Point

1. System of Co-ordinates

1.1 Cartesian Co-ordinates :

Let XOX′ and YOY′ be two perpendicular straight lines drawn through any point O in the plane of the paper. Then

1.1.1 Axis of x : The line XOX′ is called axis of x.

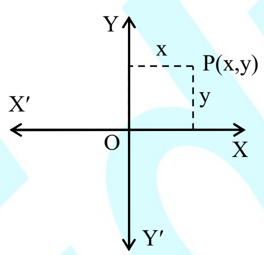
1.1.2 Axis of y : The line YOY′ is called axis of y.

1.1.3 Co-ordinate axes : x axis and y axis together are called axis of co-ordinates or axis of reference.

1.1.4 Origin : The point 'O' is called the origin of co-ordinates or the Origin.

1.1.5 Oblique axis : If both the axes are not perpendicular then they are called as Oblique axes.

1.1.6 Cartesian Co-ordinates : The ordered pair of perpendicular distance from both axis of a point P lying in the plane is called **Cartesian Co-ordinates** of P. If the Cartesian co-ordinates of a point P are (x, y) then x is called abscissa or x coordinate of P and y is called the ordinate or y co-ordinate of point P.



Note :

(i) Co-ordinates of the origin is (0, 0).

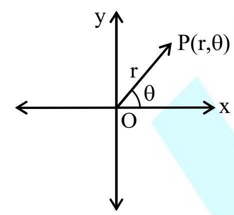
(ii) y co-ordinate on x- axis is zero.

(iii) x co-ordinate on y- axis is zero.

1.2 Polar Co-ordinates :

Let OX be any fixed line which is usually called the initial line and O be a fixed point on it. If distance of any point P from the pole O is 'r' and $\angle XOP = \theta$, then (r, θ) are called the polar co-ordinates of a point P.

If (x, y) are the Cartesian co-ordinates of a point P, then



$$x = r\cos\theta \;; \qquad y = r\sin\theta$$

and $$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

2. Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Note :

(i) Distance of a point P(x,y) from the origin $= \sqrt{x^2 + y^2}$

(ii) Distance between two polar co-ordinates $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$ is given by

$$AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$

3. Applications of Distance Formula

3.1 Position of Three Points :

Three given points A, B, C are collinear, when sum of any two distance out of AB, BC, CA is equal to remaining third distance. Otherwise the points will be vertices of a triangles.

3.1.1 Types of Triangle : If A, B and C are vertices of triangle then it would be.

(a) Equilateral triangle, when AB = BC = CA.

(b) Isosceles triangle, when any two distance are equal.

(c) Right angle triangle, when sum of square of any two distances is equal to square of the third distance.

3.2 Position of four Points :

Four given point A, B, C and D are vertices of a

(a) Square if AB = BC = CD = DA and AC = BD

(b) Rhombus if AB = BC = CD = DA & AC ≠ BD

(c) Parallelogram if AB = DC; BC = AD; AC ≠ BD

(d) Rectangle if AB = CD; BC = DA; AC = BD

	Quadrilateral	Diagonals	Angle between diagonals
(i)	Parallelogram	Not equal	$\theta \neq \frac{\pi}{2}$
(ii)	Rectangle	Equal	$\theta \neq \frac{\pi}{2}$
(iii)	Rhombus	Not equal	$\theta = \frac{\pi}{2}$
(iv)	Square	Equal	$\theta = \frac{\pi}{2}$

Note :

(i) Diagonal of square, rhombus, rectangle and parallelogram always bisect each other.

(ii) Diagonal of rhombus and square bisect each other at right angle.

(iii) Four given points are collinear, if area of quadrilateral is zero.

4. Section Formula

Co-ordinates of a point which divides the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ are.

(i) For internal division

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

(ii) For external division

$$= \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

(iii) Co-ordinates of mid point of PQ are

put $m_1 = m_2$; $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Note :

(i) Co-ordinates of any point on the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are

$$\left(\frac{x_1 + \lambda x_2}{2}, \frac{y_1 + \lambda y_2}{2}\right), (\lambda \neq -1)$$

(ii) Lines joins (x_1, y_1) and (x_2, y_2) is divided by

(a) x axis in the ratio $= -y_1 / y_2$

(b) y axis in the ratio $= -x_1 / x_2$

if ratio is positive divides internally, if ratio is negative divides externally.

(iii) Line $ax + by + c = 0$ divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio

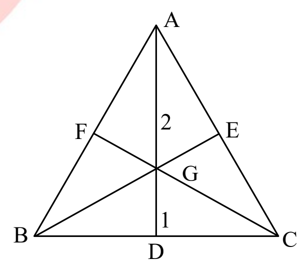
$$-\left(\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right)$$

5. Co-ordinate of some particular Point

Let $A(x_1,y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of any triangle ABC, then

5.1 Centroid :

The centroid is the point of intersection of the medians (Line joining the mid point of sides and opposite vertices).

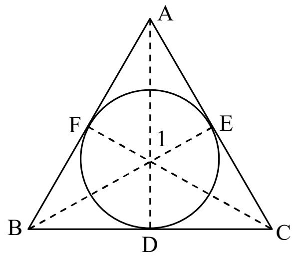


Centroid divides the median in the ratio of 2 : 1.
Co-ordinates of centroid

$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

5.2 Incentre :

The incentre of the point of intersection of internal bisector of the angle. Also it is a centre of a circle touching all the sides of a triangle.



Co-ordinates of in centre

$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$ where a, b, c are the sides of triangle ABC.

Note :

(i) Angle bisector divides the opposite sides in the ratio of remaining sides eg.

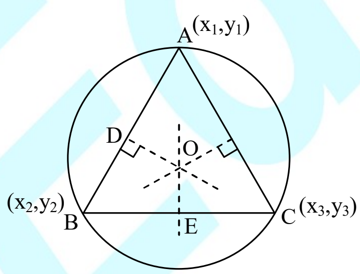
$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$$

(ii) Incentre divides the angle bisectors in the ratio $(b + c):a$, $(c + a):b$, and $(a + b):c$

(iii) **Excentre :** Point of intersection of one internal angle bisector and other two external angle bisector is called as excentre. There are three excentre in a triangle. Co-ordinate of each can be obtained by changing the sign of a, b, c respectively in the formula of In centre.

5.3 Circumcentre :

It is the point of intersection of perpendicular bisectors of the sides of a triangle. It is also the centre of a circle passing vertices of the triangle. If O is the circumcentre of any triangle ABC, then $OA^2 = OB^2 = OC^2$

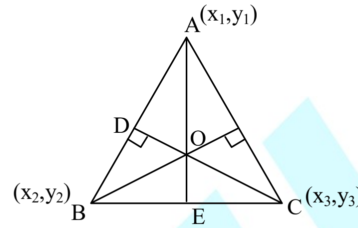


Note :

If a triangle is right angle, then its circumcentre is the mid point of hypotenuse.

5.4 Ortho Centre :

It is the point of intersection of perpendicular drawn from vertices on opposite sides (called altitudes) of a triangle and can be obtained by solving the equation of any two altitudes.



Note :

If a triangle is right angle triangle, then orthocentre is the point where right angle is formed.

Remarks :

(i) If the triangle is equilateral, then centroid, incentre, orthocentre, circumcentre, coincides

(ii) Ortho centre, centroid and circumcentre are always colinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1

(iii) In an isosceles triangle centroid, orthocentre, incentre, circumcentre lies on the same line.

6. Area of Triangle and Quadrilateral

6.1 Area of Triangle

Let $A(x_1,y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of a triangle, then -

$$\text{Area of Triangle ABC} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[\, x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

Note :

(i) If area of a triangle is zero, then the points are collinear.

(ii) In an equilateral triangle

(a) having sides 'a' area is $= \frac{\sqrt{3}}{4}a^2$

(b) having length of perpendicular as 'p' area is $\frac{p^2}{\sqrt{3}}$

6.2 Area of quadrilateral :

If (x_1,y_1), (x_2, y_2), (x_3, y_3) and (x_4, x_4) are vertices of a quadrilateral then its area

$$= \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)]$$

Note :

(i) If the area of quadrilateral joining four points is zero then those four points are colinear.

(ii) If two opposite vertex of rectangle are (x_1, y_1) and (x_2, y_2) and sides are parallel to coordinate axes then its area is

$= |(y_2 - y_1)(x_2 - x_1)|$

(iii) If two opposite vertex of a square are A (x_1, y_1) and C (x_2, y_2) then its area is

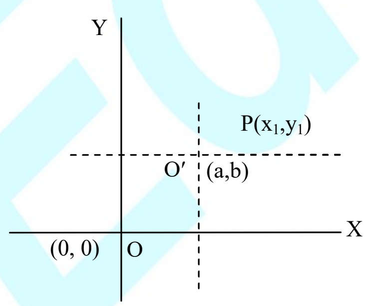
$$= \frac{1}{2}AC^2 = \frac{1}{2}[(x_2 - x_1)^2 + (y_2 - y_1)^2]$$

7. Transformation of Axes

7.1 Parallel transformation :

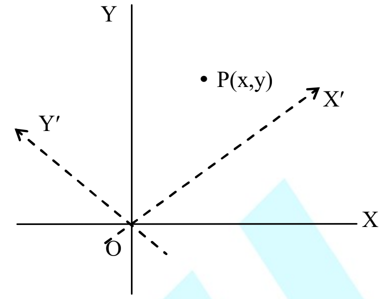
Let origin O(0, 0) be shifted to a point (a, b) by moving the x axis and y axis parallel to themselves. If the co-ordinate of point P with reference to old axis are (x_1, y_1) then co-ordinate of this point with respect to new axis will be $(x_1 - a, y_1 - b)$

$P(x', y') = P(x_1 - a, y_1 - b)$



7.2 Rotational transformation :

Let OX and OY be the old axis and OX′ and OY′ be the new axis obtained by rotating the old OX and OY through an angle θ.



Again, if co-ordinates of any point P(x, y) with reference to new axis will be (x′, y′), then

$x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

The above relation between (x, y) and (x′, y′) can be easily obtained with the help of following table.

	$x\downarrow$	$y\downarrow$
$x'\rightarrow$	$\cos\theta$	$\sin\theta$
$y'\rightarrow$	$-\sin\theta$	$\cos\theta$

7.3 Reflection (Image) of a Point :

Let (x, y) be any point, then its image w.r.t.

(i) x-axis $\Rightarrow (x, -y)$

(ii) y-axis $\Rightarrow (-x, y)$

(iii) origin $\Rightarrow (-x, -y)$

(iv) line $y = x \Rightarrow (y, x)$

8. Locus

A locus is the curve traced out by a point which moves under certain geometrical conditions. To find a locus of a point first we assume the Co-ordinates of the moving point as (h, k) then try to find a relation between h and k with the help of the given conditions of the problem. In the last we replace h by x and k by y and get the locus of the point which will be an equated between x and y.

Note :

(i) Locus of a point P which is equidistant from the two point A and B is straight line and is a perpendicular bisector of line AB.

(ii) In above case if

PA = kPB where k ≠ 1

then the locus of P is a circle.

(iii) Locus of P if A and B is fixed.

(a) Circle if ∠APB = Constant

(b) Circle with diameter AB if $\angle ABB = \frac{\pi}{2}$

(c) Ellipse if PA + PB = Constant

(d) Hyperbola if PA – PB = Constant

9. Some important Points

(i) Quadrilateral containing two sides parallel is called as Trapezium whose area is given by $\frac{1}{2}$ (sum of parallel sides) × (Distance between parallel sides)

(ii) A triangle having vertices $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ and $(at_3^2, 2at_3)$, then area is

$$\Delta = a^2[(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)]$$

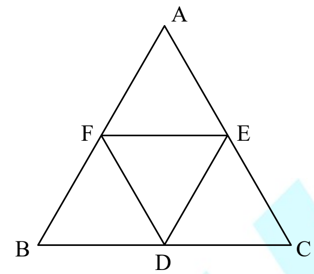
(iii) Area of triangle formed by Co-ordinate axis and the line ax + by + c = 0 is equal to $\frac{c^2}{2ab}$

(iv) When x co-ordinate or y co-ordinate of all vertex of triangle are equal then its area is zero.

(v) In a Triangle ABC, of D, E, F are midpoint of sides AB, BC and CA then

$EF = \frac{1}{2}BC$ and

$\Delta DEF = \frac{1}{4}(\Delta ABC)$



(vi) Area of Rhombus formed by

$ax \pm by \pm c = 0$ is $\frac{2c^2}{ab}$

(vii) Three points (x_1, y_1), (x_2, y_2), (x_3, y_3) are collinear if

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$$

(viii) When one vertex is origin then area of triangle

$$\frac{1}{2} = (x_1y_2 - x_2y_1)$$

(ix) To remove the term of xy in the equation $ax^2 + 2hxy + by^2 = 0$, the angle θ through which the axis must be turned (rotated) is given by

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$$