Recapitulation of Rational Numbers

Introduction

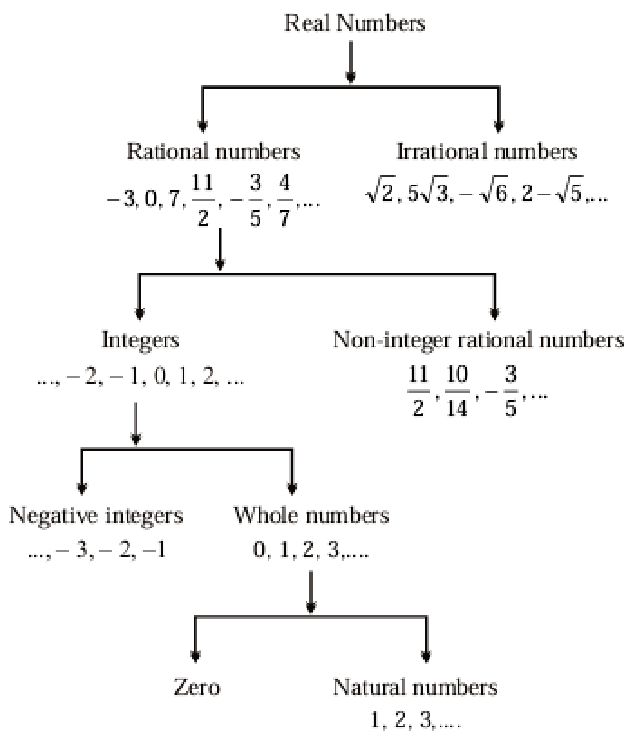
We know that for two given integers p and q, their sum p + q, difference p – q and product pq is always integer. But the system of integers suffered from the defect that division is not always possible within the system. For example, to problems such as 3 ÷ 5 or – 4 ÷ 3 there was no answer. That is to say no integer could be found to fill in the blank 5 × ....= 3 or 3 × ....= –7. Therefore, need was felt to go beyond integers and construct a new number system which include integers and in which all division could be carried out. The numbers that were created were called Rational Numbers. The word 'rational' is derived from the word ratio.

Definition:

A rational number is any number that can be written in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0. For example, $\frac{5}{6}, -\frac{6}{11}, \frac{8}{-9}$, are rational numbers.

Numbers

In Hindu Arabic system we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits to represent any number. A group of figures denoting a number is called a numeral.



i. Natural Numbers: (N): The set of numbers 1, 2, 3, 4…… ∞ are called natural numbers and decimal numbers are not allowed in natural numbers. N = {1, 2, 3…∞} Addition and multiplication are closure, commutative and associative for natural numbers.

ii. Whole Numbers: (W): The set of natural numbers with '0' is called set of whole numbers W = {0, 1, 2, 3, ∞} Addition and multiplication are closure, commutative and associative for whole numbers.

iii. Integers (I or z): The set of positive and negative without decimal numbers, is called integers. z = I = {–∞ …… –3, –2, –1, 0, 1, 2, 3…∞} Addition and multiplication are closure, commutative and associative for Integers.

iv. Even Numbers: All integers which are divisible by 2 are called even numbers and denoted by 2n where n is integer. So, E = {....... –4, –2, 0, 2, 4 ....}

v. Odd Numbers: All integers which are not divisible by 2 are called odd numbers and denoted by 2n + 1, where n is integer. So, O = {–7, –5, –3, –1, 1, 3 ...}

vi. Prime Numbers: All natural numbers that have one & itself only as their factors are called prime numbers. So, P = {2, 3, 5, 7, 11, 13, 17, 19, 23,} * 2 is only even prime number & it is smallest prime number.

vii. Composite Numbers: All natural numbers which are not prime called composite numbers. So, C = {4, 6, 8, 9, 10, 12, 14........} * 1 is neither prime nor composite number.

viii. Co–prime Numbers: If the H.C.F. (or G.C.D.) of the given numbers is 1 then they are known as co–prime numbers. Eg. 5, 8 are co–prime …. θ Their HCF is 1.

* Any Two consecutive numbers are always co–prime.

ix. Real Numbers: Numbers which can represent actual physical quantities in a meaningful way are known as real numbers. These can represent on the number line. Number line is geometrical straight line with arbitrarily defined zero (origin).

x. Rational Numbers: The real numbers which are in form of q /p where p and q are integers and q ≠ 0 Eg. $\frac{5}{7}, -\frac{3}{8}, \frac{11}{1} = 11, 0, \frac{2}{71}$..... Etc.

- All natural numbers, whole numbers and integers are rational.
- Rational numbers, includes all integers, terminating fractions (if the decimal parts are terminating like 0.2, 0.5, –3.5 etc.) and non–terminating recurring decimals (like 6. 0, 3.777 .... etc.)

- Rational number is in standard form or simplest form if H.C.F. of numerator and denominator is 1.

Note:

(i) 1 is neither prime nor Composite.

(ii) 2 is the only even number which is prime.

(iii) There are 25 prime numbers between 1 & 100.

(iv) The sum (or difference) of a rational number and an irrational number is irrational.

(v) The product of a rational and an irrational number is irrational (Except 0).

Reciprocal or multiplicative inverse

If $\frac{a}{b}$ be a rational number then $b\ /a$ is called multiplicative inverse if: $\frac{a}{b} \times \frac{b}{a}$ = 1

Ex.1

Is $\frac{8}{9}$ the multiplicative inverse of $\left(-1\,\frac{1}{8}\right)$=? Why or why not?

Sol. $\left(-1\,\frac{1}{8}\right)$ = $-\frac{9}{8}$

$\frac{8}{9} \times \left(-1\,\frac{1}{8}\right)$= $\frac{8}{9} \times -\frac{9}{8}$ = 1 $\neq$ 1

$\frac{8}{9}$ is not the multiplicative inverse of $\left(-1\,\frac{1}{8}\right)$ =.

(i) Zero has no reciprocal.

(ii) Reciprocal of 1 is 1.

(iii) Reciprocal of –1 is –1.

ABSOLUTE VALUE OF A RATIONAL NUMBER Absolute value of a rational number is its numerical value (value without signs) For example,

$$\left|-\frac{3}{5}\right| = \frac{3}{5} \ \& \ \left|\frac{7}{9}\right| = \frac{7}{0}$$

Properties: The absolute value of the sum of two rational numbers is always less than or equal to the sum of the absolute values of the given numbers. $|x + y| \leq |x| + |y|$

The absolute value of the product of two rational numbers is equal to the product of the absolute values of the given numbers.

$|x \times y| = |x| \times |y|$