Integers

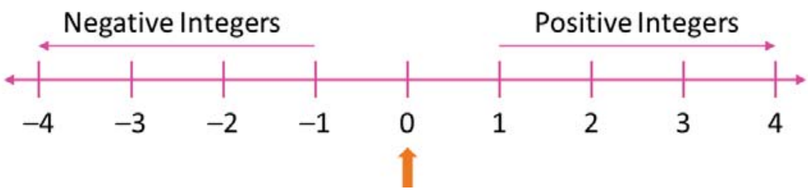
Introduction

The system of whole numbers together with negative numbers are called integers i.e., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4……

- **Positive Integers:** The natural numbers 1, 2, 3, 4… are called positive integers.
- **Negative Integers:** The numbers –1, –2, –3, –4, are called negative integers.
- 'Zero' is an integer which is neither positive nor negative. In real life, integers are used to represent opposite situations.

For Example:

An aeroplane is 1000 m above the ground level	=	+1000
A fish is 1000 m below the ground level	=	–1000
Deposit of Rs.7000 in your bank account	=	+7000



Addition and subtraction of integers

Addition and Subtraction of Integer

Adding integers is the process of finding the sum of two or more integers.

Adding Integers on a Number Line

The addition of integers on a number line is based on the given principles:

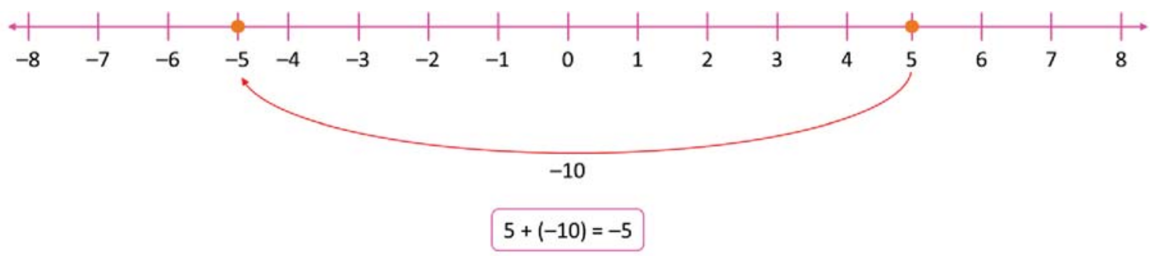
- Adding a positive number is done by moving towards the right side (or the positive side) of the number line.
- Adding a negative integer is done by moving towards the left side (or the negative side) of the number line.
- Any one of the given integers is taken as the base point from where we start moving on the number line.

Let us understand with an example:

Example: Use the number line and add the following integers: 5 + (–10)

Solution:

Since we need to add a negative number (–10), we will move towards the left on the number line. Starting from 5, we will take 10 steps towards the left which will bring us to –5.



Rules of addition of Integers

Rule 1: To add two integers of like signs, add their values regardless of their signs and give the sum their common sign.

For Example: (i) 28 + 35 = 63 (ii) (–15) + (–19) = – (15 + 19) = –34

Rule 2: To add two integers of unlike signs, find the difference between their numerical values regardless of their signs and give the sign of the greater integer to this difference.

For Example: (i) –35 + 12 = –23 (ii) 68 + (–37) = 31

Properties of Addition

- **Closure Property:** Let a and b be any two integers, then a + b will always be an integer. This is called the closure property of addition of integers.

  Examples: 8 + 5 = 13, (–12) + 6 = –6, 9 + (–15) = –6

- **Commutative Property:** If a and b are two integers, then a + b = b + a, i.e., on changing the order of integers, we get the same result. This is called the commutative property of addition of integers.

  Example: 4 + 6 = 6 + 4 =10, (–3) + (12) = (12) + (–3) = 9

- **Associative Property:** If a, b, and c are three integers, then a + (b + c) = (a + b) + c, i.e., on the addition of integers, we get the same result, even if the grouping is changed. This is called the associative property of addition of integers

  Example: [(–3) + (–4)] + (8) = (–3) + [(–4) + 8]

  Or (–7) + 8 = (–3) + 4

  Or 1 = 1

- **Additive Identity:** If zero is added to any integer, the value of the integer does not change. If a is an integer, then a + 0 = a = 0 + a

Hence, zero is called the additive identity of integers.

Examples: 12 + 0 = 12 = 0 + 12

(–3) + 0 = (–3) = 0 + (–3)

- **Additive Inverse:** When an integer is added to its opposite, we get the result as zero (Additive identity). If a is an integer, then (–a) is it's opposite (or vice versa) such that

  a + (–a) = 0 = (–a) + a

Thus, an integer and its opposite are called the additive inverse of each Other.

Example:

9 + (–9) = 0 (–9) + 9, Here 9 and –9 are the additive inverse of each other.

Property of 1: Addition of 1 to any integer gives its successor.

Example: 12 + 1 =13. Hence, 13 is the successor of 12.

Subtraction of Integer

The method of finding the difference between two integers is known as subtracting integers. Depending on whether the numbers are positive, negative, or a mix, the value may increase or decrease.

Let us understand with some examples:

Example: Subtract the following:

(i) 15 from 7 (ii) –7 from 3 (iii) 3 from –7

Solution:

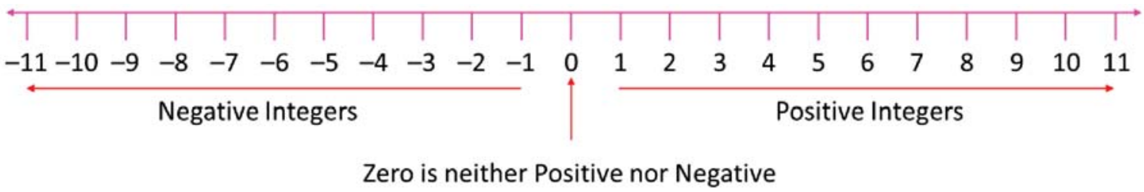
(i) 7 – 15 = 7 + (additive inverse of 15) = 7 + (–15) = –8

(ii) 3 – (– 7) = 3 + (additive inverse of –7) = 3 + 7 = 10

(iii) – 7 – 3 = (–7) + (additive inverse of 3) = (– 7) + (– 3) = –10

To represent integers on number line, we have to follow some basic rules:

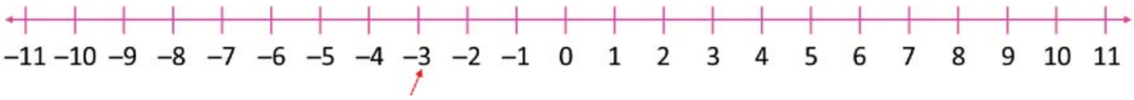
- **Step 1:** Draw a line and mark some points each at equal distance.
- **Step 2:** Mark a point on it.
- **Step 3:** Points to the right of 0 are positive integers. Mark them as: +1, +2, +3, +4 etc.
- **Step 4:** Points to the left of 0 are negative integers. Mark them as: –1, –2, –3, –4, etc.



Let us understand with some examples:

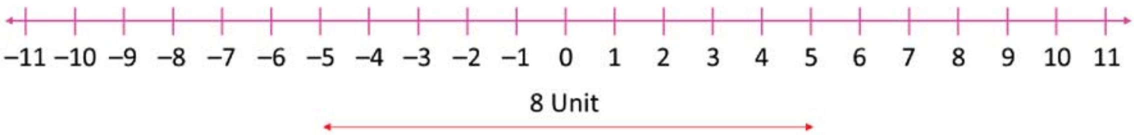
Example: Represent –3 on number line.

Solution:



Example: Find the distance between –4 and 4 on the number line.

Solution:



Properties of Subtraction

- **Closure Property:** Let a and b be any two integers, then a – b will always be an integer. This is called the closure property of subtraction of integers.

  Examples: 8 – 5 = 3, (–12) – (6) = –18

- **Commutative Property:** If a and b are two integers, then a – b ≠ b – a, i.e., commutative property does not hold good for the subtraction of integers.

  Example: 7 – (–8) = 15 but (–8) – 7 = –15

Hence, subtraction of integers is not commutative.

- **Associative Property:** If a, b and c are three integers, then (a – b) – c ≠ a – (b – c) i.e., the associative property does not hold good for the subtraction of integers.

  Example: (8 – 4) – 2 ≠ 8 – (4 – 2)

  Or 4 – 2 ≠ 8 – 2

  Or 2 ≠ 6

Hence, subtraction of integers is not associative.

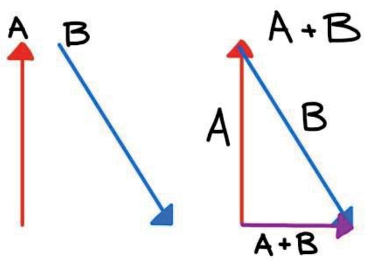
- **Property of Zero:** When zero is subtracted from an integer, we get the same integer, i.e.

  a – 0 = a, where a is an integer

  Example: 12 – 0 = 12

Property of 1: Subtraction of 1 from any integer gives its predecessor.

Example: 15 –1 = 14, Here 14 is the predecessor of 15.



Multiplication of integers

Multiplication of Integers

We can find the product of any two integers using the repeated addition method.

Multiplication of Integers Rules

- The product of a Positive Integer and a Negative Integer is negative.

  Examples: 4 × (–6) = (–24)

- The product of a Negative Integer and a Positive Integer is negative.

  Examples: (–4) × 8 = (–32)

- The product of two Positive Integers is positive.

  Examples: 5 × 10 = 505

- The product of two Negative Integers is always positive.

  Example: (–9) × (–5) = +45

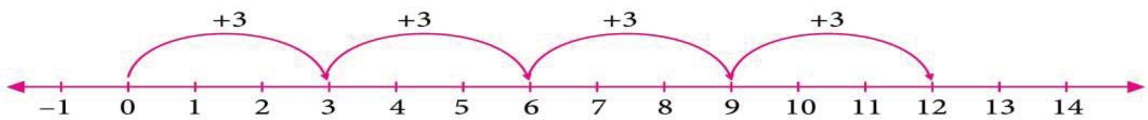
Let us understand with some examples:

Example: Find the value of (+3) × (+4)

Solution: = (+3) × 4

= (+3) + (+3) + (+3) + (+3) = (+12) or 12

On the number line, (+3) × (+4) means moving to the right of zero 4 times in steps of 3.

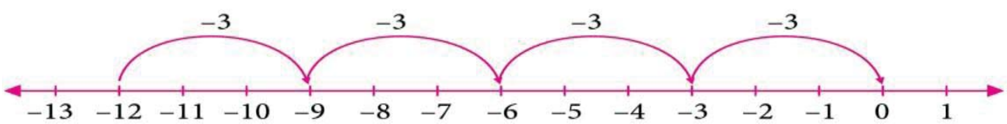


Example: Find (–3) × (+4)

Solution: = (–3) × 4

= (–3) + (–3) + (–3) + (–3) = (–12)

(–3) × (+4) means moving to the left of zero 4 times in steps of 3.



Division of integers

Division of Integers

Division of integers involves the grouping of items. It includes both positive numbers and negative numbers.

The following table will help you remember rules for dividing integers:

Types of Integers	Result	Example
Both Integers Positive	Positive	$24 \div 8 = 3$
Both Integers Negative	Positive	$-24 \div -8 = 3$
1 Positive and 1 Negative	Negative	$-24 \div 8 = -3$

Let us

Understand with some examples:

Example: $(-128) \div (-8) = 16$

$0 \div (115) = 0$

$(21) \div (-21) = -1$

$(68) \div (-1) = -68$

Properties of multiplication of integers

Properties of multiplication of integers

- **Closure Property:** If a and b are two integers, then a × b will also be an integer. This is called the closure property of multiplication of integers.

  Example: 7 × (-5) = (-35)

- **Commutative Property:** If a and b are two integers, then a × b = b × a, i.e., on changing the order of integers, we get the same result. This is called the commutative property of multiplication of integers.

  Example: 6 × 9 = 9 × 6 = 54

Thus, the commutative property holds good for the multiplication of integers.

- **Associative Property:** If a, b and c are three integers, then a × (b × c) = (a × b) × c. This is called the associative property of multiplication of integers.

  Example: [(-5) × 2] × (-3) = (-5) × [2 × (-3)]

  Or (-10) × (-3) = (-5) × (-6)

  Or 30 = 30

Thus, associative property holds good for the multiplication of integers.

- **Multiplicative Identity:** The product of any integer and 1 gives the same integer. If ‘a’ is an integer, then a × 1 = a = 1 × a. Hence, 1 is called the multiplicative identity.

  Example: 19 × 1 = 1 × 19 = 19

- **Multiplicative Inverse:** If ‘a’ is an integer, then $a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$. Thus, an integer and its reciprocal are called the multiplicative inverse of each other.

  Example: $9 \times \frac{1}{9} = 1 = \frac{1}{9} \times 9$

- **Property of Zero:** The product of any integer and zero gives the result as zero. If ‘a’ is an integer, then a × 0 = 0 = 0 × a.
  Example: 9 × 0 = 0 = 0 × 9

Associative Property	(a × b) × = c = a × (b × c)
Commutative Property	a × b = b × a
Distributive Property	a(b + c) = ab + ac a(b – c) = ab – ac
Identity Property	a × 1 = a
Zero Property	a × 0 = 0

Distributive Property

Multiplication distributes over addition. If a, b, and c are three integers, then

$a \times (b + c) = ab + ac.$

This is called the distributive property of multiplication of integers.

Let us understand with some examples:

Example: $(-7) \times [3 + (-4)] = (-7) \times (3) + (-7) \times (-4)$

Or $(-7) \times (-1) = (-21) + 28$

Or $7 = 7$

Hence, integers possess the distributive property of multiplication.

Example: $3 \times [4 - 8] = (3 \times 4) - (3 \times 8)$

Or $3 \times (-4) = (12) - (24)$

Or $-12 = -12$

Properties of Division of Integers

- **Closure Property:** The closure property does not hold good for the division of integers.

  Examples: $15 \div 3 = 5$ (5 is an integer)

  $9 \div 4 = \frac{9}{4}$ ($\frac{9}{4}$ is not an integer)

- **Commutative Property:** If a and b are two integers, then $a \div b \neq b \div a$

  Example: $4 \div 2 = 2$ but $2 \div 4 = \frac{2}{4}$ or $\frac{1}{2}$

- **Associative Property:** If a, b, c are three integers, then $(a \div b) \div c \neq a \div (b \div c)$

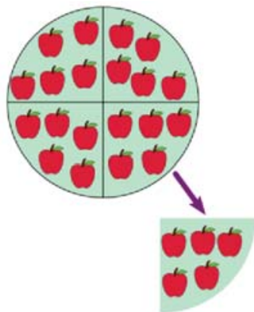
  Example: $(24 \div 2) \div (-2) \neq 24 \div [4 \div (-2)]$

  Or $6 \div (-2) \neq 24 \div (-2)$

  Or $(-3) \neq (-12)$

- **Property of 1:** Any integer divided by 1 gives the same integer as the quotient. If 'a' is an integer, then $a \div 1 = a$.

  Example: $5 \div 1 = 5$



- **Property of Zero:** When zero is divided by any integer, the result is always zero. If a is an integer, then $0 \div a = 0$

  Example: $0 \div 9 = 0$