PROBABILITY

1.1 Basic concept

(i) The probability of occurrence of an event *E*, given that an event *F* has occurred, is called conditional probability of event E and is written as P(E/F) and is given by

$$P(E/F)=\frac{P(E\cap F)}{P(F)},\ P(F)\neq 0.$$

(ii) The Probability of occurrence of an event *F*, given that an event *E* has already occurred, is called conditional probability of event *F* and is written as *P*(*F*/*E*), and is given by

$$P(F/E)=\frac{P(E\cap F)}{P(F)},\ P(E)\neq 0.$$

1.2 Properties of conditional probability :

(i) The Conditional probability of an event E, given that an event F has already occurred is always greater than or equal to 0 and less than or equal to 1.

i.e., $0\leq P(E/F)\leq 1.$

(ii) If S is the sample space of an experiment and F is an event which has already occurred, then P(S/F)=1.

(iii) If A and B are any two events of an experiment of sample space S and F is an event of S which has already occurred, such that $P(F)\neq 0$, then $P(A\cup B/F)=P(A/F)+P(B/F)-P(A\cap B/F).$

If events A and B are mutually exclusive. then $P(A\cup B/F)=P(A/F)+P(B/F).$

(iv) If E is an event of an experiment, whose sample space is S and F is another event which has already occurred , then $P(\overline{E}/F)=1-P(E/F).$

1.3 Multiplication theorem of probability.

Multiplication theorem of probability. If E and F are two given events of an experiment, whose sample space is S, then probability of simultaneous happening of E and F is given by

$P(E\cap F)=P(F).P(F/E), P(E)\neq 0$ or $P(E\cap F)=P(F).P(E/F), P(F)\neq 0.$

- Multiplication rule of probability for more than two events. If E, F and G are three events of a sample space, then $P(E\cap F\cap G)=P(E).P(F/E).P(G/E\cap F).$
- Two events are said to be independent if happening of one does not affect the probability of happening of the other.
  Also two events E and F are said to be independent, if
  $P(F/E)=P(F),\quad P(E)\neq 0.$
  $P(E/F)=P(E),\quad P(F)\neq 0.$
- Events E and F associated with the same random experiment are said to be independent, if $P(E\cap F)=P(E).P(F).$
- Three events E, F and G of an experiment are said to be independent, if
  (i) $P(E\cap F)=P(E).P(F).$ (ii) $P(F\cap G)=P(F).P(G).$
  (iii) $P(E\cap F)=P(E).(G).$ (iv) $P(E\cap F\cap G)=P(E).P(F).P(G).$
- **Multiplication theorem of probability for independent events.** If A and B are two independent events. then probability of simultaneous happening of A and B is given by
  $P(A\cap B)=P(A).P(B)$ or $P(A \text{ and } B)=P(A).P(B).$
- $P(\text{only } A)=P(A\cap\overline{B})=P(A)-P(A\cap B).$
- P(at least one)=1 – P (none) = 1 – P(0)
- (neither A nor B) = $P(\overline{A}\cap\overline{B})=1-P(A\cup B)=1-P(A \text{ or } B)$

- **Partition of a sample space.** A set of events $E_1, E_2, \ldots, E_n$, is said to represent partition of the sample space S, of an experiment, if

  (i) $P(E_i) > 0, \forall i = 1,2, \ldots, n$. (ii) $E_i \cap E_j = \Phi\, i \neq j, i, j = 1, 2, \ldots, n$. (iii) $E_1 \cup E_2 \cup E_3 \ldots \cup E_n = S$

  We can also say that the events $E_1, E_2, \ldots E_n$ represent a partition of a sample space if they are mutually exclusive exhaustive events, with non zero probability.
- **Theorem of total probability.** Let events $E_1, E_2, \ldots, E_n$ from a partition of the sample space S, of an experiment. If A is any event associated with the sample space S, then

$$P(A) = P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + \ldots + P(E_n).P(A/E_n) = \sum_{i=1}^{n} P(E_i).P(A/E_1).$$

1.4 Baye's Theorem. Let $E_1, E_2, \ldots E_n$ be n exhaustive events, with non-zero probabilities, of a random experiment, i.e., events $E_1, E_2, \ldots E_n$ constitute partition of sample space S. If A be any arbitrary event of the sample space of the above experiment with P(A)>0, then

i.e., $$P(E_i/A) = \frac{P(E_i).P(A/E_i)}{\sum_{d=1}^{n} P(E_j).P(A/E_j)}; 1 \le i \le n.$$

1.5 A probability distribution represent that how probability of an experiment is distributed over different exhaustive events of the experiment. If $x_1, x_2 \ldots, x_n$ are the possible real number values associated to different exhaustive events of an experiment and $p_1, p_2, \ldots p_n$, are their respective probabilities, then distribution is represented as

x	x_1	x_2	x_3	...	x_n	
P(x)	P_1	P_2	P_3	...	P_n	$\sum P_i = 1$

- For a given probability distribution

  (i) Mean $(\mu) = \sum_{i=1}^{n} x_i P_i; \mu$ is also called expected value of x, E (x).

  (ii) Variance $(\sigma^2) = \sum_{i=1}^{n} p_i x_i^2 - \mu^2$. (iii) Standard Deviation = σ = $\sqrt{\text{Variance}}$.
- Different trials of a random experiment are called Bernoulli trials, if :
  (i) The number of trials n is finite.
  (ii) Each trial has exactly two outcomes known as success and non-success. We have
  P (success) + P (non-success) =1.
  (iii) The trial are independent .
  (iv) The probability of success remains same for each trial , denoted by 'p' and the of non-success is denoted by 'q'. We have p + q = 1.

- Binomial Distibution, denoted by B (n,p) is given by $(q+p)^n$,
  where p represents probability of success, q represents probability of non-success and n is number of trials.
  Probability of r successes, P (r) si given by P(r) = ${}^nC_r q^{n-r} p^r$.
  n, p are parameters of binomial distribution.

- For Binomial Distibution

  (i) mean = np (ii) Variance = npq (iii) Standard Deviation = $\sqrt{npq}$

  (iv) Recurrence formula : $P(r+1) = \frac{n-r}{r+1}.\frac{p}{q}.P(r); \text{for } 0 \le r < n; p(0) = q^n$

SOLVED PROBLEMS

Ex.1 ***Mother, father and son line up at random for a family picture.***
E : Son on one end F : Father in middle
Find P (E | F).

Sol. Total number of equally likely ways for placing mother, father and son in a row =3 ! =6.

E = {SMF, SFM, MFS, FMS } F = {SMF, FMS}

$E \cap F$ = {SMF, FMS}

$$P(E) = \frac{4}{6} = \frac{2}{3};\ P(F) = \frac{2}{6} = \frac{1}{3};\ P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

Now, $$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{6}}{\frac{2}{6}} = 1$$

Ex.2 ***Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'***

Sol. Here $n(S) = 6 \times 6 = 36$

∴ Let, E : Two numbers appearing on throwing two dice are different.

∴ n(E) = 6 × 6 – 6 = 30

$[\because (1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \text{ are ruled out}]$

Let, F The sum of numbers on the dice is 4,

F ={(1,3), (2,2),(3,1)}

∴ $n(F) = 3$

$E \cap F = \{(1,3,),(31)\}$

∴ $n(E \cap F) = 2$ Now,

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{1}{15}$$

Hence, the probability of the event "the sum of numbers on the dice is 4" is $\frac{1}{15}$.

Ex.3 ***An instructor has a question bank consisting of 300 easy True/False question, 200 difficult True/False question, 500 easy multiple choice questions and 400 difficult multiple choice question. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question ?***

Sol. Here, total number of question

= 300 + 200 + 500 + 400 =1400

Let, E : an easy question Then,

n (E) = 300 + 500 =800

F: a multiple choice question Then,

n (F) = 500 + 400 = 900

∴ $E \cap F$ = an easy and a multiple choice question

∴ $n(E \cap F) = 500$

Now, $$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{500}{1400}}{\frac{900}{1400}} = \frac{5}{9}$$

Ex.4 ***Consider an experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.***

Sol. Here

S={31, 32, 33, 34, 35, 36, 61, 62, 63, 64, 65, 66, 1T, 1H, 2T, 2H, 4T, 4H, 5T, 5H}

∴ $n(S) = 20$

Let, E: The coin shows a tail. Then

E ={1T, 2T, 4T, 5T}

n (E) =4

Let, F = At least one die shows a 3. Then

F = {31, 32, 33, 34, 35, 36, 63}

n (F) = 7

∴ $E \cap F = \phi$

$n(E \cap F) = 0$

Now, $$P(E|F) = \frac{P(E \cap F)}{P(E)} = \frac{0}{\frac{7}{20}} = 0$$

Ex.5 ***Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the younger is a girl, (ii) at least one is a girl ?***

Sol. S = {BB, BG, GB, GG}, where the first letter in a pair denotes younger child.

Lat A : Event that both are girls. Than

A = {GG}

(i) Let B : The younger child is a girl.
Then B= {GB, GG}
∴ $A \cap B = \{GG\}$

Now, $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{\frac{1}{4}}{\frac{2}{4}} = \dfrac{1}{2}$

(ii) Let C : Event that at least one child is a girl
Then C = {BG, GB, GG}
$A \cap C = \{GG\}$

Now,

$$P(A|C) = \frac{P(A\cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Ex.6 ***A die is thrown three times. Events A and B are defined as below :***
A : 4 on the third throw
B : 6 on the first and 5 on the second throw
Find the probability of A given that B has already occurred.

Sol. The sample space has 216 outcomes.

Now, $A = \begin{Bmatrix} (1,1,4) & (1,2,4)... & (1,6,4) & (2,1,4) & (2,2,4)... & (2,6,4) \\ (3,1,4) & (3,2,4)... & (3,6,4) & (4,1,4) & (4,2,4) & (4,6,4) \\ (5,1,4) & (5,2,4)... & (5,6,4) & (6,1,4) & (6,2,4) & (6,6,4) \end{Bmatrix}$

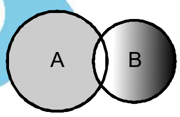
B = {(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)} and $A \cap B = \{(6,5,4)\}$

Now, $P(B) = \dfrac{6}{216}$ and $P(A\cap B) = \dfrac{1}{216}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

Ex.7 ***Given two independent events A and B such that P (A)=0.3, P (B) = 0.6. Find :***
(i) P (A and B) (ii) P (A and not B)
(iii) P (A or B) (iv) P (neither A nor B)

Sol. (i) P (A and B)
= $P(A\cap B)$
= P(A). P (B)
[∴ A and B are independent events]
= 0.3 × 0.6 = 0.18



$A\cap B' = A - (A\cap B)$

(ii) P (A and not B) = P(A) - $P(A\cap B)$
= 0.3 - 0.18 = 0.12

(iii) P (A or B) = $P(A\cup B)$
= P(A)+P(B)− $P(A\cap B)$
= 0.3 + 0.6 - 0.18
= 0.9 - 0.18 = 0.72

(iv) P (neither A nor B) = $P(A'\cap B')$
= 1 - $P(A\cup B)$
= 1 - 0.72
= 0.28

Ex.8 If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by 1 - P (A′) P (B′)

Sol. We have P (at least one of A and B)
= $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
= P (A) + P (B) - P (A) P (B)
[∴ $P(A\cap B)$ = P (A). P (B)]
= P (A) + P (B) [1−P (A)] = P (A) + P (B). P(A′)
= [1 − P(A′)] + P(B) P(A′)
= 1- P (A′) [1-`P(B)] = 1- P (A′) P (B′)

Ex.9 ***A die is tossed thrice. Find the probability of getting an odd number at least once.***

Sol. If A, B, C are three independent events, then the probability of occurrence of at least one of A, B, and C is given by 1-P(A′) P (B′)P (C′).

Let, A (or B or C) : Event of getting an odd number

⇒ A′ (or B or C) : Event of getting an even number

⇒ P (A′) = P (B′) = P(C′) = $\dfrac{1}{2}$

Hence, the required probability is

$$1 - P(A')P(B')P(C') = 1 - \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$$

Ex.10 Two balls are drawn at random with replacement form a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red
(ii) first ball is black and second is red
(iii) one of them is black and other is red.

Sol. Here,

(i) P (a red ball) = P (R) = $\dfrac{8}{18} = \dfrac{4}{9}$

Since, the ball is replaced

P (R) for the second draw = $\dfrac{8}{18} = \dfrac{4}{9}$

∴ P (RR) = P (both the balls red) = $\dfrac{4}{9}\times\dfrac{4}{9} = \dfrac{16}{81}$

(ii) P (B) for first draw = $\frac{10}{18} = \frac{5}{9}$

P (R) for second draw = $\frac{8}{18} = \frac{4}{9}$

$\therefore$ Required probability = P (BR) = $\frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$

(iii) P (one of them is black and other is red)

= P (BR) + P (RB) = $\frac{20}{81} \frac{4}{9} \times \frac{5}{9} = \frac{40}{81}$

Ex.11 One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent ?

(i) E: 'the card drawn is a spade'
F: 'the card drawn is an ace'

(ii) E: 'the card drawn is black'
F: 'the card drawn is a king'

(iii) E: 'the card drawn is a king or queen'
F: 'the card drawn is a queen or jack'.

Sol. (i) E : the card drawn is spade

Then, P (E) = $\frac{13}{52} = \frac{1}{4}$

F: the card drawn is an ace

then, P (F) = $\frac{4}{52} = \frac{1}{13}$

$\Rightarrow$ $E \cap F$: the card drawn is an ace or a spade

$\therefore$ P ($E \cap F$) = $\frac{1}{52}$

Now, P (E) P (F) = $\frac{1}{4} \times \frac{1}{13} = \frac{1}{52} = P(E \cap F)$

$\Rightarrow$ E and F are independent events.

(ii) E : the ard drawn is black P (E)= $\frac{26}{52} = \frac{1}{2}$

F : the card is a king P (F) = $\frac{4}{52} = \frac{1}{13}$

$\Rightarrow$ $E \cap F$: The card is a black king

P ($E \cap F$) = $\frac{2}{52} = \frac{1}{26}$

Now, P (E) P (F) = $\frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$ = P ($E \cap F$) $\Rightarrow$ E and F are independent events.

(iii) E: the card drawn is a king or a queen

P (E) = $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

F: the card drawn is a queen or a jack

P (F) = $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

$\Rightarrow$ $E \cap F$ = the card drawn is a queen

$\therefore$ P ($E \cap F$) = $\frac{4}{52} = \frac{1}{13}$

Now, P (E) P (F) = $\frac{2}{13} \times \frac{2}{13} = \frac{4}{169} \neq P(E \cap F)$ Hence, E and F are not independent events.

Ex.12 Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem.

Sol. Let, E : Solving problem by A
F : Solving problem by B

It is given that P (E) = $\frac{1}{2}$ and

P (F) = $\frac{1}{3}$

(i) P (The problem is solved)

= P ($E \cup F$) = P (E) + P (F) - P ($E \cap F$)

= P (E) +P (F) - P (E) P (F)

[$\because$ E and F are independent events]

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{2} \times \frac{1}{3} = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

(ii) P (exactly one of them solves the problem)

= P ($E \cap F$) + P ($E \cap F'$) = P (E') P (F) + P (E) P (F')* = [1-P (E)] P (F) + P (E) [1-P (F)]

$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

Ex.13 In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspapers also.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper also.

Sol. Here, $P(H) = \frac{60}{100} = \frac{3}{5}$

[H stands for Hindi newspaper]

$P(E) = \frac{40}{100} = \frac{2}{5}$

[E stands for English newspaper]

and $P(H \cap E) = \frac{20}{100} = \frac{1}{5}$

(a) $P(H' \cap E') = 1 - P(H \cup E)$

$= 1 - [P(H) + P(E) - P(H \cap E)]$

$= 1 - \left[\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right] = 1 - \frac{4}{5} = \frac{1}{5}$

(b) $P(E|H) = \frac{P(H \cap E)}{P(H)} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$

(c) $P(H|E) = \frac{P(H \cap E)}{P(E)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$

* If E and F are independent events, then
(i) E and F′ (ii) E′ and F
(iii) E′ and F″ are also independent events.

Ex.14 From a lot of 30 bulbs which include 6 defective, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Sol. Let X denote the random variable "the number of defective bulbs". Then X can take values 0, 1, 2, 3 and 4.

Now, P (X = 0) = P (No defective bulb)

$= \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30} = \left(\frac{4}{5}\right)^4 = \frac{256}{625}$

P (X = 1) = P (only one defective bulb)

$= {}^4C_1 \left(\frac{6}{30}\right)\left(\frac{24}{30}\right)^3 = 4 \times \frac{1}{5} \times \left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P (X = 2) = P (only 2 defective bulbs)

$= {}^4C_2 \left(\frac{6}{30}\right)^2\left(\frac{24}{30}\right)^2 = 6 \times \left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P (X = 3) = P (only 3 defective bulbs)

$= {}^4C_3 \left(\frac{6}{30}\right)^3\left(\frac{24}{30}\right) = 4 \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right) = \frac{16}{625}$

P(x = 4) = P(all defective bulbs)

$= {}^4C_4 \left(\frac{6}{30}\right)^4 = 1 \times \left(\frac{1}{5}\right)^4 = \frac{1}{625}$ 2

Ex.15 Let X be the random variable which assumes values 0, 1, 2, 3, such that 3P (X = 0) = 2P (X = 1) = p(X = 2) = 4p(X = 3) Find the probability distribution of X.

Sol. Let P (X = 2) = p. Then, $P(X = 0) = \frac{p}{3}$.

$P(X = 1) = \frac{p}{2}$ and $P(X = 3) = \frac{p}{4}$

Now, we know that

P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1

$\therefore \quad \frac{p}{3} + \frac{p}{2} + p + \frac{p}{4} = 1$

$\Rightarrow \quad 4p + 6p + 12p + 3p = 12$

$\Rightarrow \quad 25p = 12 \quad \Rightarrow \quad p = \frac{12}{25}$

Thus, the probability distribution of X therefore

is

X	0	1	2	3
P(X)	$\frac{4}{25}$	$\frac{6}{25}$	$\frac{12}{25}$	$\frac{3}{25}$

Ex.16 A die is tossed once If the random variable X is defined as

$$X = \begin{cases} 1, \text{ if the die results in an even number} \\ 0, \text{ if the die results in an odd number} \end{cases}$$ ***then***

find the mean and variance number of X.

Sol. In tossing a die once, the sample space S is given by

S = {1, 2, 3, 4, 5, 6}

$\therefore$ P (getting an even number) = $\frac{3}{6} = \frac{1}{2}$

P (getting an odd number) = $\frac{3}{6} = \frac{1}{2}$

As given, X takes the value 0 or 1.

P(X = 0) = P (getting an odd number) = $\frac{1}{2}$

P(X = 1) = P (getting an even number) = $\frac{1}{2}$

Thus, the probability distribution of X is given by

X	0	1
P(X)	$\frac{1}{2}$	$\frac{1}{2}$

Now, Mean$=\mu=E(X)=\Sigma x_i p_i = \left(0\times\frac{1}{2}\right)+\left(1\times\frac{1}{2}\right)$

$= 0 + \frac{1}{2} = \frac{1}{2}$

Variance$=E(X^2)-[E(X)]^2 = \left(0^2\times\frac{1}{2}\right)+\left(1^2\times\frac{1}{2}\right)-\left(\frac{1}{2}\right)^2$

$= 0 + \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

Ex.17 An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of 3 balls. Find the mean and variance of X.

Sol. When 3 balls are drawn at random, there may be no red ball, 1 red ball, 2 red ball or 3 red balls. Let X be the random variable showing the number of red balls in a draw of 3 balls. Then, X can take the value 0, 1, 2 or 3.

P (X = 0) = P (getting no red ball)

$= \frac{^4C_3}{^7C_3} = \frac{4\times3\times2}{7\times6\times5} = \frac{4}{35}$

P(X = 1) = P (getting 1 red ball)

$= \frac{^4C_2\times{}^3C_1}{^7C_3} = \frac{\frac{4\times3}{2}\times3\times3\times2}{7\times6\times5} = \frac{18}{35}$

P (X = 2) = P (getting two red balls)

$= \frac{^4C_1\times{}^3C_2}{^7C_3} = \frac{4\times3\times3\times2}{7\times6\times5} = \frac{12}{35}$

P (X = 3) = P (getting 3 red balls)

$= \frac{^3C_3}{^7C_3} = \frac{3\times2\times1}{7\times6\times5} = \frac{1}{35}$

Thus, the probability distribution of X is given below :

X	0	1	2	3
P(X)	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

Mean = E (X) = μ Σ xi pi

$= \left(0\times\frac{4}{35}\right)+\left(1\times\frac{18}{35}\right)+\left(2\times\frac{12}{35}\right)+\left(3\times\frac{1}{35}\right) = 0$

$+ \frac{18}{35} + \frac{24}{35} + \frac{3}{35} = \frac{45}{35} = \frac{9}{7}$

Variance = E (X^2) – $[E(X)]^2$

$= \sum x_i^2 p_i - \mu^2 = \left(0^2\times\frac{4}{35}\right)+\left(1^2\times\frac{18}{35}\right)+\left(2^2\times\frac{12}{35}\right)$

$+ \left(3^2\times\frac{1}{35}\right) - \frac{81}{49}$

$= 0 + \frac{18}{35} + \frac{48}{35} + \frac{9}{35} - \frac{81}{49} = \frac{75}{35} - \frac{81}{49} = \frac{15}{7}$

$- \frac{81}{49} = \frac{24}{49}$

Ex.18 Find the binomial distribution whose
(i) mean and standard derivations are 9 and $\frac{3}{2}$ respectively.
(ii) mean and variance are 9 and 6 respectively.

Sol. Let p, q be the probabilities of success and failure in only one trial and n be the number of trials. Then, the binomial distribution is $(q + p)^n$.

EXERCISE – I UNSOLVED PROBLEMS

(i) For binomial distribution, we have

Mean = np = 9 and S.D = $\sqrt{npq} = \sqrt{\frac{3}{2}}$

$\Rightarrow \sqrt{9q} = \frac{3}{2} \Rightarrow 9q = \frac{9}{4} \Rightarrow q = \frac{1}{4}$

But $p + q = 1 \quad \Rightarrow p = 1 - q = 1 - \frac{1}{4} = \frac{3}{4}$

Since, np = 9 and $p = \frac{3}{4}$, n = 12

Hence, the required B.D. is $\left(\frac{1}{4}+\frac{3}{4}\right)^{12}$

(ii) For binomial distribution, we have

Mean = np = 9 and Variance = npq = 6 $\therefore$ $q = \frac{6}{9} = \frac{2}{3} \Rightarrow p = \frac{1}{3}$ and n = 27

Hence, the B.D. is $\left(\frac{2}{3}+\frac{1}{3}\right)^{27}$

Q.1 A pair of dice is tossed. Find the probability of getting a total of 10 if it is known that the two numbers appeared are different.

Q.2 A dice is tossed. If a number greater than 2 is obtained, a coin is tossed. FInd the probability of getting a head if it is given that even number has occurred on the dice.

Q.3 If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl and (ii) at least one is a girl ?

Q.4 From a bag containing 7 white and 5 red balls, 5 balls are drawn at random one by one. What is the probability that the balls drawn are alternately of different colour.

Q.5 A problem in Mathematics is given to A, B and C. Their respective probabilities of solving the problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. What is the probability that (a) the problem is solved and (b) exactly one of them solves it.

Q.6 A pair of dice to tossed twice. Find the probability of getting a total of 7 on each toss.

Q.7 A bag contains 7 white and 3 red balls. Two balls are drawn with replacement. What is the probability that (a) the first ball is white and the second ball is red. (b) one ball is white and the other red.

Q.8 A pair of dice is tossed once. Find the probability of getting an even number on both the dice if it is known that 2 has occurred on one of the dice.

Q.9 From a deck of 52 cards, first one card is drawn and then two cards are drawn. What is the probability of getting two aces and a king when first card is (a) replaced (b) not replaced, before taking out next two cards.

Q.10 Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male ? Assume that there are 60% males and 40% females.

Q.11 Three urns, A, B and C, contain 8 white and 4 red; 6 white and 6 red; and 4 white and 8 red balls respectively. One of the urns is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the urn A.

Q.12 A company has two plants to manufacture scooters. Plant I manufactures 70% of the scooters and plant II manufactures 30%, At plant I, 80% of the scooters produced are rated to be of standard quality, and at plant II, 90% of the scooters are rated to be of standard quality. A scooter is chosen at random and is found to be of standard quality. What is the probability that it has come from plant II ?

Q.13 A card is accidentally dropped from a pack of 52 cards. From the remaining cards two cards are drawn and both are found to be spades. What is the probability that the card dropped was also a spade ?

Q.14 In an objective test, an examinee either guesses or copies or knows the answer to a multiple-choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered it.

Q.15 A biased dice, which always shows a six when rolled, is mixed with three fair dice. One of the dice is rolled twice and a six appears each time. What is the probability that the dice rolled is the biased one ?

Q.16 Suppose 5 men out of 100 and 25 women out of 100 are orators. An orator is chosen at random from a group consisting of 60 men and 40 women. Find the probability that the orator chosen is a man.

Q.17 Three bags, A, B and C contain 6 white and 4 black; 7 white and 3 black; and 8 white and 2 black balls. Two ball are drawn at random from one of the bags. The balls drawn are one white and one black. What is the probability that the balls drawn are from bag A if the probabilities of selecting bags A, B and C are $\frac{1}{2}, \frac{3}{10}$ and $\frac{2}{5}$ respectively ?

Q.18 In a college there 1800 boys and 1200 girls. If 60% of the boys and 20% of the girls are taller than 1.7 m, find the probability that a randomly selected 1.75 m-tall student is a boy.

Q.19 Bag A contains 4 red and 2 black balls. Bag B contains 3 red and 3 black balls. One ball is transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red. Find the

EXERCISE – II BOARD PROBLEMS

Q.1 A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact ?

Q.2 A bag contains 30 tickets numbered from 1 to 30. Five tickets are drawn at random and arranged in ascending order. Find the probability that the third number is 20.

Q.3 A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both spades. Find the probability of the lost card being a spade.

Q.4 An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers and their probabilities of accidents are 0·01, 0·03 and 0·15 respectively. One of the insured persons meets with an accident. Find the probability that he is a scooter driver.

Q.5 There are two identical boxes containing respectively 4 white and 3 red balls, 3 white and 7 red balls. A box is chosen at random and a ball is drawn from it. If the ball drawn is white, what is the probability that it is from the first box ?

Q.6 A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{6}$ respectively. Find the probability that one of them is able to solve the problem correctly.

Q.7 A box contains 2 gold and 3 silver coins. Another box contains 3 gold and 3 silver coins. A box is chosen at random and coin is drawn from it. If the selected coin is gold coin, find the probability that it was drawn from the second box.

Q.8 A can solve 90% of the problems given in a book and B can solve only 70% problems. What is the probability that atleast one of them will solve the problem selected at random from the book ?

Q.9 Two cards are drawn successively without replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of spades.

Q.10 A pair of dice is thrown 200 times. If getting a sum of 9 is considered a success, find the mean and variance of the number of successes.

Q.11 Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the mean and variance for the number of aces.

Q.12 The probability that a person will get an electric contract is $\frac{2}{5}$ and the probability that he will not get the plumbing contract is $\frac{4}{7}$. If the probability of getting atleast one contract is $\frac{2}{3}$, what is the probability that he will get both ?

Q.13 One bag contains 1 red and 3 blue balls, a second bag contains 2 red and 1 blue ball and a third bag contains 4 red and 3 blue balls. One bag is chosen at random and two balls are drawn from it. If one ball is red and the other is blue, find the probability that they were picked up from the second bag.

Q.14 A student is given a test with 8 items of true-false type. If he gets 6 or more items correct, he is declared a pass. Given that he guesses the answer to each item, compute the probability that he will pass the test.

Q.15 In a single throw of three dice find the probability of getting
(i) a total of 5 (ii) a total of atmost 5

Q.16 Find the probability distribution of the number of successes in two tosses of a die where a success is defined as a number less than 3. Also find mean and variance of the distribution.

Q.17 A and B throw two dice simultaneously turn by turn. A wins if be throws a total of 5, B will win if he throws a doublet. Find the probability that B will win the game, though A started it.

Q.18 Two cards are drawn from a well shuffled pack of 52 cards one after the other without replacement. Find the probability that one of these is a queen and the other is a king of opposite colour.

Q.19 Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of jacks.

Q.20 An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting
(i) 2 red ballsi (ii) 2 blue balls (iii) one red and one blue ball.

Q.21 Find the binomial distribution for which mean is 4 and variance 3.

Q.22 A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a 6. Find the probability that it is actually 6.

Q.23 In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their output 5, 4, and 2 percent are respectively defective bolts. A bolt is drawn at random from the total production and is found to be defective. Find the probability that it is manufactured by the machine B.

Q.24 In a bulb factory, meachines A, B and C manufacture 60%, 30% and 10% bulbs respectively. 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A.

Q.25 A doctor is to visit a patient. From the past experience it is known that the probabilities of the doctor coming by train, bus, scooter or taxi are $\frac{1}{10}$, $\frac{1}{5}$, $\frac{3}{10}$ and $\frac{2}{5}$ respectively. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{12}$ if he comes by train, bus or scooter respectively but by taxi he will not be late. When he arrives, he is late. What is the probability that he came by bus ?

Q.26 Three bags contain balls as shown in the table below :

Bag	Number of white balls	Number of black balls	Number of red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at ran and two balls are drawn from it. They happen to be white and red. What is the probability that they came from bag III ?

Q.27 From a lot of 30 bulbs which includes 6 defective, a sample of 4 bulbs is drawn at random with replacement. Find the mean and variance of the number of defective bulbs.

Q.28 A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the mean and variance of the number of successes.

Q.29 Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

Q.30 On a multiple choice examination with three possible answers (out of which only one is correct) for each of five questions, what is the probability that a candidate would get four or more correct answer just by guessing ?

Q.31 From a lot of 10 bulbs, which includes 3 defectively, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

Q.32 Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Q.33 Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ?

Q.34 Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the the mean and variance of the number of red cards.

Q.35 Suppose a girl throws a die. If the gets a 5 or 6, she tosses a coin 3 times and notes the number of heads. If the gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained excatly one head, what is the probability that she threw 1, 2, 3, or 4 with the die ?

Q.36 The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are indenpendent, find the probability of only one of them coming to the school in time.
Write at least one advantage of coming to school in time.

Q.37 In a hockey metach, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.

Answers

EXERCISE – 1 (UNSOLVED PROBLEMS)

1. $\frac{1}{15}$ **2.** $\frac{1}{3}$ **3.** $\frac{1}{2}$ **4.** $\frac{7}{99}$ **5. (a)** $\frac{3}{4}$ **(b)** $\frac{11}{24}$ **6.** $\frac{1}{36}$ **7.** $\frac{21}{100}$ **8.** $\frac{5}{11}$

9. (a) $\frac{11}{8619}$ **(b)** $\frac{6}{5525}$ **10.** $\frac{3}{4}$ **11.** $\frac{2}{9}$ **12.** $\frac{27}{83}$ **13.** $\frac{11}{50}$ **14.** $\frac{24}{29}$

15. $\frac{12}{13}$ **16.** $\frac{3}{13}$ **17.** $\frac{24}{43}$ **18.** $\frac{9}{11}$ **19.** $\frac{3}{11}$ **20.** Mean = $\frac{7}{2}$; variance = $\frac{35}{12}$

21. Mean = 1; variance = $\frac{3}{4}$

22.

X	0	1	2	3	Total
P(X)	1/30	3/10	1/2	1/6	1

23.

X	0	1	2	Total
P(X)	6/11	9/22	1/22	1

24.

X	0	1	2	Total
P(X)	81/169	72/169	16/169	1

25. $\frac{2}{27}$ **26.** $\frac{101(4)^8}{(5)^{10}}$

27.

No. of Tails	0	1	2	Total
Probability	4/9	4/9	1/9	1

28. $\frac{7}{10}, \frac{21}{100}$ **29.** more than 3 times **30.** $\frac{3}{16}$

EXERCISE – 2 (BOARD PROBLEMS)

1. 42% **2.** $\frac{285}{5278}$ **3.** $\frac{11}{50}$ **4.** $\frac{1}{52}$ **5.** $\frac{40}{61}$ **6.** $\frac{19}{45}$ **7.** $\frac{5}{9}$ **8.** $\frac{97}{100}$

9.

X	0	1	2
P(X)	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{1}{17}$

10. $\frac{200}{9}$; $\frac{1600}{81}$ **11.** $\frac{2}{13}$; $\frac{24}{169}$ **12.** $\frac{17}{105}$ **13.** $\frac{28}{73}$ **14.** $\frac{37}{256}$ **15.** (i) $\frac{1}{36}$ (ii) $\frac{5}{108}$

16.

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

17. $\frac{4}{7}$ **18.** $\frac{4}{663}$

19.

X	0	1	2
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{69}$

20. (i) $\frac{49}{121}$ (ii) $\frac{16}{121}$ (iii) $\frac{56}{121}$

21. $\left(\frac{3}{4}+\frac{1}{4}\right)^{16}$ **22.** $\frac{3}{8}$ **23.** $\frac{28}{69}$ **24.** $\frac{2}{5}$ **25.** $\frac{4}{7}$ **26.** $\frac{5}{17}$ **27.** Mean : $\frac{4}{5}$, Variance : $\frac{16}{25}$

28. Mean : $\frac{2}{3}$, Variance : $\frac{5}{9}$ **29.** Mean : 1, Variance : $\frac{25}{51}$ **30.** $\frac{11}{3}\left(\frac{1}{3}\right)^4$

31.

X	0	1	2	
P(X)	7/15	7/15	1/15	1

32. $\frac{7}{3}\left(\frac{5}{6}\right)^5$ **33.** $\frac{2}{3}$ **34.** 1, $\frac{25}{51}$ **35.** 8/11