PROBABILITY

CONTENTS

- Definiation
- Terms
- Approaces Probability
- Types of experimental Probability

DEFINITIONS

Words 'chance' probably, or most probably etc. shows uncertainty in our statements.

The uncertainity of 'probably' etc. can be measured numerically by means of 'probability'.

TERMS

Definitions :

Trial and Event :

An experiment is called a **trial** if it results in anyone of the possible outcomes and all the possible outcomes are called **events**.

For Example

(i) Participation of player in the game to win a game, is a trial but winning or losing is an event.

(ii) Tossing of a fair coin is a trial and turning up head or tail are events.

(iii) Throwing of a dice is a trial and occurrence of number 1 or 2 or 3 or 4 or 5 or 6 are events.

(iv) Drawing a card from a pack of playing cards is a trial and getting an ace or a queen is an event.

Favourable Events :

Those outcomes of a trial in which a given event may happen, are called **favourable cases** for that event.

For Example -

(i) If a coin is tossed then favourable cases of getting H is 1.

(ii) If a dice is thrown then favourable case for getting 1 or 2 or 3 or 4 or 5 or 6, is 1.

(iii) If two dice are thrown, then favourable cases of getting a sum of numbers as 9 are four i.e (4,5), (5,4), (3,6), (6,3).

Sample Space :

The set of all possible outcomes of a trial is called its **sample space**. It is generally denoted by S and each outcome of the trial is said to be a point of sample of S.

For example

(i) If a die is thrown once, then its sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

(ii) If two coins are tossed together then its sample space

$$S = \{HT, TH, HH, TT\}.$$

APPROACHES PROBABILITY

(i) Experimental or Empirical or observed frequency approach.

(ii) Classical approach

(iii) Axiomatic approach

(i) Experimental Probability : Let there be n trials of an experiment and A be an event associated to it such that A happens in m-trials. Then the empirical probability of happening of event A is denoted by P(A) and is given by

$$P(A) = \frac{m}{n} \quad \text{i.e.,}$$

$$P(A) = \frac{\text{Number of trials in which the event happens}}{\text{Total number of trails}}$$

Clearly, $0 \le m \le n$. Therefore,

$$0 \le \frac{m}{n} \le 1 \quad \Rightarrow 0 \le P(A) \le 1$$

Thus, the probability of happening of an event always lies between 0 and 1. If P(A) = 1, then A is called a certain event and A is known as an impossible event, If P(A) = 0.

❖ Further, if $\overline{A}$ denotes negative of A i.e. event that A doesn't happen, then for above cases m, n ; we shall have

$$P(\overline{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$$\therefore P(A) + P(\overline{A}) = 1$$

Playing Cards :

(i) Total : 52 (26 red, 26 black)

(ii) Four suits : Heart (♥) , Diamond (♦), Spade (♠), Club (♣) - 13 cards each

(iii) Court Cards : 12 (4 Kings, 4 queens, 4 jacks)

(iv) Honour Cards:16 (4 aces, 4 kings, 4 queens, 4 jacks)

➤ TYPES OF EXPERIMENTAL PROBABILITY

1. **Deterministic :** Deterministic experiments are those experiments which when repeted under identical conditions produce the same result or outcome. For example, if we mark head (H) on both sides of a coin and it is tossed, then we always get the same outcome assuming that it does not stand vertically.

2. **Random or probabilistic :** If an experiment, when repeated under identical conditions, do not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes, then it is known as a random or probabilistic experiment. For example, in the tossing of a coin one is not sure if a head (H) or tail (T) will be obtained, so it is a random experiment. Similarly, rolling an unbiased die is an example of a random experiment.

❖ EXAMPLES ❖

Ex.1 A coin is tossed 500 times with the following frequencies of two outcomes :

Head : 240 times, tail : 260 times

Find the probability of occurrence of each of these event.

Sol. It is given that the coin is tossed 500 times.

$\therefore$ Total number of trials = 500

Let us denote the event of getting a head and of getting a tail by A and B respectively. Then,

Number of trials in which the event A happens = 240.

and, Number of trials in which the event B happens = 260.

$$\therefore P(A) = \frac{\text{Number of trials in which the event A happens}}{\text{Total number of trials}}$$

$$= \frac{240}{500} = 0.48$$

$$\therefore P(B) = \frac{\text{Number of trials in which the event B happens}}{\text{Total number of trials}}$$

$$= \frac{260}{500} = 0.52$$

Note : We note that P(A) + P(B) = 0.48 + 0.52. Therefore, A and B are the only two possible outcomes of trials.

Ex.2 A die is thrown 1000 times with the following frequency for the outcomes 1, 2, 3, 4, 5 and 6 as given below :

Outcome :	1	2	3	4	5	6
Frequency:	179	150	157	149	175	190

Find the probability of happening of each outcome. **[NCERT]**

Sol. Let A_i denote the event of getting the outcome i, where i = 1, 2, 3, 4, 5, 6. Then,

$P(E_i)$ = Probability of getting outcome 1

$$= \frac{\text{Frequency of 1}}{\text{Total number of times the die is thrown}}$$

$$= \frac{179}{1000} = 0.179$$

$P(E_2)$ = Probability of getting outcome 2

$= \dfrac{\text{Frequency of 2}}{\text{Total number of times the die is thrown}}$

$= \dfrac{150}{1000} = 0.15$

Similarly, we have,

$P(E_3) = \dfrac{157}{1000} = 0.157$,

$P(E_4) = \dfrac{149}{1000}$

$= 0.149$, $P(E_5)$

$= \dfrac{175}{1000}$

$= 0.175$

and, $P(E_6) = \dfrac{190}{1000}$

$= 0.19$

Ex.3 The percentage of marks obtained by a student in the monthly unit tests are given below :

Unit test :	I	II	III	IV	V
Percentage of marks obtained	58	64	76	62	85

Find the probability that the student gets :

(i) a first class i.e. at least 60 % marks

(ii) marks between 70 % and 80 %

(iii) a distinction i.e. 75 % or above

(iv) less than 65 % marks.

Sol. Total number of unit tests held = 5

(i) Number of unit test in which the student gets a first class i.e. at least 60 % marks = 4.

$\therefore$ Probability that the student gets a first class

$= \dfrac{4}{5} = 0.8$

(ii) Number of tests in which the student gets between 70 % and 80 % = 1.

$\therefore$ Probability that a student gets marks between 70 % and 80 % $= \dfrac{1}{5} = 0.2$.

(iii) Number of tests in which the student gets distinction $= \dfrac{2}{5} = 0.4$

(iv) Number of tests in which the student gets less than 65 % marks = 3

$\therefore$ Probability that a student gets less than 65 % marks $= \dfrac{3}{5} = 0.6$.

Ex.4 On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25828573, the unit place digit is 3) is given in the table below :

Digit :	0	1	2	3	4	5	6	7	8	9
Frequency:	22	26	22	22	20	10	14	28	16	20

A number is chosen at random, find the probability that the digit at its unit's place is :

(i) 6

(ii) a non-zero multiple of 3

(iii) a non-zero even number

(iv) an odd number.

Sol. We have,

Total number of selected telephone numbers = 200

(i) It is given that the digit 6 occurs 14 times at unit's place.

$\therefore$ Probability that the digit at unit's place is $6 = \dfrac{14}{200}$

$= 0.07$

(ii) A non-zero multiple of 3 means 3, 6 and 9.

Number of telephone number in which unit's digit is either 3 or 6 or 9 = 22 + 14 + 20 = 56.

$\therefore$ Probability of getting a telephone number having a multiple of 3 at unit's place $= \dfrac{56}{200} = 0.28$

(iii) Number of telephone number having an even number (2 or 4 or 6 or 8) at unit's place

$= 22 + 20 + 14 + 16 = 72$

$\therefore$ Probability of getting a telephone number having an even number at units place

$= \frac{72}{200}$

$= 0.36$

(iv) Number of telephone number having an odd digit (1 or 3 or 5 or 7 or 9) at units' place

$= 26 + 22 + 10 + 28 + 20 = 106$

$\therefore$ Probability of getting a telephone number having an odd numbr at unit's place

$= \frac{106}{200}$

$= 0.53$

Ex.5 A tyre manufacturing company kept a record of the distance covered before a tyre to be replaced. Following table shows the resuts of 1000 cases.

Distance in km :	Less than 400	400 to 900	900 to 1400	More than 1400
Number of tyres :	210	325	385	80

If you buy a tyre of this company, what is the probability that :

(i) it will need to be replaced before it has covered 400 km ?

(ii) it will last more that 900 km ?

(iii) it will need to be replaced after it has covered somewhere between 400 km and 1400 km ?

(iv) it will not need to be replaced at all ?

(v) it will need to be replaced ?

Sol. We have,

(i) The number of trials = 1000

$\therefore$ Probability that a tyre will need to be replaced before it has covered

$400 \text{ km} = \frac{210}{1000} = 0.21$

(ii) The number of tyres that last more than 900 km = 385 + 80 = 465

$\therefore$ Probability that a tyre will last more than

$900 \text{ km} = \frac{465}{1000} = 0.465$

(iii) The number of tyres which require replacement after covering distance between 400 km and 1400 km = 325 + 385 = 710.

$\therefore$ Probability that a tyre require replacement

$400 \text{ km and } 1400 \text{ km} = \frac{710}{1000} = 0.71$

(iv) The number of tyres that do not need to be replaced at all = 0

$\therefore$ Probability that a tyre does not need be replaced $= \frac{0}{1000} = 0$

(v) Since all the tyres we have considered to be replaced, so

Probability that a tyre needs to be replaced

$= \frac{1000}{1000} = 1$

Ex.6 Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions favourable to germination. After 20 days the number of seeds which had germinated in each collection were counted and recorded as follows :

Bag :	1	2	3	4	5
Number of seeds germinated :	40	48	42	39	41

What is the probability germinated of :

(i) more than 40 seeds is a bag ?

(ii) 49 seeds in a bag ?

(iii) more than 35 seeds in a bag ?

(iv) at least 40 seeds in a bag ?

(v) at most 40 seed in a bag ?

Sol. Total number of bags = 5

(i) Number of bags in which more than 40 seeds germinated out of 50 seeds = 3.

$\therefore$ Probability of germinated of more than 40 seeds in a bag $= \frac{3}{5}$.

(ii) Number of bags in which 49 seeds germinated = 0.

$\therefore$ Probability of germination of 49 seeds $= \frac{0}{5} = 0$

(iii) Number of bags in which more than 35 seeds germinated = 5.

$\therefore$ Probability of germination of more than 35 seeds = $\frac{5}{5}$ = 1.

(iv) Numbr of bags in which at least 40 seeds germinated = 4

$\therefore$ Probability of germination of at least 40 seeds = $\frac{4}{5}$

(v) Number of bags in which at most 40 seeds germinated = 2.

$\therefore$ Probability of germination of at most 40 seeds = $\frac{2}{5}$

Ex.7 The distance (in km) of 40 female engineers from their residence to their place of work were found as follows -

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	2

Find the probability that an engineer lives :

(i) less than 7 km from her place of work ?

(ii) at least 7 km from her place of work ?

(iii) within $\frac{1}{2}$ km from her place of work ?

(iv) at most 15 km from her place of work ?

Sol. Total number of female engineers = 40

(i) Number of female engineers living at a distance less than 7 km from their place of work = 10.

$\therefore$ Probability that a female engineer lives at a distance less than 7 km from her place of work $= \frac{10}{40} = \frac{1}{4} = 0.25$

(ii) Number of female engineers living at least 7 km away from her place of work = 30

$\therefore$ Probability that a female engineer lives at least 7 km away from her place of work $= \frac{30}{40} = 0.75$

(iii) Since there is no engineer living at a distance less than $\frac{1}{2}$ km from her place of work.

$\therefore$ Probability that an engineer within $\frac{1}{2}$ km from her place of work = $\frac{0}{40}$ = 0.

(iv) Number of engineers living at a distance of 15 km or less away from her place of work = 30.

$\therefore$ Probability that an engineer lives at most 15 km away from her place of work = $\frac{30}{40}$ = 0.75

Ex.8 An insurance company selected 2000 drivers at random in a particular city to find a relationship between age and accidents. The data obtained are given in the following table:

Age of drivers (in years)	Accidents in one year				
	0	1	2	3	Over 3
18-29	440	160	110	61	35
30-50	505	125	60	22	18
Above 50	360	45	35	15	9

Find the probabilities of the following events for a driver chosen at random form the life city:

(i) being 18-29 years of age and having exactly 3 accidents in one year.

(ii) being 30-50 years of age and having one or more accidents in a year.

(iii) having no accidents in one year.

Sol. Total number of drivers = 2000

(i) The number of drivers who are 18-29 years old and have exactly 3 accidents in one year is 61.

$\therefore$ Probability of a driver being 18-29 years of age and has exactly 3 accidents = $\frac{61}{2000}$ = 0.0305

(ii) The number of drivers 30-50 years of age and having one or more accidents in one year = 125 + 60 + 22 + 18 = 225.

$\therefore$ Probability of a driver being 30-50 years of age and having one or more accidents

$= \frac{225}{2000} = 0.1125$

(iii) The number of drivers having no accidents in one year

= 440 + 505 + 360 = 1305

$\therefore$ probability of a driver having no accident in one year = $\frac{1305}{2000}$ = 0.653

Ex.9 Find the probability that a number selected at random from the numbers 1 to 25 is not a prime number when each of the gievn number is equally likely to be selected.

Sol. Here S = {1, 2, 3, 4, ..., 25}

Let E = event of getting a prime number = {2, 3, 5, 7, 11, 13, 17, 19, 23}.

Then, n (E) = 9

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{9}{25}$.

Required probability

$$= 1 - P(E) = \left(1 - \frac{9}{25}\right) = \frac{16}{25}.$$

Ex.10 Eleven bags of wheat flour, each marked 5 kg. actually contained the following weights of flour (in kg.) :

4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Sol. Total number of bags = 11

Number of bags containing more than 5 kg of flour = 7

Therefore, probability of bags containing more then 5 kg of flour

$$= \frac{\text{Number of bags containing more than 5kg flour}}{\text{Total number of bags}}$$

$$= \frac{7}{11}$$

Ex.11 The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.

(i) What is the probability that on a given day it was correct ?

(ii) What is the probability that it was not correct on a given day ?

Sol. The total number of days for which the record is available = 250

(i) P(correct forecast)

$$= \frac{\text{Number of days when the forecast was correct}}{\text{Total number of days for which the record is available}}$$

$$= \frac{175}{250} = 0.7$$

(ii) The number of days when the forecast was not correct = 250 – 175 = 75.

P(not correct forecast) = $\frac{75}{250}$

= 0.3

Ex.12 If the probability of winning a game is 0.3, what is the probability of lossing it ?

Sol. Probability of winning a game = 0.3.

Probability of losing it = q (say).

$\Rightarrow$ 0.3 + q = 1 $\Rightarrow$ q = 1 – 0.3

$\Rightarrow$ q = 0.7

Ex.13 Two coins are tossed simulataneously. Find the probability of getting

(i) two heads (ii) at least one head

(iii) no head

Sol. Let H denotes head and T denotes tail.

$\therefore$ On tossing two coins simultaneously, all the possible outcomes are

(i) The probability of getting two heads = P(HH)

$$= \frac{\text{Event of occurence of two heads}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$

(ii) The probability of getting at least one head = P(HT or TH or HH)

$$= \frac{\text{Event of occurence of at least one head}}{\text{Total number of possible outcomes}} = \frac{3}{4}$$

(iii) The probability of getting no head = P(TT)

$$= \frac{\text{Event of occurence of no head}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$

Ex.14 On tossing three coins at a time, find -

(i) All possible outcomes.

(ii) events of occurence of 3 heads, 2 heads, 1 head and 0 head.

(iii) probabilty of getting 3 heads, 2 heads, 1 head and no head.

Sol. Let H denotes head and T denotes tail. On tossing three coins at a time,

(i) All possible outcomes = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}. These are the 8 possible outcomes.

(ii) An event of occurence of 3 heads

= (HHH) = 1

An event of occurence of 2 heads

= {HHT, HTH, THH} = 3

An event of occurence of 1 head = {HTT, THT, TTH} = 3

An event of occurence of 0 head = {TTT} = 1

(iii) Now, probability of getting 3 heads = P (HHH)

$$= \frac{\text{Event of occurence of 3 heads}}{\text{Total number of possible outcomes}} = \frac{1}{8}$$

Simultaneously, probability of getting 2 heads

= P(HHT or THH or HTH)

$$= \frac{\text{Event of occurence of 2 heads}}{\text{Total number of possible outcomes}} = \frac{3}{8}$$

Probability of getting one head

= P (HTT or THT or TTH)

$$= \frac{\text{Event of occurence of 1 head}}{\text{Total number of possible outcomes}} = \frac{3}{8}$$

Probability of getting no head = P (TTT)

$$= \frac{\text{Event of occurence of no head}}{\text{Total number of possible outcomes}} = \frac{1}{8}$$

Ex.15 A bag contains 12 balls out of which x are white,

(i) If one ball is drawn at random, what is the probability that it will be a white ball ?

(ii) If 6 more white balls are put in the bag, the probability of drawing a white ball will double than that in (i). Find x.

Sol. Random drawing of balls ensures equally likely outcomes

Total number of balls = 12

$\therefore$ Total number of possible outcomes = 12

Number of white balls = x

(i) Out of total 12 outcomes, favourable outcomes = x

$$P(\text{White ball}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{x}{12}$$

(ii) If 6 more white balls are put in the bag, then

Total number of white balls = x + 6

Total number of balls in the bag

= 12 + 6 = 18

$$P(\text{White ball}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{x+6}{12+6}$$

According to the question,

Probability of drawing white ball in second case

= 2 × probability drawing of white ball in first case

$$\Rightarrow \frac{x+6}{18} = 2\left(\frac{x}{12}\right)$$

$$\Rightarrow \frac{x+6}{18} = \frac{x}{6}$$

$\Rightarrow$ 6x + 36 = 18x

$\Rightarrow$ 12x = 36

$\Rightarrow$ x = 3

Hence, number of white balls = 3

Ex.16 What is the probability that a leap year, selected at random will contain 53 Sundays ?

Sol. Number of days in a leap year = 366 days

Now, 366 days = 52 weeks and 2 days

The remaining two days can be

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

For the leap year to contain 53 Sundays, last two days are either Sunday and Monday or Saturday and Sunday.

$\therefore$ Number of such favourable outcomes = 2

Total number of possible outcomes = 7

$\therefore$ P(a leap year contains 53 sundays) = $\frac{2}{7}$

Ex.17 Three unbiased coins are tossed together. Find the probability of getting :

(i) All heads,

(ii) Two heads

(iii) One head

(iv) At least two heads.

Sol. Elementary events associated to random experiment of tossing three coins are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$\therefore$ Total number of elementary events = 8.

(i) The event "Getting all heads" is said to occur, if the elementary event HHH occurs i.e. HHH is an outcome. Therefore,

$\therefore$ Favourable number of elementary events = 1

Hence, required probability = $\frac{1}{8}$

(ii) The event "Getting two heads" will occur, if one of the elementary events HHT, THH, HTH occurs.

$\therefore$ Favourable number of elementary events = 3

Hence, required probability = $\frac{3}{8}$

(iii) The events of getting one head, when three coins are tossed together, occurs if one of the elementary events HTT, THT, TTH happens.

$\therefore$ Favourable number of elementary events = 3

Hence, required probability

$= \frac{3}{8}$

(iv) If any of the elementary events HHH, HHT, HTH and THH is an outcome, then we say that the event "Getting at least two heads" occurs.

$\therefore$ Favourable number of elementary events = 4

Hence, required probability

$= \frac{4}{8} = \frac{1}{2}$.

Ex.18 A piggy bank contains hundred 50 p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) will be a 50 p coin ?

(ii) will not be a ₹ 5 coin ?

Sol. Number of ₹ 50 coins = 100

Number of ₹ 1 coins = 50

Number of ₹ 2 coins = 20

Number of ₹ 5 coins = 10

(i) The number of favourable outcomes of 50 p coin to fall = 100

Total number of coins = 100 + 50 + 20 + 10 = 180

Total number of possible outcomes = 180

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P\,(50\text{ p}) = \frac{100}{180} = \frac{5}{9}$$

(ii) Number of favourable outcomes of 5 Rs coin to not fall = 180 – 10 = 170

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P\,(\text{not Rs. } 5) = \frac{170}{180} = \frac{17}{18}$$

Ex.19 A box contains 20 balls bearing numbers, 1, 2, 3, 4, ... 20. A ball is drawn at random from the box. What is the probability that the number on the balls is

(i) An odd number (ii) Divisible by 2 or 3

(iii) Prime number (iv) Not divisible by 10

Sol. Total number of possible outcomes = 20

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

(i) Number of odds out of first 20 numbers = 10

Favourable outcomes by odd = 10

$$P(\text{odds}) = \frac{\text{Favourable outcomes of odd}}{\text{Total number of possible outcomes}}$$

$$= \frac{10}{20} = \frac{1}{2}$$

(ii) The numbers divisible by 2 or 3 are 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

Favourable outcomes of numbers divisible by 2 or 3 = 13

P (numbers divisible by 2 or 3)

$$= \frac{\text{Favourable outcomes of divisible by 2 or 3}}{\text{Total number of possible outcomes}}$$

$$= \frac{13}{20}$$

(iii) Prime numbers out of first 20 numbers are 2, 3, 5, 7, 11, 13, 17, 19

Favourable outcomes of primes = 8

$$P(\text{primes}) = \frac{\text{Favourable outcomes of primes}}{\text{Total number of possible outcomes}}$$

$$= \frac{8}{20} = \frac{2}{5}$$

(iv) Numbers not divisible by 10 are 1, 2, .. 9, 11, ...19

Favourable outcomes of not divisible by 10 = 18

P(not divisible by 10)

$$= \frac{\text{Favourable outcomes of not divisible by 10}}{\text{Total number of possible outcomes}}$$

$$= \frac{18}{20} = \frac{9}{10}$$

IMPORTANT POINTS TO BE REMEMBERED

1. In the experimental approach to probability, we find the probability of the occurrence of an even by actually performing the experiment a number of times and adequate recording of the happening of event.

2. In the theoretical approach to probability, we try to predict what will happen without actually performing the experiment.

3. An outcome of a random experiment is called an elementary event.

4. An event associated to random experiment is a compound event if it is obtained by combining two or more elementary events associated to the random experiment.