PIPE AND CISTERN

What a typical question looks like?

In such questions, basically a tank has to be filled by two (or more) pipes and we are given:
1) Time is taken by each pipe to fill the tank.
2) Total time is taken to fill the tank

Inlet pipes are responsible for filling the tank. They, basically, **bring the water in**. *The work done by them is positive*.

Then we an **Outlet pipe**, there can be any number of outlet pipes too. Outlet pipes are responsible for emptying the tank. They, basically, **put the water out.** *The work done by them is negative.*

Rules for solving such questions:
1. If a pipe can fill the tank in 'x ' hours then, the part filled in 1 hour = 1/x

2. If a pipe can empty the tank in ' y' hours then, the part emptied in 1 hour = 1/y

3. If a pipe can fill the tank in 'x ' hours and another can empty it in '*y*' hours then, the **net part filled in 1 hour = 1/x – 1/y; Total time is taken to fill such tank = XY/y-x**

4. A pipe can fill the tank in 'x' hrs. Due to leak, it is filled in 'y' hrs, time is taken by a leak to empty the tank **=xy/y – x hrs**
5. If leak time > Inlet pipe then the tank will be filled; If leak time < Inlet pipe then the tank will be emptied.

Sample Questions:

Qs. 1 – Pipe A can fill the tank in 20 hours while Pipe B alone can fill it in 30 hours and Pipe C can empty the tank in 40 hours. If all the pipes are opened together, in how long will the tank be full?

Solutions – Net part filled in 1 hour = 1/20 + 1/30 – 1/40 (as work done by C is negative)
= 7/120
⇒ A full tank will be full in **120/7 = 17 1/7 hours.**

Q2. There's a leak in the bottom of the tank. When the tank is thoroughly repaired, it would be filled in 3.5 hours. It now takes half an hour longer. If the tank is full, how long would it take to leak the tank?

Sol. Here, clearly the '**leak'** is working as an **Outlet pipe**.
Done using rule 5)

We need to find the time taken to an empty tank by the leak (or outlet pipe) if the tank is full

The repaired tank is filled in 3.5 hours ⇒ Inlet pipe takes 3.5 hours

Un-repaired tank takes 3.5+0.5 = 4 hrs ⇒ time taken 4 hours to fill the tank.

Total time taken to empty such tank **= xy/y-x = 3.5 x 4 / 4 – 3.5 = 28 hrs.**

A leak would empty the cistern in 28 hours.

Q3. Two pipes P and Q would fill the tank in 24 hours and 32 hrs respectively. If both pipes are opened together, find when the first pipe must be turned off so that the tank may be just filled in 16 hrs?

Sol. Suppose the pipe P is closed after 'x' hours.
Then, P pipe would fill in 1 hr = 1/24 and in x hrs = x / 24

Pipe Q would fill in 1 hour = 1 / 32 and in 16 hrs (as tank is full in 16 hrs) = 16 /32 = 1/2

Pipe P work in 'x' hr + Pipe Q work in 16 hrs = 1 (as they complete the 1 unit of work) = x/24 + 16/32 = 1
⇒ x = 12 hours.

Q4. Three pipes A, B and C can fill a cistern in 6 hrs. After working together for 2 hrs, C is closed and A & B fill it in 8 hrs. Then find the time in which cistern can be filled by pipe C.

Sol: A + B + C work in 1 hr = 1/6 of cistern
A+B+C work in 2 hr = 1/6
A+B+C work in 2 hr = 1/6 × 2 = 1/3 of cistern
Unfilled part after 2 hrs = 1 – 1/3 = 2/3 of Cistern
This 2/3 of cistern is filled by A & B in 8 hrs.
⇒ A & B can fill the full cistern in = 8×3/2 = 12 hrs
We know that A+B+C = 6 hrs
C = (A+B+C) – (A+B) = (1/6) – (1/12) = 1/12
⇒ C alone would fill it in 12 hrs.

Q5. A tank has a leak that would empty it in 8 hrs. A tap is turned on which admits 6 liters a minute into the tank, and it's now emptied in 12 hrs. How many liters does the tank hold?

Sol. Time by Outlet Pipe = 8 hrs
Tank emptied in = 12 hrs
Done using rule 5)

Time by Inlet pipe = (12×8)/(12 – 8) = 24 hrs.
Also given: Inlet pipe takes 6 liters in a minute ⇒ In 1 hr, intake = 6×60 = 360L
⇒ Intake in 24 hrs = **360×24 = 8640 liters**
Hence, the total capacity of the tank is **8,640 L.**

Note: *If it's given that tank takes 8 hrs to get full but with a leak, it takes 2 hrs more, then 8 hrs is the time taken by Inlet pipe and 10 hrs is total time to fill with the leak.*

Qs. 6*.* A can fill the tank in 12 minutes, B in 15 minutes and C empties it in 6 minutes. A and B are opened for 5 minutes then C is also opened. At what time is the tank empty?

Sol. A + B in 5 minutes = [1/12 + 1/15]×5 = ¾
⇒ 3/4th part of tank is filled in 5 minutes.
When C is also opened, work done by all pipes in 1 minute = **1/12 + 1/15 – 1/6 = 1/60**

When all three are opened, the tank is emptied in 60 minutes.
So, 3/4 part will be emptied in **= 60 x 3/4 = 45 minutes**

Q7. Two pipes can separately fill a tank in 20 hrs and 30 hrs respectively. Both the pipes are opened to fill the tank but when the tank is 1/3 full a leak is developed in the tank through which 1/3 of water supplied by both the tank leak out. What is the total time taken to fill the tank?

Sol. Time taken by two pipes to fill the tank = (20×30)/(20+30) = 12 hrs.
1/3rd tank is filled in = 12×1/3 = 4 hrs; Left time = 12 – 4 = 8 hrs.

Now, leakage develops which empties 1/3rd of water supplied (by both pipes)
⇒ Now, efficiency of Inlet pipes = 1 – 1/3 = 2/3rd .

Earlier, at 1 efficiency they were taking 8 hrs
now at 2/3rd efficiency they will take 8 ÷ 2/3 = 12 hrs

⇒ Total time taken to fill the tank= 4 + 12 = 16 hrs.

⇒ Time taken to fill after leakage = **12×3 = 36 hrs.**