$

- (iv)  $C = \frac{Q}{\mu \Delta T} = \frac{QR}{\mu R \Delta T} = \frac{QR}{(P_2 V_2 - P_1 V_1)} = \frac{6.6 \times 10^5 \times 8.31}{1 \times 3.2 \times 10^5} = 17.14 \text{ J/mol K}$

### Illustration 54.

As shown in figure when a system is taken from state a to state b, along the path



$A \rightarrow c \rightarrow b$ , 60 J of heat flow into the system and 30 J of work is done :

- (i) How much heat flows into the system along the path  $a \rightarrow d \rightarrow b$  if the work is 10 J.  
(ii) When the system is returned from b to a along the curved path, the work done by the system is -20 J. Does the system absorb or liberate heat and how much ?  
(iii) If,  $U_a = 0$  and  $U_d = 22 \text{ J}$ , find the heat absorbed in the process  $a \rightarrow d$  and  $d \rightarrow b$ .

### Solution:

$$\text{For the path abc} \quad \Delta U = Q - W = 60 - 30 = 30 \text{ J or } U_b - U_a = 30 \text{ J}$$

- (i) Along the path adb  $Q = \Delta U + W = 30 + 10 = 40 \text{ J}$

- (ii) Along the curved path ba  $Q = (U_a - U_b) + W = (-30) + (-20) = -50$ , heat liberates from system

$$(iii) Q_{ad} = U_d - U_a + W_{ad}$$

$$\text{but } W_{ad} = W_{adb} - W_{db} = 10 - 0 = 10 \text{ Hence}$$

$$Q_{ad} = 22 - 0 + 10 = 32 \text{ J}$$

$$\text{and } Q_{db} = U_b - U_d + W_{db} = 30 - 22 + 0 = 8 \text{ J}$$

### Illustration 55.

1 kg of water at 373 K is converted into steam at same temperature. Volume of 1 cm<sup>3</sup> of water becomes 1671 cm<sup>3</sup> on boiling. What is the change in the internal energy of the system if the latent heat of vaporization of water is  $5.4 \times 10^5 \text{ cal kg}^{-1}$ ?

### Solution:

Volume of 1 kg of water = 1000 cm<sup>3</sup> =  $10^{-3} \text{ m}^3$ , Volume of 1 kg of steam =  $10^3 \times 1671 \text{ cm}^3 = 1.671 \text{ m}^3$

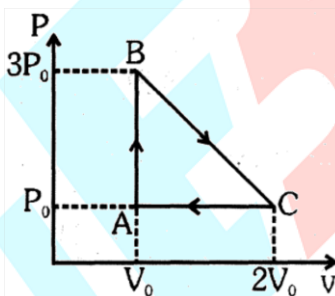
Change in volume,  $\Delta V = (1.671 - 10^{-3}) \text{ m}^3 = 1.670 \text{ m}^3$ , Pressure,  $P = 1 \text{ atm.} = 1.01 \times 10^5 \text{ N m}^{-2}$

In expansion work done,  $W = P\Delta V = 1.01 \times 10^5 \times 1.67 \text{ J} = \frac{1.686 \times 10^5}{4.2} \text{ cal} = 4.015 \times 10^4 \text{ cal}$

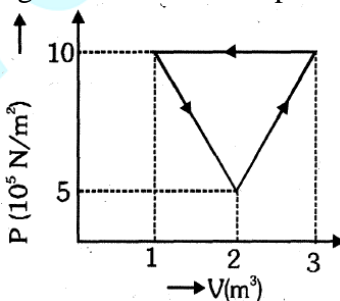
But  $\Delta U = Q - W$  (first law of thermodynamics) or  $\Delta U = (5.4 \times 10^5 - 0.4015 \times 10^5) \text{ cal} = 4.9985 \times 10^5 \text{ cal}$

### BEGINNER'S BOX - 7

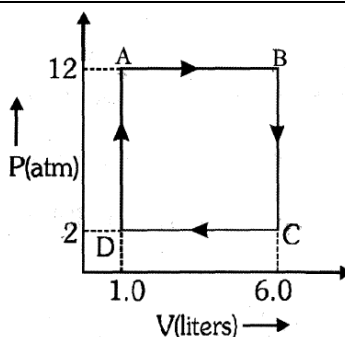
- One mole of an ideal monoatomic gas is taken round the cyclic process ABCA as shown in fig. calculate



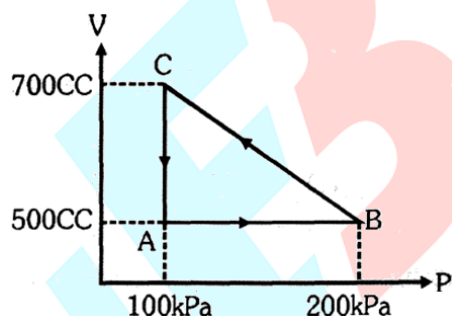
- The work done by the gas.
  - The heat rejected by the gas in the path CA and heat absorbed in the path AB.
  - The net heat absorbed by the gas in the path BC.
- Calculate the heat supplied in figure shown for complete cycle.



- Given figure shows a P-V graph of the thermodynamic behaviour of an ideal gas. Find out work in the processes  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $C \rightarrow D$  and  $D \rightarrow A$  from this graph.



4. At 8 atm pressure a monoatomic, gas expands from 2 litre to 5 litre then calculate the following
- Work done by the gas
  - Increase in internal energy
  - Amount of heat supplied
5. A gas is taken through a cyclic process ABCA as shown in figure. If 2.4 cal of heat is given in the process. Calculate the value of J.



## 6. DIFFERENT PROCESSES

### 6.1 Isometric or Isochoric Process

Isochoric process is a thermodynamic process that takes place at constant volume of the system, but pressure and temperature varies for change in state of the system.

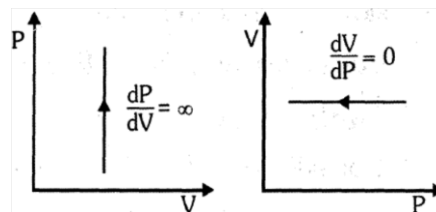
**Equation of state**  $P/T = \text{constant}$  ( $P$  and  $T$  are variable,  $V$  is constant)

**Work done** In this process volume remains constant so  $dV = 0 \Rightarrow W = \int_{V_i}^{V_f} P dV = 0$

**Form of first Law**  $Q = \Delta U$

It means whole of the heat supplied is utilized for change in internal energy of the system  $Q = \Delta U = \mu C_v \Delta T$

**Slope of the P-V curve**  $\frac{dP}{dV} = \infty$



### Specific heat or molar heat capacity at constant volume ( $C_v$ )

The quantity of heat required to raise the temperature of 1 gram mole of gas through  $1^\circ\text{C}$  at constant volume is equal to the specific heat at constant volume.

- A gas enclosed in a cylinder having rigid walls and a fixed piston. When heat is added to the gas, there would be no change in the volume of the gas.

- When a substance melts, the change in volume is negligibly small. So, this may be regarded as a nearly isochoric process.
- Heating process in pressure cooker is an example of isometric process.

## 6.2 Isobaric Process

Isobaric process is a thermodynamic process that takes place at constant pressure, but volume and temperature varies for change in state of the system.

- **Equation of state**  $V/T = \text{Constant}$  or  $V \propto T$
- **Work done** In this process pressure remain constant  $\therefore dP = 0$

$$\text{Work done } W = \int_{V_i}^{V_f} P dV = P(V_f - V_i)$$

$\Rightarrow$  Fraction of Heat given which –

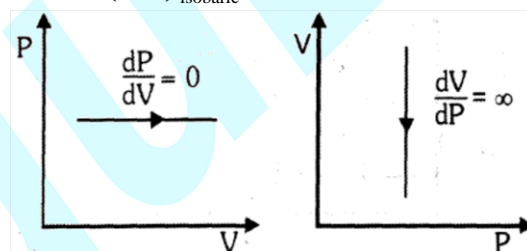
- (i) is converted into internal energy  $\frac{\Delta U}{\Delta Q} = \frac{1}{\gamma}$
- (ii) does work against external pressure  $\frac{\Delta W}{\Delta Q} = 1 - \frac{1}{\gamma}$

- **Form of first Law**  $Q = \Delta U + P(V_f - V_i)$   
 $\mu C_p (T_f - T_i) = \mu C_v (T_f - T_i) + P(V_f - V_i)$

It is clear that heat supplied to the system is utilized for :

- (i) Increasing internal energy and (ii) Work done against the surrounding atmosphere.

- **Slope of the PV curve :**  $\left(\frac{dP}{dV}\right)_{\text{isobaric}} = 0$



- **Specific heat at constant pressure ( $C_p$ )**  
 The quantity of heat required to raise the temperature of 1 gram mole gas through  $1^\circ\text{C}$  at constant pressure is equal to the specific heat.  
 Ex. Heating of water at atmospheric pressure.
  - Melting of solids and boiling of liquids at atmospheric pressure.

## 6.3 Isothermal Process

In this process pressure and volume of system change but temperature remains constant. In an isothermal process, the exchange of heat between the system and the surroundings is allowed. Isothermal process is carried out by either supplying heat to the substance or by extracting heat from it. A process has to be extremely slow to be isothermal.

- **Equation of state :**  $PV = \text{constant}$  ( $\mu RT$ ) ( $T$  is constant)



### • Work Done

Consider  $\mu$  moles of an ideal gas, enclosed in a cylinder, at absolute temperature  $T$ , fitted with a frictionless piston. Suppose that gas undergoes an isothermal expansion from the initial state  $(P_1, V_1)$  to the final state  $(P_2, V_2)$ . Work done :  $W = \int_{V_1}^{V_2} PdV$   $\therefore PV = \mu RT$  then  $P = \frac{\mu RT}{V}$

$$\therefore W = \int_{V_1}^{V_2} \frac{\mu RT}{V} dV = \mu RT \int_{V_1}^{V_2} \frac{dV}{V} = \mu RT [\log_e V]_{V_1}^{V_2} = \mu RT [\log_e V_2 - \log_e V_1] = \mu RT \log_e \left[ \frac{V_2}{V_1} \right]$$

$$\Rightarrow W = 2.303 \mu RT \log_{10} \left[ \frac{P_1}{P_2} \right] \quad [ \text{ } \ominus P_1 V_1 = P_2 V_2 ]$$

### Form of First Law

There is no change in temperature and internal energy of the system depends on temperature only

So  $\Delta U = 0, Q = 2.303 \mu RT \log_{10} \left[ \frac{V_2}{V_1} \right]$

It is clear that Whole of the heat energy supplied to the system is utilized by the system in doing external work. There is no change in the internal energy of the system.

### Slope of the isothermal curve

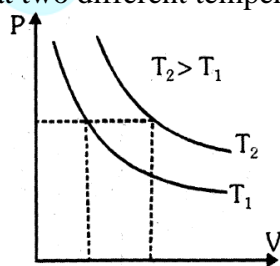
For an isothermal process,  $PV = \text{constant}$

Differentiating,  $PdV + VdP = 0 \Rightarrow VdP = -PdV \Rightarrow \frac{dP}{dV} = -\frac{P}{V}$

Slope of isothermal curve,  $\left[ \frac{dP}{dV} \right]_{\text{isothermal}} = -\frac{P}{V}$

### For a given system :

- The product of the pressure and volume of a given mass of a perfect gas remains constant in an isothermal process.
- Boyle's law is obeyed in an isothermal process .
- A graph between pressure and volume of a given mass of a gas at constant temperature is known as isotherm or isothermal curve of the gas.
- Two isotherms for a given gas at two different temperatures  $T_1$  and  $T_2$  are shown in figure.



at const  $P$ ;  $V \propto T$  so  $T_2 > T_1$

- The curves drawn for the same gas at different temperatures are mutually parallel and do not cut each other.
- If two isotherms intersect each other at a single point we get same value of  $P$  and  $V$  at intersection point.  
 $PV = \mu RT_1$  for temperature  $T_1$  and  $PV = \mu RT_2$  for temperature  $T_2$ .  
 It means  $T_1 = T_2$  which is not possible.
- An ideal gas enclosed in a conducting cylinder fitted with a conducting piston.

- Let the gas be allowed to expand very-very slowly. This shall cause a very slow cooling of the gas, but heat will be conducted into the cylinder from the surrounding. Hence the temperature of the gas remains constant. If the gas is compressed very-very slowly, heat will be produced, but this heat will be conducted to the surroundings and the temperature of the gas shall remain constant.
- The temperature of a substance remains constant during melting. So, the melting process is an isothermal process.
- Boiling is an isothermal process, when a liquid boils, its temperature remains constant.
- If sudden changes are executed in a vessel of infinite conductivity then they will be isothermal.

#### 6.4 Adiabatic Process

It is that thermodynamic process in which pressure, volume and temperature of the system do change but there is no exchange of heat between the system and the surroundings.

A sudden and quick process will be adiabatic since there is no sufficient time available for exchange of heat so process adiabatic.

**Equation of state :**  $PV = \mu RT$

Equation for adiabatic process  $PV^\gamma = \text{constant}$ ,  $TV^{\gamma-1} = \text{constant}$ ,  $T^\gamma P^{1-\gamma} = \text{constant}$

#### Work done

Let initial state of system is  $(P_1, V_1, T_1)$  and after adiabatic change final state of system is  $(P_2, V_2, T_2)$  then we can write  $P_1 V_1^\gamma = P_2 V_2^\gamma = K$  (here  $K$  is const.)

$$\text{So } W = \int_{V_1}^{V_2} P dV = K \int_{V_1}^{V_2} V^{-\gamma} dV = K \left( \frac{V^{-\gamma+1}}{-\gamma+1} \right)_{V_1}^{V_2} = \frac{K}{(-\gamma+1)} [V_2^{-\gamma+1} - V_1^{-\gamma+1}] \quad (QK = P_1 V_1^\gamma = P_2 V_2^\gamma)$$

$$\Rightarrow W = \frac{1}{(\gamma-1)} [P_1 V_1^\gamma V_1^{-\gamma} \cdot V_1 - P_2 V_2^\gamma V_2^{-\gamma} \cdot V_2] = \frac{1}{(\gamma-1)} [P_1 V_1 - P_2 V_2]$$

$$\Rightarrow W = \frac{\mu R}{(\gamma-1)} (T_1 - T_2) \quad (\because PV = \mu RT)$$

**From of first law :**  $DU = -\delta W$

It means the work done by an ideal gas during adiabatic expansion (or compression) is on the cost of change in internal energy proportional to the fall (or rise) in the temperature of the gas.

If the gas expands adiabatically, work is done by the gas. So,  $W_{\text{adia}}$  is positive.

The gas cools during adiabatic expansion and  $T_1 > T_2$ .

If the gas is compressed adiabatically, work is done on the gas. So,  $W_{\text{adia}}$  is negative.

The gas heats up during adiabatic compression and  $T_1 < T_2$ .

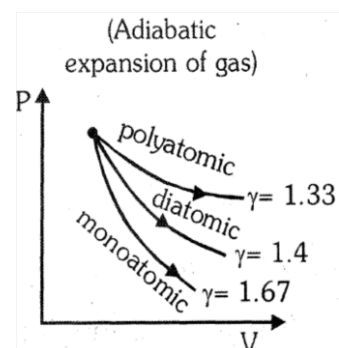
#### Slope of the adiabatic curve

For an adiabatic process,  $PV^\gamma = \text{constant}$

Differentiating,  $\gamma P V^{\gamma-1} dV + V^\gamma dP = 0$

$$\Rightarrow V^\gamma dP = -\gamma P V^{\gamma-1} dV \Rightarrow \frac{dP}{dV} = \frac{\gamma P V^{\gamma-1}}{V^\gamma} = -\gamma \frac{P}{V} = \gamma \left( -\frac{P}{V} \right)$$

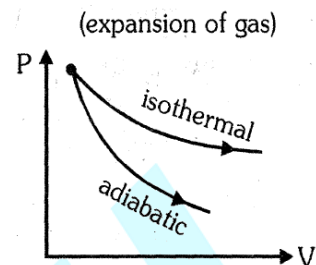
$$\text{Slope of adiabatic curve, } \left[ \frac{dP}{dV} \right]_{\text{adiabatic}} = -\frac{\gamma P}{V}$$



### Magnitude of slope of adiabatic curve is greater than the slope of isotherm

$$\left| \frac{dP}{dV} \right|_{\text{adia}} = \gamma \frac{P}{V} = \gamma \left| \frac{dP}{dV} \right|_{\text{iso}} \Rightarrow \frac{\text{slope of adiabatic changes}}{\text{slope of isothermal change}} = \gamma$$

Since  $\gamma$  is always greater than one so an adiabatic curve is steeper than an isothermal curve

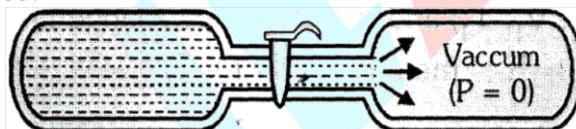


### Examples of adiabatic process

- A gas enclosed in a thermally insulated cylinder fitted with a non-conducting piston. If the gas is compressed suddenly by moving the piston downwards, some heat is produced. This heat cannot escape from the cylinder. Consequently, there will be an increase in the temperature of the gas.
- If a gas is suddenly expanded by moving the piston outwards, there will be a decrease in the temperature of the gas.
- Bursting of a cycle tube.
- Propagation of sound waves in a gaseous medium.
- In diesel engines burning of diesel without spark plug is done due to adiabatic compression of diesel vapour and air mixture

### Free Expansion

Take a thermally insulated bottle with ideal gas at some temperature  $T_1$  and by means of a pipe with a stopcock, connect this to another insulated bottle which is evacuated. If we suddenly open the stopcock, the gas will rush from the first bottle into the second until the pressures are equalized.



Experimentally, we find that this process of free expansion does not change the temperature of the gas - when the gas attains equilibrium and stops flowing, the final temperature of both bottles are equal to the initial temperature  $T_1$ . This process is called a free expansion.

The change in the internal energy of the gas can be calculated by apply the first law of thermodynamics to the free-expansion process.

The process is adiabatic because of the insulation, So  $Q = 0$ .

No part of the surroundings moves so the system does no work on its surroundings.

- **For ideal gas**

$$(\delta W)_{\text{ext.}} = \text{Work done against external atmosphere} = P dV = 0$$

$$(\because P = 0) \Rightarrow \text{because vacuum}$$

$$(\delta W)_{\text{int.}} = \text{Work done against internal molecular forces} = 0$$

$$\delta Q = dU + \delta W \Rightarrow 0 = dU + 0$$

The internal energy does not change  $dU = 0$  So  $U$  and  $T$  are constant.

The initial and final states of this gas have the same internal energy.

Which implies that the internal energy of an ideal gas does not depend on the volume at all.

The free-expansion process has led us to the following conclusion :

The internal energy  $U(T)$  of an ideal gas depends only on the temperature.

**Relation between degrees of freedom and specific heat of gas**

Energy related with each degree of freedom for a gas molecule =  $\frac{1}{2} kT$ , Energy related with all degree of freedom for a gas molecule =  $\frac{f}{2} kT$

Internal energy of one mole of ideal gas (total K.E.)  $U = \frac{f}{2} kT$  for Isometric process (volume constant)  $\delta W = 0$

By first law of thermodynamics  $\delta Q = \delta Q + dU \Rightarrow C_V dT = dU \Rightarrow C_V = \frac{dU}{dT}$

$$C_V = \frac{dU}{dT} = \frac{f}{2} R = \frac{R}{\gamma - 1}, C_P = C_V + R = \left[ \frac{f}{2} + 1 \right] R = \frac{\gamma R}{\gamma - 1} \text{ and } \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

$$C_V = \frac{R}{\gamma - 1}, C_P = \frac{\gamma R}{\gamma - 1} \text{ and } \gamma = 1 + \frac{2}{f}$$

**General expression for  $C$  ( $C_P$  or  $C_V$ ) in the process  $PV^x = \text{constant}$**   $C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$

For isobaric process  $P = \text{constant}$  so  $x = 0 \therefore C = C_P = \frac{R}{\gamma - 1} + R = C_V + R$

For isothermal process,  $PV = \text{constant}$  so  $x = 1 \therefore C = \infty$

For adiabatic process  $PV^\gamma = \text{constant}$  so  $x = \gamma \therefore C = 0$

**Values of  $f$ ,  $U$ ,  $C_V$ ,  $C_P$  and  $\gamma$  for different gases are shown in table below.**

Atomicity of gas	$f$	$C_V$	$C_P$	$\gamma$
Monoatomic	3	$\frac{3}{2} R$	$\frac{5}{2} R$	$\frac{5}{3} = 1.67$
Diatomic, Triatomic and Triatomic linear (at normal temperature)	5	$\frac{5}{2} R$	$\frac{7}{2} R$	$\frac{7}{5} = 1.4$
Poly atomic Triangular Non-linear	6	$\frac{6}{2} R = 3R$	$\frac{8}{2} R = 4R$	$\frac{4}{3} = 1.33$

- $1 < \gamma < 2$
- If atomicity of gases is same  $U$ ,  $C_P$ ,  $C_V$  and  $\gamma$  is same for gas mixture.
- If in a gas mixture gases are of different atomicity, then for gas mixture  $\gamma$  changes according to following condition. Diatomic  $\gamma_1 \leq \text{mixture} \leq \gamma_2$  mono atomic where  $\gamma_1 < \gamma_2$ .
- If ' $f$ ' is the degree of freedom per molecule for a gas, then
  - \* Total energy of each molecule =  $\frac{fkT}{2}$
  - \* Total energy per mole of gas =  $N_0 \frac{f}{2} kT = \frac{f}{2} RT$
- According to kinetic theory of gases, the molecule are not interacting with each other. So potential energy is zero and internal energy of gas molecules is only their kinetic energy.
- For ' $\mu$ ' mole of a gas : Internal energy at temperature  $T$  is  $U = \frac{\mu f RT}{2} = \mu C_V T$

- Change in internal energy is given by  $dU = \frac{\mu f R}{2} (dT) = \mu C_V dT$ . This change is process independent.

### **$C_P$ is greater than $C_V$**

If a gas is heated at constant volume, the gas does no work against external pressure. In this case, the whole of the heat energy supplied to the gas is spent in raising the temperature of the gas.

If a gas is heated at constant pressure, its volume increases. In this case, heat energy is required for the following two purposes :

- to increase the volume of the gas against external pressure.
- To increase the temperature of 1 mole of gas through 1 K.

Thus, more heat energy is required to raise the temperature of 1 mole of gas through 1 K when it is heated at constant pressure than when it is heated at constant volume.  $\therefore C_P > C_V$

The difference between  $C_P$  and  $C_V$  is equal to thermal equivalent of the work done by the gas in expanding against external pressure.

**Mayer's formula :**  $C_P - C_V = R$

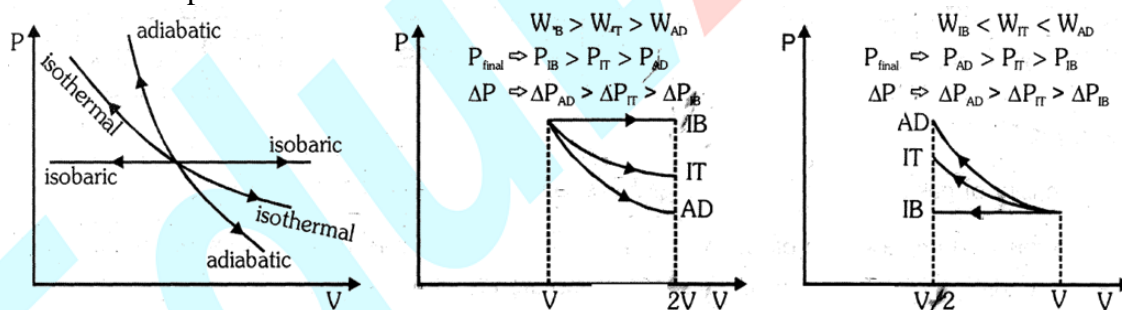
At constant pressure  $dQ = \mu C_P dT$ ,  $dU = \mu C_V dT$  &  $dW = PdV = \mu R dT$

Now from first law of thermodynamics  $dQ = dW + dU$

$$\Rightarrow \mu C_P dT = \mu R dT + \mu C_V dT \Rightarrow C_P = R + C_V \Rightarrow C_P - C_V = R$$

### **GOLDEN KEY POINTS**

- When a gas expands its volume increases, then final pressure is less for adiabatic expansion. But, when a gas compresses its volume decreases, then the final pressure is more in case of adiabatic compression.



### **First Law of Thermodynamics Applied to Different Processes**



Process	Q	$\Delta U$	W
Cyclic	W	0	Area of the closed curve
Isochoric	$\Delta U$	$\mu C_V \Delta T$ ( $\mu$ mole of gas)	0
Isothermal	W	0	$\mu RT \log_e \left[ \frac{V_f}{V_i} \right] = \mu RT \log_e \left[ \frac{P_i}{P_f} \right]$
Adiabatic	0	-W	$\frac{\mu R(T_f - T_i)}{1 - \gamma}$
Isobaric	$\mu C_P \Delta T$	$\mu C_V \Delta T$	$P(V_f - V_i) = \mu R(T_f - T_i)$

### Illustrations

#### Illustration 56.

An ideal gas has a specific heat or molar heat capacity at constant pressure  $C_P = 5R/2$ . The gas is kept in a closed vessel of volume  $0.0083 \text{ m}^3$  at a temperature of  $300\text{K}$  and a pressure of  $1.6 \times 10^6 \text{ Nm}^{-2}$ . An amount of  $2.49 \times 10^4 \text{ J}$  of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

#### Solution:

$$C_V = C_P - R = \frac{5R}{2} - R = \frac{3R}{2}, \Delta V = 0, T_1 = 300 \text{ K}, V = 0.0083 \text{ m}^3, P_1 = 1.6 \times 10^6 \text{ Nm}^{-2}$$

$$\text{From first law of thermodynamics } Q = \Delta U + P\Delta V \Rightarrow \Delta U = Q = 2.49 \times 10^4 \text{ J}$$

$$\text{From gas equation } n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3}$$

$$\therefore \Delta U = nC_V \Delta T \Rightarrow \Delta T = \frac{\Delta U}{\mu C_V} = \frac{2.49 \times 10^4 \times 6}{3 \times 8.3 \times 16} = 375 \text{ K}$$

$$\text{Final temperature} = 300 + 375 = 675 \text{ K}$$

$$\text{According to pressure law } P \propto T \Rightarrow P_2 = \frac{T_2}{T_1} \times P_1 = \frac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6 \text{ Nm}^{-2}$$

#### Illustration 57.

5 moles of oxygen is heated at constant volume from  $10^\circ\text{C}$  to  $20^\circ\text{C}$ . What will be change in the internal energy of the gas? The gram molecular specific heat of oxygen at constant pressure.

$$C_p = 8 \text{ cal/mole and } R = 8.36 \text{ joule/mole } ^\circ\text{C}.$$

#### Solution:

$$\Theta C_V = C_P - R = 8 - 2 = 6 \text{ cal/mole } ^\circ\text{C}$$

$$\therefore \text{Heat absorbed by 5 moles of oxygen at constant volume}$$

$$\Theta = \mu C_V \Delta T = 5 \times 6 (20 - 10) = 30 \times 10 = 300 \text{ cal}$$

$$\text{At constant volume } \Delta V = 0. \quad \therefore \Delta W = 0$$

$$\therefore \text{From first law of thermodynamics } Q = \Delta U + W \Rightarrow 300 = \Delta U + 0 \Rightarrow \Delta U = 300 \text{ cal}$$

#### Illustration 58.

What amount of heat must be supplied to  $2.0 \times 10^{-2} \text{ kg}$  of nitrogen (at room temperature) to raise the temperature by  $45^\circ\text{C}$  at constant pressure. Molecular mass of  $\text{N}_2=28$ ,  $R=8.3 \text{ J mol}^{-1} \text{ K}^{-1}$ .



**Solution:**

$$\text{Here } m = 2 \times 10^{-2} \text{ kg, } \Rightarrow \mu = \frac{M}{M_w} = \frac{2 \times 10^{-2}}{28 \times 10^{-3}} = \frac{5}{7} \text{ \& } C_p = \frac{7}{2} R$$

$$\therefore Q = \mu C_p \Delta T = \frac{5}{7} \times \frac{7}{2} \times 8.3 \times 45 = 933.75 \text{ J}$$

**Illustration 59.**

Two moles of a gas at  $127^\circ\text{C}$  expand isothermally until its volume is doubled. Calculate the amount of work done .

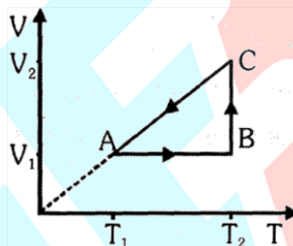
**Solution:**

$$\mu = 2, \quad T = 127 + 273 = 400\text{K}, \quad \frac{V_2}{V_1} = 2$$

$$\begin{aligned} \text{From formula } W &= 2.3026 \mu R T \log_{10} \frac{V_2}{V_1} = 2.3026 \times 2 \times 8.3 \times 400 \times \log_{10} 2 \\ &= 2.3026 \times 2 \times 8.3 \times 400 \times 0.3010 \approx 4.6 \times 10^3 \text{ J} \end{aligned}$$

**Illustration 60.**

Figure shows a process ABCA performed on an ideal gas. Find the net heat given to the system during the process.

**Solution:**

Since the process is cyclic, hence the change in internal energy is zero.

The heat given to the system is then equal to the work done by it.

The work done in part AB is  $W_1 = 0$  (the volume remains constant). The part BC represents an isothermal process so that the work done by the gas during this part is  $W_2 = \mu R T_2 \ln \frac{V_2}{V_1}$

During the part CA :  $V \propto T$  So,  $V/T$  is constant and hence,  $P = \frac{\mu R T}{V}$  is constant

The work done by the gas during the part CA is  $W_3 = P(V_1 - V_2) = \mu R T_1 - \mu R T_2 = -\mu R (T_2 - T_1)$ .

The net work done by the gas in the process ABCA is  $W = W_1 + W_2 + W_3 = \mu R \left[ T_2 \ln \frac{V_2}{V_1} - (T_2 - T_1) \right]$

The same amount of heat is given to the gas.

**Illustration 61.**

During an experiment an ideal gas is found to obey an additional law  $VP^2 = \text{constant}$ . The gas is initially at temperature  $T$  and volume  $V$ . What will be the temperature of the gas when it expands to a volume  $2V$ .

**Solution:**

By gas equation  $PV = \mu R T$  and  $VP^2 = \text{constant}$  on eliminating  $P$

$$\left[ \frac{A}{\sqrt{V}} \right] V = \mu RT \Rightarrow \sqrt{V} = \frac{\mu R}{A} T \quad \therefore \frac{\sqrt{V_1}}{\sqrt{V_2}} = \left[ \frac{T_1}{T_2} \right] \Rightarrow \frac{\sqrt{V}}{\sqrt{2V}} = \frac{T}{T'} \Rightarrow T' = (\sqrt{2}) T$$

**Illustration 62.**

$2\text{m}^3$  volume of a gas at a pressure of  $4 \times 10^5 \text{ Nm}^{-2}$  is compressed adiabatically so that its volume becomes  $0.5 \text{ m}^3$ . Find the new pressure. Compare this with the pressure that would result if the compression was isothermal. Calculate work done in each process.  $\gamma = 1.4$

**Solution:**

$$V_1 = 2\text{m}^3, P_1 = 4 \times 10^5 \text{ Nm}^{-2}, \quad V_2 = 0.5 \text{ m}^3$$

$$\text{In adiabatic process } P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_2 = 4 \times 10^5 \left[ \frac{2}{0.5} \right]^{1.4} = 4 \times 10^5 (4)^{1.4} = 2.8 \times 10^6 \text{ Nm}^{-2}$$

$$\text{In isothermal process } P_1 V_1 = P_2 V_2 \Rightarrow P_2 = \frac{P_1 V_1}{V_2} = \frac{4 \times 10^5 \times 2}{0.5} = 1.6 \times 10^6 \text{ Nm}^{-2}.$$

Now work done in adiabatic process

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{(4 \times 10^5 \times 2 - 2.8 \times 10^6 \times 0.5)}{1.4 - 1} = -1.48 \times 10^6 \text{ J}$$

$$\begin{aligned} \text{Work done in isothermal process } W &= 2.303 \mu RT \log \frac{V_2}{V_1} = 2.3026 P_1 V_1 \log \frac{V_2}{V_1} \\ &= 2.3026 \times 4 \times 10^5 \times 2 \times \log \left[ \frac{0.5}{2.0} \right] = 2.3026 \times 4 \times 10^5 \times 2 \log \left( \frac{1}{4} \right) = -1.1 \times 10^6 \text{ J} \end{aligned}$$

**Illustration 63.**

Two samples of a gas initially at same temperature and pressure are compressed from a volume  $V$  to  $\frac{V}{2}$ . One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?

**Solution:**

Let initial volume,  $V_1 = V$  and pressure,  $P_1 = P$ , final volume,  $V_2 = \frac{V}{2}$  and final pressure,  $P_2 = ?$

$$\text{For isothermal compression } P_2 V_2 = P_1 V_1 \text{ or } P_2 = \frac{P_1 V_1}{V_2} = \frac{PV}{\frac{V}{2}} = 2P$$

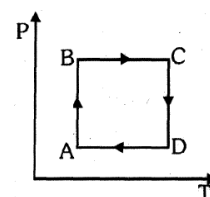
For adiabatic compression

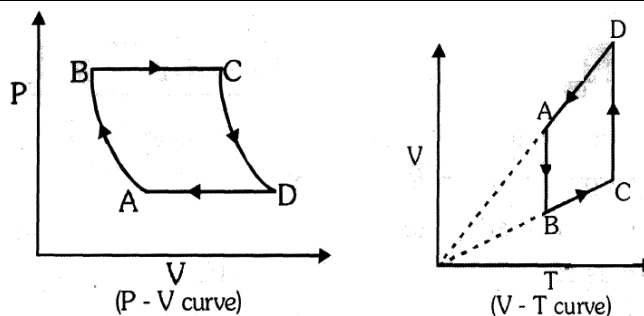
$$P_2' = P_1 \left[ \frac{V_1}{V_2} \right]^\gamma \Rightarrow P_2' = P \left[ \frac{V}{V/2} \right]^\gamma = 2^\gamma P \Rightarrow P_2' = 2^\gamma P \quad \gamma > 1 \quad \therefore 2^\gamma > 2 \text{ and } P_2' > P_2$$

Pressure during adiabatic compression is greater than the pressure during isothermal compression.

**Illustration 64.**

Plot  $P - V$ ,  $V - T$  graph corresponding to the  $P - T$  graph for an ideal gas shown in figure. Explain your answers.

**Solution:**



For process AB ;  $T = \text{constant}$  so  $P \propto \frac{1}{V}$

For process BC  $P = \text{constant}$  so  $V \propto T$

For process CD ;  $T = \text{constant}$  so  $V \propto \frac{1}{P}$

For process DA  $P = \text{constant}$  so  $V \propto T$

### Illustration 65.

The specific heat of argon at constant volume is  $0.075 \text{ kcal/kg K}$ . Calculate its atomic weight,  $[R = 2 \text{ cal/mol K}]$

#### Solution:

As argon is monoatomic, its molar specific heat at constant volume will be

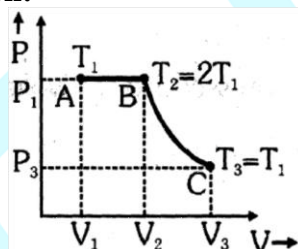
$$C_V = \frac{3}{2} R = \frac{3}{2} \times 2 = 3 \text{ cal/mol K}, C_V = M_w c_V \text{ and } c_V = 0.075 \text{ cal/g K}$$

$$\text{So } 3 = M_w \times 0.075 \Rightarrow M_w = \frac{3}{0.075} = 40 \text{ gram/mole}$$

### Illustration 66.

Two moles of helium gas ( $\gamma = 5/3$ ), assumed ideal are initially at  $27^\circ\text{C}$  and occupy a volume of 20 litres. The gas is first expanded at constant pressure till its volume is doubled. It then undergoes an adiabatic change until the temperature return to its initial value. Determine the final pressure and volume of the gas. Also, calculate the work done under isobaric and adiabatic processes.  $[R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}]$  [AIPMT (Mains) 2005]

#### Solution:



$$V_1 = 20 \times 10^{-3} \text{ m}^3, \\ T_3 = T_1 = 300 \text{ K}$$

For process AB ( $P = \text{constant}$ )  $T_2 = 2T_1 = 600 \text{ K}$  ( $\because V_2 = 2V_1$ )

For process BC  $TV^{\gamma-1} = \text{constant}$  Therefore  $(2T_1)(V_2)^{5/3-1} = (T_1)(V_3)^{5/3-1}$

$$\Rightarrow V_3 = (V_2)(2)^{3/2} = 2\sqrt{2} V_2 = 80\sqrt{2} \times 10^{-3} \text{ m}^3 = 80\sqrt{2} \text{ litres}$$

$$P_3 = \frac{\mu RT_3}{V_3} = \frac{(2)(8.3)(300)}{80\sqrt{2} \times 10^{-3}} = 0.44 \times 10^5 \text{ N/m}^2$$

Work done under isobaric process  $= P\Delta V = P_1(V_2 - V_1)$

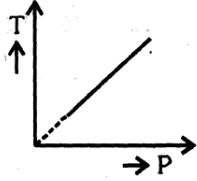
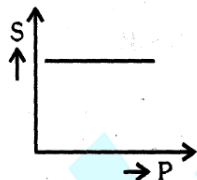
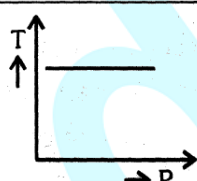
$$= \frac{\mu RT_1}{V_1}(V_2 - V_1) = \frac{2 \times 8.3 \times 300}{20 \times 10^{-3}} (40 \times 10^{-3} - 20 \times 10^{-3}) = 4980 \text{ J}$$

$$\text{Work done under adiabatic process} = \frac{\mu R(T_2 - T_3)}{(\gamma - 1)} = \frac{2 \times 8.3 \times (300)}{(5/3 - 1)} = 7470 \text{ J}$$

### BEGINNER'S BOX - 8

- 32 gram of oxygen gas at temperature  $27^\circ\text{C}$  is compressed adiabatically to  $1/3$  of its initial volume. Calculate the change in internal energy ? ( $\gamma = 1.5$  for oxygen)
- How much heat energy should be added to the gaseous mixture consisting of one gram of hydrogen and one gram helium to raise its temperature from  $0^\circ\text{C}$  to  $50^\circ\text{C}$ .
  - at constant volume and.
  - at constant pressure, assuming both the to be ideal. $R = 2 \text{ cal/mol-K}$ ,  $\gamma_{\text{H}_2} = 1.41$ ,  $\gamma_{\text{He}} = 1.67$

- Match the column-I with column-II

Column - I	Column - II
(A) 	(p) Isometric process
(B) 	(q) Adiabatic process
(C) 	(r) Isothermal process

- Match the column-I with column-II

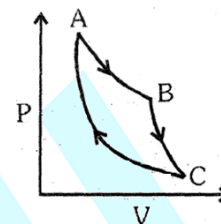
Column-I	Column-II
(A) Isobaric process	(p) No heat exchange
(B) Isothermal process	(q) Constant pressure
(C) Isoentropic process	(r) constant internal energy
(D) Isochoric process	(s) Work done is zero

- Match the column-I with column-II

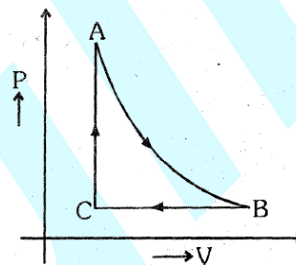
Column-I	Column-II
(A) Adiabatic expansion	(p) $Q = 0$
(B) Adiabatic free expansion	(q) $\Delta U = 0$

(C) Isobaric expansion	(r) $W = 0$
(D) Isothermal expansion	(s) $Q = W + \Delta U$
(E) Isochoric cooling	

6. An ideal gas expands isothermally along AB and does 700 J of work.  
 (a) How much heat does the gas exchange along AB.  
 (b) The gas expands adiabatically along BC and does 400 J of work. When the gas returns to A along CA, it exhusts 100 J of heat to its surroundings. How much work is done on the gas along this path.



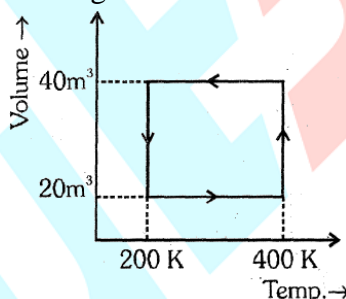
7. For one mole of an ideal gas at 300 K occupies a volume of  $0.36 \text{ m}^3$  at a pressure of 2 atm. The gas expands adiabatically until its volume becomes  $1.44 \text{ m}^3$ . Next gas is compressed isobarically to its original volume. Finally the pressure is increased isochorically until the gas return to its initial state. (see figure).



- (a) Determine the temp. at the point B and C  
 (b) The work done during the process A to B

(Assume  $\gamma = 1.5$ ,  $R = 8 \frac{\text{J}}{\text{mol} \cdot \text{K}}$ )

8. Following figure is given for 1 mole gas.



Then find out net work done in the cyclic process.

### EFFECIENCY OF A CYCLE ( $\eta$ ) :

$$\eta = \frac{\text{Total Mechanical work done by the gas in the whole process}}{\text{Heat absorbed by the gas (only +ve)}}$$

$$= \frac{\text{Area of the P - V cycle}}{\text{Heat injected into the system}}$$

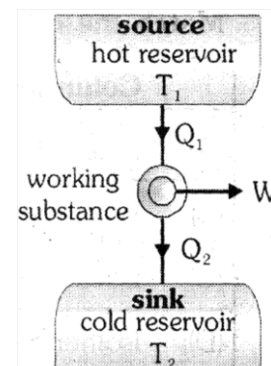
### HEAT ENGINE

Heat engine is a device which converts heat into work.

Three parts of a heat engine:

- (i) Source of high temperature reservoir at temperature  $T_1$
- (ii) Sink of low temperature reservoir at temperature  $T_2$
- (iii) Working substance .

In a cycle of heat engine the working substance extracts heat  $Q_1$  from source, does some work and rejects remaining heat  $Q_2$  to the sink.





Efficiency of heat engine  $\eta = \frac{\text{work done (W)}}{\text{heat taken from source (Q)}} \Rightarrow \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$

### REFRIGERATOR

It is inverse of heat engine. It extracts heat ( $Q_2$ ) from a cold reservoir, some external work  $W$  is done on it and rejects heat ( $Q_1$ ) to hot reservoir.

The coefficient of performance of a refrigerator (C.O.P. or  $\beta$ ).

$$\beta = \frac{\text{Heat extracted from cold reservoir}}{\text{Work done on refrigerator}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

For Carnot reversible refrigerator  $\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \therefore \beta = \frac{Q_2}{W} = \frac{1}{\left[\frac{Q_1}{Q_2} - 1\right]} = \frac{1}{\left[\frac{T_1}{T_2} - 1\right]}$

$$\Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

**Relation between  $\beta$  and  $\eta$  :**

(Refrigerator and heat engine

Are maintained at same thermal reservoirs)

$$\eta = \frac{W}{Q_1} \quad [\text{Efficiency of heat engine}]$$

$$\beta = \frac{Q_2}{W} \quad [\text{C.O.P. of refrigerator}]$$

$$\text{and } Q_1 = Q_2 + W$$

$$\text{then } \beta = \frac{1}{\eta} - 1$$

### SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is the principle of conservation of energy. Common experience shows that there are many conceivable processes that are perfectly allowed by the First law and yet are never observed. For example, nobody has ever seen a book lying on a table jumping to a height by itself. Would be possible if the principle of conservation of energy were the only restriction. This principle, which disallows many phenomena consistent with the first law of thermodynamics is known as the second law of thermodynamics.

The Second Law of Thermodynamics gives a fundamental limitation to the efficiency of a heat engine and the co-efficient of performance of refrigerator.

#### Second Law of Thermodynamics

##### Kelvin Planck Statement

No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work.

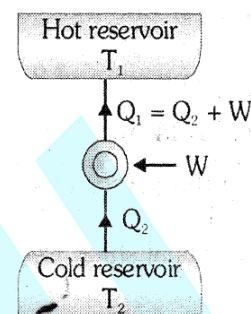
##### Claussius Statement

No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.

It can be proved that the two statements above are completely equivalent.

The second law of thermodynamics is a general principle which places constraints upon the direction of heat transfer and the attainable efficiencies of heat engines.

Entropy Statement of the second Law of Thermodynamics



The entropy of the Universe never decrease. It either increases (for irreversible processes) or remains the same (for reversible processes) •

## REVERSIBLE AND IRREVERSIBLE PROCESS

### (i) Reversible Process

A reversible process is defined as a process that can be reversed without leaving any trace on the surroundings. Both the system and the surroundings are returned to their initial status at the end of the reverse process.

#### Conditions for a Reversible process

- (a) The process should take place very slowly, i.e. quasi-statically, i.e. seemingly static (from the Latin word qasi meaning 'as if') so that it satisfies the following requirement at each stage of the process.
  - The system should be in mechanical equilibrium.
  - The System be in thermal equilibrium.
  - The system should be in chemical equilibrium, i.e., no new products should be formed.
- (b) There should be no friction losses etc. It should be remembered that a complete reversible process or cycle of operations is only an ideal case. In actual practice, there is always a loss of heat due to friction, conduction and radiation. However, it is possible to approximate reversible processes through carefully controlled procedures, but they can never be achieved.

#### Examples

- Frictionless pendulum.
- Quasi-equilibrium expansion and compression of a gas.

### (ii) Irreversible Process

A process that is not reversible is called an irreversible process.

The spontaneous processes occurring in nature are irreversible. In fact irreversibility is a rule rather than an exception in nature.

#### Examples

- (i) Sudden unbalanced expansion of a gas.
- (ii) Heat produced by friction.
- (iii) Heat generated when a current flows through an electric resistance in any direction.
- (iv) Cooking gas leaking from a cylinder.
- (v) Diffusion of liquids or gases.
- (vi) Breaking of an egg.
- (vii) The growth of a plant.

A system can be restored to its initial state following a process, regardless of whether the process is reversible or irreversible. But for reversible process, this restoration is made without leaving any net change on the surroundings whereas for irreversible processes, the surroundings usually do some work on the system and therefore will not return to their original state.

## ENTROPY AND THE SECOND LAW OF THERMODYNAMICS

To have a better understanding of thermodynamic process and the second and third laws of thermodynamics, let us introduce the concept of entropy. It was introduced by Clausius in 1865. A quantity that denotes the amount of disorder is called entropy and is denoted by S.

The total energy is always conserved in any process but the total entropy always increases or remains the same in any process (i.e., the disorder increases or remains constant).

$$dS = \frac{dQ}{T} \Rightarrow \Delta S = \int \frac{dQ}{T}$$

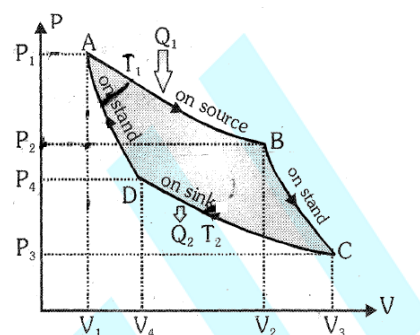
For reversible processes,  $\Delta S = 0$ . For irreversible process  $\Delta S > 0$

A process where  $\Delta S < 0$  is not possible.

### 3. Carnot cycle

Carnot devised an ideal engine which is based on a reversible cycle of four operations in succession:

- |                              |                   |
|------------------------------|-------------------|
| (i) Isothermal expansion,    | A $\rightarrow$ B |
| (ii) Adiabatic expansion,    | B $\rightarrow$ C |
| (iii) Isothermal compression | C $\rightarrow$ D |
| (iv) Adiabatic compression   | D $\rightarrow$ A |



#### Main parts of Carnot's engine are

- **Source of heat :**

It is a hot body of very large heat capacity kept at a constant high temperature  $T_1$ . Its upper surface is perfectly conducting so that working substance can take heat from it.

- **Mechanical arrangements and working substance :**

It is a hollow cylinder whose walls are perfectly non-conducting and its base is perfectly conducting fitted with non-conducting piston. This piston move without any friction. Ideal gas enclosed in cylinder as a working substance.

- **Heat sink :**

It is a cold body at low temperature  $T_2$ . It is a body of large heat capacity and its upper surface is highly conducting so that working substance can reject heat to it.

- **Stand :**

It is made by perfectly insulating material. When cylinder placed on it working substance can expanded or compressed adiabatic.

- **Working :**

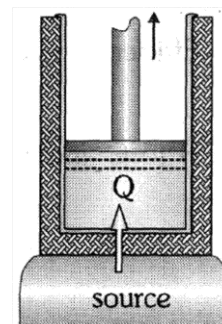
A set of reversible processes through them working substance is taken back to initial condition to get maximum work from this type of ideal engine.

Processes of Carnot's cycle can be denoted by an indicator diagram.

- **Isothermal expansion A  $\rightarrow$  B**

Initially the cylinder is taken to be in thermal equilibrium with the high temperature  $T_1$ , this is initial state of working substance denoted by point A ( $P_1, V_1, T_1$ ). After that the piston is allowed to move outward slowly. With the movement of the piston. The process is very slow so that it is isothermal. Heat from reservoir flows through the base of cylinder into the gas so temperature of the gas remains  $T_1$ . Gas expand and receive heat  $Q_1$  from source and gets state B ( $P_2, V_2, T_1$ )

This heat input  $Q_1$  to the gas from path A to B is utilized for doing work  $W_1$ . By path A to B the heat input to the gas = the work done against the external pressure.

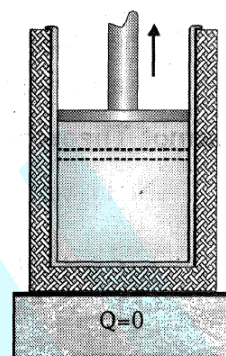


$$W_1 = Q_1 = \frac{100 \times 3.4 \times 10^5 \times 27}{273} dV = \mu RT_1 \ln \frac{V_2}{V_1}$$

• **Adiabatic expansion B → C**

Now cylinder is put in contact with a non-conducting stand and piston is allowed to move outward, because no heat can enter in or leave out so the expansion of gas is adiabatic. The temperature falls to  $T_2$ K and gas describes the adiabatic from B to point C ( $P_3, V_3, T_2$ ). During this expansion more work is done ( $W_2$ ) at the expense of the internal energy.

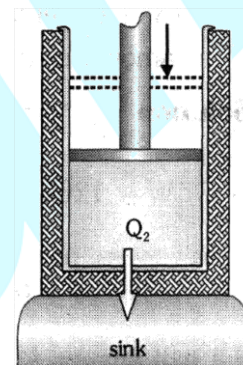
$$\text{Work done in adiabatic path BC is } W_2 = \frac{\mu R}{\gamma - 1} (T_1 - T_2)$$



• **Isothermal compression C → D**

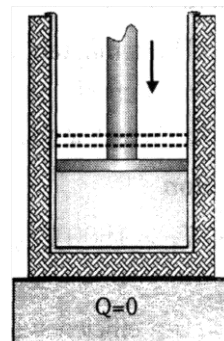
Now the gas cylinder is placed in contact with sink at temperature  $T_2$ . The piston is moved slowly inward so that heat produced during compression passes to the sink. The gas is isothermally compressed from C to point D ( $P_4, V_4, T_2$ ). The heat reject  $Q_2$  to the cold reservoir (sink) at  $T_2$  occurs over this path. Amount of work done on gas  $W_3$  = amount of heat rejected to the sink

$$Q_2 = W_3 = \mu RT_2 \ln \left( \frac{V_4}{V_3} \right) \Rightarrow Q_2 = \mu RT_2 \ln \left( \frac{V_4}{V_3} \right)$$



• **Adiabatic compression D → A**

The cylinder is removed from the sink and is put in contact with insulating stand now piston moves inward. Heat is not allowed to go out and it increases the internal energy of the system. Now work is done on the gas during adiabatic compression from state D to initial point A ( $P_1, V_1, T_1$ ). No heat exchanges occur over the adiabatic path. Work done on the system  $W_4 = \frac{\mu R}{\gamma - 1} (T_2 - T_1)$ .



This cycle of operations is called a Carnot cycle.

In first two steps work is done by engine  $W_1$  and  $W_2$  are positive

In last two steps work is done on gas  $W_3$  and  $W_4$  are negative

The work done in complete cycle  $W$  = the area of the closed part of the P-V cycle.

$$W = W_1 + W_2 + W_3 + W_4$$

$$\therefore W = \mu RT_1 \ln \frac{V_2}{V_1} + \frac{\mu R}{\gamma - 1} (T_1 - T_2) + \mu RT_2 \ln \frac{V_4}{V_3} - \frac{\mu R}{\gamma - 1} (T_2 - T_1) = \mu RT_1 \ln \frac{V_2}{V_1} + \mu RT_2 \ln \frac{V_4}{V_3}$$

$$\text{Efficiency of Carnot Engine, } \eta = \frac{W}{Q_1} \Rightarrow \eta = \frac{\mu RT_1 \ln \frac{V_2}{V_1} + \mu RT_2 \ln \frac{V_4}{V_3}}{\mu RT_1 \ln \frac{V_2}{V_1}}$$

B to C and D to A are adiabatic paths

$$\begin{aligned} \text{so } T_1 V_2^{(\gamma-1)} &= T_2 V_3^{(\gamma-1)} \text{ and } T_1 V_1^{(\gamma-1)} = T_2 V_4^{(\gamma-1)} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4} \\ \Rightarrow \eta &= \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1} \Rightarrow \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \frac{Q_1}{T_1} = \frac{Q_2}{T_2} \\ \eta &= \frac{T_1 - T_2}{T_1} \times 100\% \Rightarrow \eta = \frac{Q_1 - Q_2}{Q_1} \times 100\% \end{aligned}$$

The efficiency for the Carnot engine is the best that can be obtained for any heat engine. The efficiency of a Carnot engine is never 100% because it is 100% only if temperature of sink  $T_2 = 0$  which is impossible.

### Carnot Theorem

No irreversible engine (I) can have efficiency greater than Carnot reversible engine (R)

working between same hot and cold reservoirs.  $\eta_R > \eta_I \Rightarrow 1 - \frac{T_2}{T_1} > 1 - \frac{Q_2}{Q_1}$

## Illustrations

### Illustration 67.

A Carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

**Solution:**

$$\begin{aligned} W &= 1200 \text{ J, } T_1 = 800 \text{ K, } T_2 = 400 \text{ K} \therefore \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} \Rightarrow 1 - \frac{400}{800} = \frac{1200}{Q_1} \\ \Rightarrow 0.5 &= \frac{1200}{Q_1} \end{aligned}$$

$$\text{Heat energy supplied by source} = Q_1 = \frac{1200}{0.5} = 2400 \text{ joule per cycle}$$

### Illustration 68.

The temperatures  $T_1$  and  $T_2$  of the two heat reservoirs in an ideal Carnot engine are  $1500^\circ\text{C}$  and  $500^\circ\text{C}$  respectively. Which of the following : increasing  $T_1$  by  $100^\circ\text{C}$  or decreasing  $T_2$  by  $100^\circ\text{C}$  would result in a greater improvement in the efficiency of the engine?

**Solution:**

$$T_1 = 1500^\circ\text{C} = 1500 + 273 = 1773 \text{ K and } T_2 = 500^\circ\text{C} = 500 + 273 = 773 \text{ K.}$$

$$\text{The efficiency of a Carnot's engine } \eta = 1 - \frac{T_2}{T_1}$$

When the temperature of the source is increased by  $100^\circ\text{C}$ , keeping  $T_2$  unchanged the new temperature of the source is  $T_1' = 1500 + 100 = 1600^\circ\text{C} = 1873 \text{ K.}$

$$\text{The efficiency becomes } \eta' = 1 - \frac{T_2}{T_1'} = 1 - \frac{773}{1873} = 0.59$$

On the other hand, if the temperature of the sink is decreased by  $100^\circ\text{C}$ , keeping  $T_1$  unchanged, the new temperature of the sink  $T_2' = 500 - 100 = 400^\circ\text{C} = 673 \text{ K.}$  The efficiency now becomes



$$\eta'' = 1 - \frac{T_2'}{T_1} = 1 - \frac{673}{1773} = 0.62$$

Since  $\eta''$  is greater than  $\eta'$ , decreasing the temperature of the sink by  $100^\circ\text{C}$  results in a greater efficiency than increasing the temperature of the source by  $100^\circ\text{C}$ .

### Illustration 69.

A heat engine operates between a cold reservoir at temperature  $T_2 = 300\text{ K}$  and a hot reservoir at temperature  $T_1$ . It takes  $200\text{ J}$  of heat from the hot reservoir and delivers  $120\text{ J}$  of heat to the cold reservoir in a cycle. What could be the minimum temperature of hot reservoir?

#### Solution:

Work done by the engine in a cycle is  $W = 200 - 120 = 80\text{ J}$ .  $\eta = \frac{W}{Q} = \frac{80}{200} = 0.4$

From Carnot's Theorem  $0.4 \leq 1 - \frac{T_2}{T_1} = 1 - \frac{300}{T_1} \Rightarrow \frac{300}{T_1} \leq 0.6 \Rightarrow T_1 \geq \frac{300}{0.6} \Rightarrow T_1 \geq 500$

### Illustration 70.

A Carnot engine works as refrigerator between  $250\text{ K}$  and  $300\text{ K}$ . If it receives  $750\text{ cal}$  of heat from the reservoir at the lower temperature. Calculate the amount of heat rejected at the higher temperature.

#### Solution

$$T_1 = 300\text{ K} \quad T_2 = 250\text{ K} \quad Q_2 = 750 \quad Q_1 = ?$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad Q_1 = \frac{300}{250} \times 750 = 900\text{ cal}$$

### Illustration 71.

The temperature inside and outside of refrigerator are  $260\text{ K}$  and  $315\text{ K}$  respectively. Assuming that the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.

#### Solution:

$$T_2 = 260\text{ K}, T_1 = 315\text{ K} \quad ; W = 1\text{ joule}$$

Coefficient of performance of Carnot refrigerator  $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\therefore \frac{Q_2}{1} = \frac{260}{315 - 260} = \frac{260}{55} \Rightarrow Q_2 = \frac{260}{55} = 4.73\text{ J} \quad \text{So} \quad Q_1 = Q_2 + W = 5.73\text{ J}$$

### Illustration 72.

A refrigerator takes heat from water at  $0^\circ\text{C}$  and transfer it to room at  $27^\circ\text{C}$ . If  $100\text{ kg}$  of water is converted in ice at  $0^\circ\text{C}$  then calculate the work done. (Latent heat of ice is  $3.4 \times 10^5\text{ J/kg}$ )

#### Solution:

Coefficient of performance (COP)  $= \frac{T_2}{T_1 - T_2} = \frac{273}{300 - 273} = \frac{273}{27}$

$$W = \frac{Q_2}{\text{COP}} = \frac{mL}{\text{COP}} = \frac{100 \times 3.4 \times 10^5}{273 / 27} = \frac{100 \times 3.4 \times 10^5 \times 27}{273} = 3.36 \times 10^6\text{ J}$$

1. An ideal engine which works between  $27^{\circ}\text{C}$  &  $227^{\circ}\text{C}$  takes 100 calorie of heat in one cycle. Calculate the work done in one cycle ?
2. The coefficient of performance of a Carnot refrigerator is  $5/3$ . If the refrigerator operating at a intake temperature  $-23^{\circ}\text{C}$  then what will be the exhaust temperature ?
3. A Carnot engine absorbs 1000 J of heat from a reservoir at  $127^{\circ}\text{C}$  and reject 600 J of heat during each cycle.  
Calculate
  - (a) The efficiency of engine ..
  - (b) The temperature of the sink
  - (c) The amount of the useful work done during each cycle.
4. Two Carnot engine A and B use one after another. Engine A absorb heat from source at  $T_1 = 800\text{ K}$  and release heat to sink of temp  $T_2\text{K}$ . Second engine B absorb heat which release by first (at  $T_2\text{K}$ ) and release heat at another sink at temperature  $T_3 = 300\text{ K}$ . If work done by two engine is same than calculate value of  $T_2$ .
5. A Carnot engine, efficiency 40% and temperature of sink 300 K to increase efficiency up to 60% calculate change in temperature of source ?
6. An ideal refrigerator is working between temperature  $27^{\circ}\text{C}$  and  $127^{\circ}\text{C}$ . If it expels 120 calorie of heat in one second then calculate its wattage ?

### ANSWERS KEY

#### BEGINNER'S BOX - 1

1.  $50\text{ K} < 50^{\circ}\text{F} < 50^{\circ}\text{C}$
2. (a) All tie (b)  $50^{\circ}\text{X}$ ,  $50^{\circ}\text{Y}$ ,  $50^{\circ}\text{W}$ .
3.  $-40^{\circ}\text{C}$  or  $-40^{\circ}\text{F}$
4. (B)
5.  $1.44 \times 10^{-2}\text{ cm}$
6. In bimetallic strips the two metals have different thermal expansion coefficient. Hence on heating it bends towards the metal with lower thermal expansion coefficient.

#### BEGINNER'S BOX - 2

1. 191.11 Cal.
2. 50 kcal
3. 1 cal
4.  $10^{\circ}\text{C}$
5. 1 : 1
6. 615 kcal
7. 4000 cal
8.  $160^{\circ}\text{C}$
9.  $7.5^{\circ}\text{C}$ , water
10. (A)

#### BEGINNER'S BOX - 3

1. (a) The thermal conductivity of brass is high, i.e., brass is a good conductor of heat. So, when a brass tumbler is touched, heat quickly flows from human body to tumbler. Consequently, the tumbler appears colder. On the other hand, wood is a bad conductor. So, heat does not flow from the human body to the wooden tray in this case. Thus, it appears comparatively hotter.  
(b) This is because the two thin cloth enlose a layer of air between them. Since air is a bad conductor of heat therefore the conduction of heat is prevented.  
(c) Because mudhouses are bad conductor of heat.  
(d) Because air is trapped between their wings since air is bad conductor of heat therefore the conduction of heat is prevented.  
(e) This is because wool contains air in its pores. Air is a bad conductor of heat. In fact, wool is also a bad conductor of heat so both the air and wool do not permit heat to be conducted away from. So woollen clothes are warm.
2. (a) 240 W (b) 1.5
3.  $40^{\circ}\text{C}$
4.  $\theta_1 = 116^{\circ}\text{C}$ ,  $\theta_2 = 74^{\circ}\text{C}$
5.  $68^{\circ}\text{C}$ ;  $0.9^{\circ}\text{C}/\text{cm}$
6. 7 : 2

**BEGINNER'S BOX - 4**

1. (a) A body whose reflectivity is large would naturally absorb less heat. So, a body with large reflectivity is a poor absorber of heat. Poor absorbers are poor emitters. So, a body with large reflectivity is a poor emitter.
- (b) Let  $T$  be the temperature of the hot iron in the furnace. Heat radiated per second per unit area,  $E = \sigma T^4$ . When the body is placed in the open at temperature  $T_0$ , then the heat radiated/second/area,  $E' = \sigma (T^4 - T_0^4)$ . Clearly  $E' < E$ . So, the optical pyrometer gives too low a value for the temperature in the open.
- (c) The lower layers of earth's atmosphere reflect infrared radiations from earth back to the surface of earth. Thus the heat radiation received by the earth from the sun during the day are kept trapped by the atmosphere. If atmosphere of earth were not there, its surface would become too cold to live.
- (d) When the electric heater is switched on, a stage is quickly reached when the rate at which heat is generated by electric. Current becomes equal to the rate at which heat is lost by radiation. This is case of thermal equilibrium.
- (e) Because black surface is good absorber of heat.
- (f) On a clear night, the earth radiates energy into space at a rate proportional to the fourth power of its temperature (about 300K). The incoming radiation from space is very small because its average temperature is nearly absolute zero. On the other hand with cloud over, the earth radiates at 300K, but the radiation is absorbed in the clouds, which radiate energy back to earth again the radiation is trapped, like the green house effect.
- (g) This is due to the fact that a vacuum is created between the two walls of the thermos flask. Heat can neither flow from inside the flask to outside nor from outside air to the liquid inside the flask. In this way, loss of heat by conduction and convection has been minimised. To minimise loss of heat by radiation the surface is made shining.

2.  $\sqrt{\frac{r_2}{r_1}}$       3.  $40^\circ\text{C}$       4.  $28^\circ\text{C}$       5.  $1927\text{ K}$

**BEGINNER'S BOX - 5**

1.  $\frac{1}{2}$  mole      2.  $120\text{ kPa}$       3.  $2.2 \times 10^{24}$       4. (a)  $\frac{N_1}{N_2} = \frac{3}{2}$  ; (b) 0.947

**BEGINNER'S BOX - 6**

1. (a)  $6.21 \times 10^{-21}\text{ J}$ ; (b)  $10.35 \times 10^{-21}\text{ J}$ ; (c)  $6231\text{ J}$       2. (a)  $127^\circ\text{C}$ , (b)  $527^\circ\text{C}$

3. (a) 3, (b)  $\frac{5}{3}$ , (c)  $450\text{ R}$

4.  $v_{\text{avg}} = \frac{v_1 + v_2 + v_3}{3}$ , Root mean square speed =  $\sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}}$

5.  $9.2\text{ m/s}$       6. (a)  $\frac{E_1}{E_2} = \frac{1}{1}$  ( $\Theta$   $T$  = same for both); (b)  $v_{\text{rms}} = \sqrt{\frac{70.9}{39.9}} = \frac{1.33}{1}$

**BEGINNER'S BOX - 7**

1. (a)  $W_{\text{ABCA}} = \frac{1}{2} \times (2V_0 - V_0) \times (3P_0 - P_0) = P_0 V_0$ , (b)  $\frac{5}{2} P_0 V_0$  and  $3P_0 V_0$ , (c)  $\frac{P_0 V_0}{2}$

2.  $-5 \times 10^5\text{ J}$       3.  $W_{\text{AB}} = 6000\text{ J}$ ,  $W_{\text{BC}} = \text{zero}$ ,  $W_{\text{CD}} = 1000\text{ J}$ ,  $W_{\text{DA}} = \text{zero}$

4. (a)  $24 \times 10^2\text{ J}$ ; (b)  $36 \times 10^2\text{ J}$ ; (c)  $60 \times 10^2\text{ J}$       5.  $4.16\text{ J/cal}$

**BEGINNER'S BOX - 8**

- |    |   |    |   |
|----|---|----|---|
| 1. | 3650 J  | 2. | (a) 162.5 cal; (b) 237.5 cal                                    |
| 3. | (A) – P, (B) – q, (C) – r                                     | 4. | (A) – q, (B) – r, (C) – p, (D) – s                              |
| 5. | (A) – p, s, (B) – p, q, r, s, (C) – s, (D) – q, s, (E) – r, s |    |   |
| 6. | (a) 700 J; (b) 500 J  | 7. | (a) $T_B = 150 \text{ K}$ ; $T_C = 37.5 \text{ K}$ ; (b) 2400 J |
| 8. | 1152.6 joule  |    |   |

**BEGINNER'S BOX - 9**

- |    |       |    |       |    |                                |
|----|-------|----|-------|----|--------------------------------|
| 1. | 168 J | 2. | 400 K | 3. | (a) 40% ; (b) 240K ; (c) 400 J |
| 4. | 550 K | 5. | 250 K | 6. | 126 watt.                      |