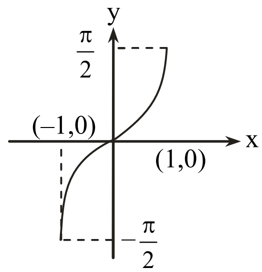
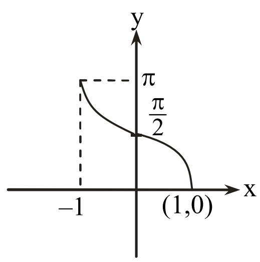
Inverse Trigonometric Function

1. Graph of different inverse Trigonometric function

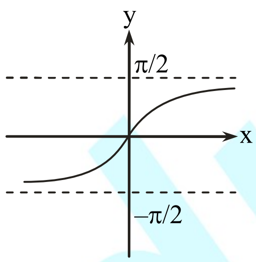
(i) $f(x) = \sin^{-1} x$



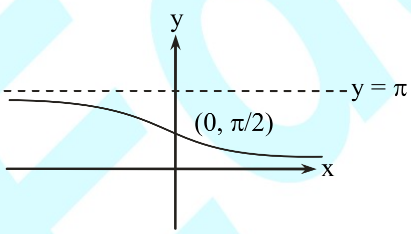
(ii) $f(x) = \cos^{-1} x$



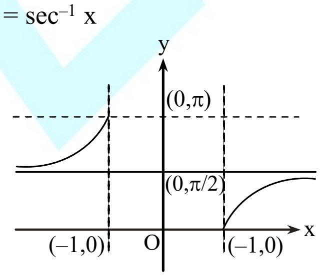
(iii) $f(x) = \tan^{-1} x$



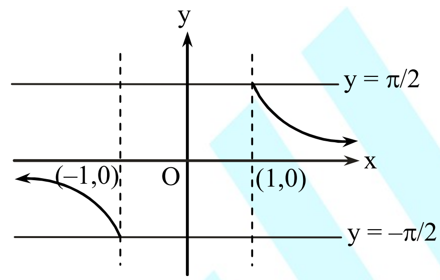
(iv) $f(x) = \cot^{-1} x$



(v) $f(x) = \sec^{-1} x$



(vi) $f(x) = \text{cosec}^{-1} x$



2. Domain & range of inverse Trigonometric function

Function	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
$\text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

Note :

(i) 1st quadrant is common to all inverse function

(ii) 3rd quadrant is not used in inverse function

(iii) 4th quadrant is used in the clockwise direction i.e. $-\frac{\pi}{2} \le y \le 0$

3. Properties of inverse Trigonometric function

P-1

(i) $\sin^{-1}(\sin\theta) = \theta$,

Provided that $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(ii) $\cos^{-1}(\cos\theta) = \theta$,

Provided that $0 \le \theta \le \pi$

(iii) $\tan^{-1}(\tan\theta) = \theta$,

Provided that $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(iv) $\cot^{-1}(\cot\theta) = \theta$,

Provided that $0 < \theta < \pi$

(v) $\sec^{-1}(\sec\theta) = \theta$,

Provided that $0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$

(vi) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$,

Provided that $-\frac{\pi}{2} \le \theta < 0$ or $0 < \theta \le \frac{\pi}{2}$

P-2

(i) $\sin(\sin^{-1}x) = x$,

Provided that $-1 \le x \le 1$

(ii) $\cos(\cos^{-1}x) = x$,

Provided that $-1 \le x \le 1$

(iii) $\tan(\tan^{-1}x) = x$,

Provided that $-\infty < x < \infty$

(iv) $\cot(\cot^{-1}x) = x$,

Provided that $-\infty < x < \infty$

(v) $\sec(\sec^{-1}x) = x$,

Provided that $-\infty < x \le 1$ or $1 \le x < \infty$

(vi) $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$,

Provided that $-\infty < x \le -1$ or $1 \le x < \infty$

P-3

(i) $\sin^{-1}(-x) = -\sin^{-1}x$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

(vi) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

P- 4

(i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(iii) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

P- 5 Conversion Property

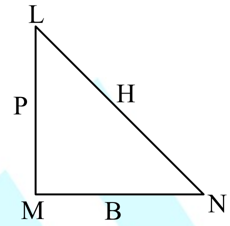
Let $\sin^{-1}x = y$

$\Rightarrow x = \sin y$

$\Rightarrow \operatorname{cosec} y = \left(\frac{1}{x}\right)$

$\Rightarrow y = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \sin^{-1}x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$. Hence



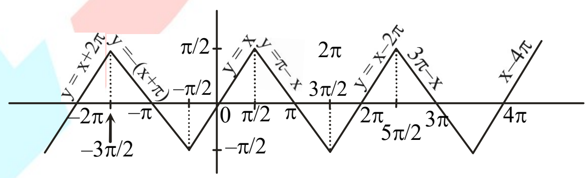
(i) $\sin^{-1}x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$ & $\operatorname{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right)$

Similarly the following results can be obtained

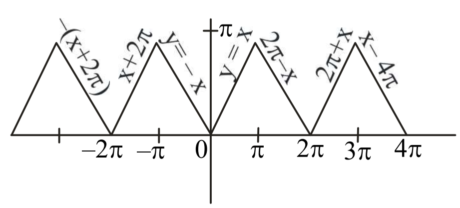
(ii) $\cos^{-1}x = \sec^{-1}\left(\frac{1}{x}\right)$ & $\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$

(iii) $\tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right)$ & $\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$

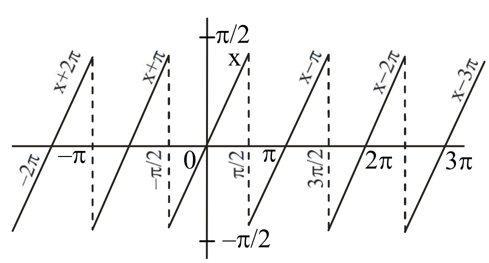
P-6 $\sin^{-1}\sin x$:



$\cos^{-1}\cos x$:



$\tan^{-1}\tan x$:



5. Formulae for sum & difference of inverse Trigonometric functions

(i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$,

if $x > 0, y > 0$ and $xy < 1$

(ii) $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$,

if $x > 0, y > 0$ and $xy > 1$

(iii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(iv) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

(vi) $\sin^{-1}x \pm \sin^{-1}y$

$= \sin^{-1}[x\sqrt{1-y^2} \pm y\sqrt{1-x^2}]$,

if $x, y \geq 0$ and $x^2 + y^2 \leq 1$.

(vii) $\sin^{-1}x \pm \sin^{-1}y$

$= \pi - \sin^{-1}[x\sqrt{1-y^2} \pm y\sqrt{1-x^2}]$,

if $x, y \geq 0$ and $x^2 + y^2 > 1$.

(viii) $\cos^{-1}x \pm \cos^{-1}y$

$= \cos^{-1}[xy \mp \sqrt{1-x^2}\, y\sqrt{1-y^2}]$,

if $x, y > 0$ and $x^2 + y^2 \leq 1$.

(ix) $\cos^{-1}x \pm \cos^{-1}y$

$= \pi - \cos^{-1}[xy \mp \sqrt{1-x^2}\, y\sqrt{1-y^2}]$,

if $x, y > 0$ and $x^2 + y^2 > 1$.

(x) $\cot^{-1}x \pm \cot^{-1}y = \cot^{-1}\left[\frac{xy \mp 1}{y \pm x}\right]$

6. Inverse Trigonometric ratios of multiple angles

(i) $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$; $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

(ii) $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$; $0 \leq x \leq 1$

(iii) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$; $-1 < x < 1$

(iv) $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$; $-\frac{1}{2} \leq x \leq \frac{1}{2}$

(v) $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$; $0 \leq x \leq \frac{1}{2}$

(vi) $3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$; $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

7. Miscelleneous results

(i) $\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right] = \sin^{-1}\left(\frac{x}{a}\right)$

(ii) $\tan^{-1}\left[\frac{3a^2x - x^3}{a(a^2-3x^2)}\right] = 3\tan^{-1}\left(\frac{x}{a}\right)$

(iii) $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

(iv) $\sin^{-1}(x) = \cos^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$= \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \text{cosec}^{-1}\left(\frac{1}{x}\right)$

(v) $\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$= \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

(vi) $\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$= \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}\left(\sqrt{1+x^2}\right)$

$= \text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$