Waves

INTRODUCTION

There are two modes of energy transfer -

- **Particle motion -** When particle is transferred from one part of space to the other it carriers energy with it.
- **Wave motion -** When no particle is transferred one part of space to the other although energy is transported.

CLASSIFICATION OF WAVES

Waves may be of following two types :

1. Electro-magnetic waves (E.M. waves)

(Medium is not necessary for propagation)

- Periodic changes takes place in electric & magnetic field hence, it is called electromagnetic wave
- In medium, E.M. waves travel with light velocity
- E.M. waves can be polarised
- E.M. waves are transverse in nature
- Medium is not required for propagation of E.M. waves
- E.M. waves has momentum

Eg. Radio waves, light waves, thermal radiation

2. Mechanical wave - (Medium is essential)

- Method of energy propagation, in which disturbance propagates with definite velocity without changing its form, is called mechanical wave.
- Energy & momentum propagates by motion of particles of medium. But medium remain at previous position, Mass transfer does not take place.
- Propagation is possible due to property of medium viz. Elasticity & inertia

Mechanical waves may be of two types :

(i) Transverse waves -

- The particles of medium vibrate in a direction perpendicular to the direction of propagation of wave.

Eg. Vibration of string, the surface waves produced on the surface of solid and liquid.

- Crest & trough are formed.

Note : Polarisation of transverse waves is possible (but it is not possible for longitudinal waves)

(ii) Longitudinal waves -

- Vibration of the particles of the medium are in the direction of weave propagation.

  Eg. Sound waves, waves in gases
- Wave proceeds in form of compression (C) & rarefaction (R)
- At places of compression the pressure and density are maximum, while at places of rarefaction those are minimum.

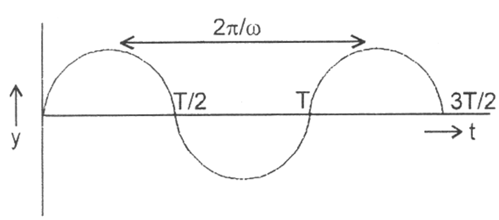
Distinguish between sound and radio waves of the same frequency (say 15 KHz)

Following are the points that distinguish a sound wave from a radio-wave

- Sound can not travel through free space while radio waves can .
- The speed of sound in air is 332 m/s while of radio waves 3×10^8 m/s
- With rise in temperature, velocity of sound increase while that of radio waves does not change.
- Sound wave can be detected by human ear if its frequency lies between 20 Hz - 20 kHz while radio wave of any frequency cannot be.
- Sound wave can never be polarised (excluding the transverse mode in solids) while ratio waves can always be polarised. .

VARIOUS TERMS RELATED TO WAVE MOTION

- Amplitude (A) - The maximum displacement of a vibrating particle of the medium from the mean position. 'A' shown amplitude in $y = A \sin \omega t$
- Periodic time (T) - Time taken to complete one oscillation and denoted by T.



$T = 1/n$ n is called frequency

- Wave frequency (n) - Number of vibrations made per second by the particles and is denoted by n.

  $n = 1/T$ unit : Hz

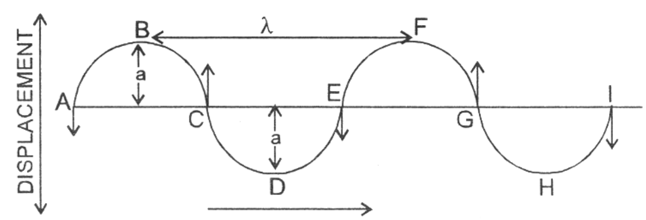
  Angular frequency : $\omega = 2\pi n$ unit : rad/sec
- Wave length (λ) - The distance between two consecutive particles in the same phase or the distance traveled by the wave in one period it is denoted by λ

  $\because$ Wave velocity = $V = n\lambda$

  Other relation :

$$V = n\lambda = \frac{\lambda}{T} = \frac{\omega}{2\pi}\lambda = \frac{\omega}{k}$$

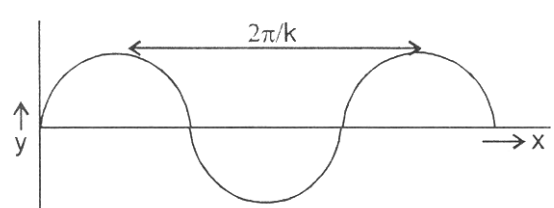
where k is defined as wave number. $k = \frac{2\pi}{\lambda}$



- Intensity of wave : In medium, propagation energy perpendicular to per unit area per second is called intensity of wave. $I = 2\pi^2 n^2 a^2 \rho v$

  where ρ = density of the medium, v = wave velocity, a = amplitude, n = wave frequency

  Unit : W/m^2 if v & ρ is constant, then $I \propto (\text{frequency})^2$ & $I \propto (\text{amplitude})^2$



Position of different particles at some instant in a string

- Energy density : (energy per unit volume is called energy density.

$$E = \text{energy density} = \frac{\text{wave Intensity}}{\text{wave velocity}} = \frac{2\pi^2 n^2 a^2 \rho v}{v}$$

$$= 2\pi^2 n^2 a^2 \rho$$

- Wave pressure - When a wave incidents on surface then it produces pressure on it, is called wave pressure.

EQUATION OF A PLANE PROGRESSIVE WAVE

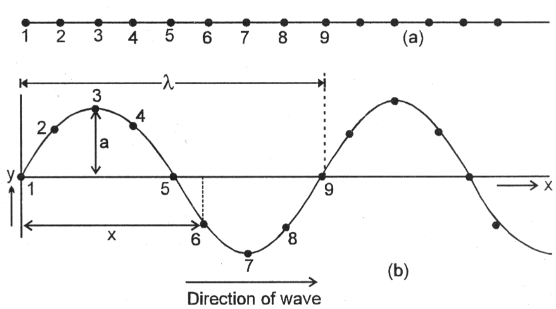
If, on the propagation of wave in a medium, the particles of the medium perform simple harmonic motion then the wave is called a 'simple harmonic progressive wave'.

Suppose, a simple harmonic progressive wave is advancing in a medium along the positive direction of the x-axis (from left to right). In fig. (a) the equilibrium positions of the particles 1, 2, 3 .... are shown. When the wave propagates, these particles oscillate about their equilibrium positions. In fig. (b) the instantaneous positions of these particles at a particular instant are shown. The curve joining these positions represents the wave.

Let the time be counted from the instant when the particle 1 situated at the origin starts oscillating. If y be the displacement of this particle after t seconds, then

$y = a \sin \omega t$, .(1)

where a is the amplitude of oscillation and $\omega = 2\pi$ n, where n is the frequency .



As the wave reaches the particles beyond the particle 1, the particles start oscillating. If the speed of the wave be ν, then it will reach particles 6, distance x from the particle 1, in x/ν sec. Therefore, the particle 6 will start oscillating x/ν sec after the particle 1. It means that the displacement of the particle 6 at a time t will be the same as that of the particle 1 at a time x/ν sec earlier i.e. at time t – (x/ν). The displacement of particle 1 at time t – (x/ν) can be the particle 6, distant x from the origin (particle 1), at time t is given by

$$y = a \sin \omega \left(t - \frac{x}{\nu} \right) \quad \text{But} \quad \omega = 2\pi\ n$$

$$y = a \sin (\omega t - kx) \quad \left(k = \frac{\omega}{v} \right) \quad ....(2)$$

$$y = a \sin \left[\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x \right] \quad \text{also } k = \frac{2\pi}{\lambda}$$

$$y = a \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda} \right]$$

This is the equation of a simple harmonic wave travelling along +x direction. If the wave is travelling along the –x direction then inside the brackets in the above equation, instead of minus sign there will be plus sign. For example, equation (4) will be of the following form :

$$y = a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$$

If ϕ be the phase difference between the above wave travelling along the +x direction an another wave, then the equation of that wave will be

$$y = a \sin \left\{ 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \pm \phi \right\}$$

When equation of wave is $y = a \sin (\omega t - kx)$

velocity of particle

$$= \frac{dy}{dt} = a\omega \cos (\omega t - kx) \quad ...(1)$$

slope of wave

$$= \frac{dy}{dx} = -ak \cos (\omega t - kx) \quad ...(2)$$

divide eq. (1) by eq. (2)

$$\frac{\frac{dy}{dt}}{\frac{dy}{dx}} = -\frac{\omega}{k}, \qquad \frac{dy}{dt} = -\frac{\omega}{k} \frac{dy}{dx}, \qquad \frac{dy}{dt} = -v \frac{dy}{dx}$$

velocity of particle = – velocity of wave × slope of wave

Solved Examples

Ex.1 A progressive wave of frequency 500Hz is travelling with a velocity of 360 meter/sec. How far part are two points 60º out of phase.

Sol. Velocity of the wave, v = 360 m/sec

frequency, n = 500 Hz

According to the formula $v = n\lambda$, the wavelength is

$$\lambda = v/n = \frac{360}{500} = 0.72\text{meter}$$

If Δx be the maximum distance between two points, then the phase difference between them

$\Rightarrow$ $\Delta\phi = 2\pi/\lambda \times \Delta x$ here, $\Delta\phi = 60^\circ = \pi/3$

Therefore the path difference

$$\Delta x = \lambda/2\pi \times \Delta\phi = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12\text{m}$$

Ex.2 The equation of a simple harmonic progressive wave is y = 8 sin (0.628x – 12.56t), where y and x are in cm and t in second. The amplitude, frequency and speed of the wave and the phase difference between two particles at a distance of 2.0 cm apart at any instant will be -

Sol. The equation of the wave is , y = 8sin (0.628x – 12.56t) and y = 8 sin 2π (0.1x – 2t) cm

Comparing it with the std. equation y = a sin 2π

$$\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

amplitude a = 8 cm

time period T = 1/2 sec

frequency n = 1/T = 2.0 sec

wavelength λ = 1/0.1 = 10 cm

$\therefore$ Wave speed $v = n\lambda$ = 2.0 × 10 = 20 cm/sec. If the distance two points is Δx, the phase distance between them is given by ,

$\Delta\phi = 2\pi/\lambda \; \Delta x$

Here, λ = 10cm and Δx = 2.0 cm,

$\therefore$ $\Delta\phi = 2\pi/10 \times 2.0 = 2\pi/5$ radian

$\Delta\phi = 2/5 \times 180^\circ = 72^\circ$

Ex.3 The distance between two particles on string is 10 cm. If the frequency of wave propagating in it is 400Hz and its speed is 100 m/s then the phase difference between he particles will be -

Sol. Phase difference = $\frac{2\pi}{\lambda}$ path difference

$$\delta = \frac{2\pi n}{V}\Delta x$$

$$\delta = \frac{2\pi \times 400}{100} \times 0.1 = 0.8\pi \text{ radian}$$

Ex.4 For the travelling harmonic wave y = 2.0 cos (10t – 0.0080x + 0.35). Where x and y are in cm and t in sec What is the phase difference between oscillatory motion at two points separated by a distance of -

(i) 4m, (ii) 0.5 m,

(iii) A/2, (iv) 3λ/4

Sol. The given equation of harmonic wave is,

y = 2.0 cos (10t – 0.0080x + 0.35) ....(1)

The standard equation of harmonic wave is

$$y = a \cos\left[2\pi\left(\frac{1}{T} - \frac{x}{\lambda}\right) + \phi\right] \quad ...(2)$$

Comparing equations (1) and (2) ,

$\frac{2\pi}{\lambda} = 0.0080$ or

$$\lambda = \frac{2\pi}{0.0080} \text{ cm} = \frac{2\pi}{0.0080 \times 100}\text{m}$$

(i) Phase difference = $\frac{2\pi}{\lambda}$ × path difference

$$= \frac{2\pi}{2\pi} \times 0.0080 \times 100 \times 4 = 3.2 \text{ rad}$$

(ii) Phase difference = $\frac{2\pi}{\lambda}$ × path difference

$$= \frac{2\pi}{2\pi} \times 0.0080 \times 100 \times 0.5 = 0.40 \text{ rad}$$

(iii) Phase difference = $\frac{2\pi}{\lambda}$ × path difference

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ rad}$$

(iv) Phase difference = $\frac{2\pi}{\lambda}$ × path difference

$$= \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2} \text{ rad}$$

Ex.5 Two particle C and D are executing simple harmonic motion with same amplitude a and same frequency along same straight line. The maximum distance between two particles is $a\sqrt{2}$. The initial phase difference between the two will be-

Sol. $y_1 = a \sin 2\pi nt$ ...(1)

$y_2 = a \sin (2\pi nt + \theta)$ ....(2)

$y = y_2 - y_1$ ....(3)

From equation (1), (2) & (3)

$y = 2a \sin \theta/2 \cos [2\pi nt + \theta/2]$ ...(4)

from equation (4) $y_{max} = 2a \sin \theta/2$ ....(5)

According to equation $y_{max} = a\sqrt{2}$ ....(6)

From equation (5) & (6)

$a\sqrt{2} = 2a \sin \theta/2$

$\Rightarrow \sin \theta/2 = 1/\sqrt{2}$

$\Rightarrow \theta = 90^o = \pi/2$ radian

Ex.6 If equation of transverse wave is $y = x_0 \cos 2\pi \left(nt - \frac{x}{\lambda}\right)$. Maximum velocity of particle is twice of wave velocity, if λ is -

Sol. $y = x_0 \cos 2\pi \left(nt - \frac{x}{\lambda}\right)$

$y = x_0 \cos \frac{2\pi}{\lambda} (vt - x)$ [$\because$ $v = n\lambda$]

$\left(\frac{dy}{dt}\right)_{max} = x_0 \times \frac{2\pi}{\lambda} v = 2v$ (given) $\therefore$ $\lambda = \pi x_0$

VELOCITY OF SOUND IN DIFFERENT MEDIA

Newton Formula: Use for every medium

$$V_{medium} = \sqrt{\frac{E}{\rho}}$$

where E = elastically coefficient of medium

& ρ = Density of medium

(a) For solid medium

$$V_{solid} = \sqrt{\frac{y}{\rho}}$$

E = y = Young's modules

for soft iron

$V_{soft\ iron} = 5150$ m/s

(b) For liquid medium

$$V_{liquid} = \sqrt{\frac{B}{\rho}}$$

E = B, where B = volume elasticity coefficient of liquid

for water $V_{water} = 1450$ m/s

(c) For gas medium

The formula for velocity of sound in air was first obtained by Newton. He assumed that sound propagates through air temperature remains constt. (i.e. process is isothermal) so

Isothermal Elastically = P

$\therefore$ $v_{air} = \sqrt{(P/\rho)}$

At NTP for air $P = 1.01 \times 10^5$ N/m^2 and

$\rho = 1.3$ kg/m^3

so $V_{air} = \sqrt{\frac{1.01 \times 10^5}{1.3}} = 279$ m/s

However, the experimental value of sound in air is 332 m/s which is much higher than given by Newton's formula

Laplace Correction

In order to remove the discrepancy between theoretical and experimental values of velocity of sound, Laplace modified Newton's formula assuming that propagation of sound in air is adiabatic process, i.e.

Adiabatic Elastically = γP

so that $v = \sqrt{\frac{\gamma P}{\rho}}$ ....(3)

i.e. $v = \sqrt{1.41} \times 279 = 331.3$ m/s which is in good agreement with the experimental value (332 m/s). This in turn establishes that sound propagates adiabatically through gases.

The velocity of sound in air at NTP is 332 m/s which is much lesser than that of light and radio-waves (= 3×10^8 m/s). This implies that

(a) If we set our watch by the sound of a distant siren it will be slow.

(b) If we record the time in a race by hearing sound from starting point it will be lesser than actual.

(c) In a cloud-lighting, through light and sound are produced simultaneously but as c > v, light proceeds thunder.

An in case of gases -

$$v_s = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{E_\phi}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

i.e., $v_s = \sqrt{\frac{\gamma PV}{mass}}$ $\left[as\ \rho = \frac{mass}{volume} = \frac{M}{V}\right]$

or $v_s = \sqrt{\frac{\gamma\ \mu RT}{M}}$ [as PV = μRT]

or $v_s = \sqrt{\frac{\gamma\ RT}{M_w}}$ $\left[as\ \mu = \frac{mass}{M_w} = \frac{M}{M_w}\right]$

M_w = molecular weight

and from kinetic theory of gases

$$V_{rms} = \sqrt{(3RT/M_w)}$$

so $\frac{v_s}{v_{rms}} = \sqrt{\frac{\gamma}{3}}$, i.e., $v_s = \left[\frac{\gamma}{3}\right]^{1/2} v_{rms}$

i.e, velocity of sound in a gas is of the order of rms speed of gas molecules ($v \sim v_{rms}$)

As velocity of sound in a according to Eqn.(4) is

$$v_s = \sqrt{\frac{\gamma RT}{M_w}}$$

velocity of sound in case of gases at constant temperature depends on the nature of gas i.e., its atomicity (γ) and molecular weight.

Effect of various quantities -

(1) Effect of temp.

For a gas γ & M_W is constant

$$v \propto \sqrt{T}$$

$$\frac{v_t}{v_0} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_1}{v_0} = \sqrt{\frac{t+273}{273}}$$

$$v_t = v_0\left[1+\frac{t}{273}\right]^{\frac{1}{2}}$$

By applying Binomial theorem

(i) For any gas medium $v_t = v_0 \left[1+\frac{t}{546}\right]$

(ii) For air v_0 = 332 m/sec

$$v_t = v_0 + 0.61t \quad m/sec$$

(2) Effect of Relative Humidity

With increases in humidity, density decreases so in the light of -

$$v = \sqrt{\gamma P/\rho}$$

We conclude that with rise in humidity velocity of sound increases. This is why sound travels factor in humid air (rainy season) than in dry air (summer) at same temperature.

(3) Effect of Pressure

As velocity of sound

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

So pressure has no effect on velocity of sound in gas as long as temperature remain constant. This is why in going up in the atmosphere, through both pressure and density decreases, velocity of sound remains constant as long as temperature remains constant. Further more it has also been established that all other factors such as amplitude, frequency, phase, loudness pitch, quality etc. has no effect on velocity of sound.

Velocity of sound in air is measured by resonance tube or Hebb's method while in gases by Quinke's tube. Kundt's tube is used to determine velocity of sound in any medium solid, liquid or gas.

(4) Effect of Motion of air -

If air is blowing then the speed of sound changes. If the actual speed of sound is v and the speed of air is w, then the speed of sound in the direction in which air is blowing will be (v + w), and in the opposite direction it will be (v – w).

(5) Effect of frequency -

There is no effect of frequency on the speed of sound. Sound waves of different frequencies travel with the same speed in air although their wavelength is in air are different. If the speed of sound were dependent on the frequency, then we could not have enjoyed orchestra.

VELOCITY OF TRANSVERSE WAVE

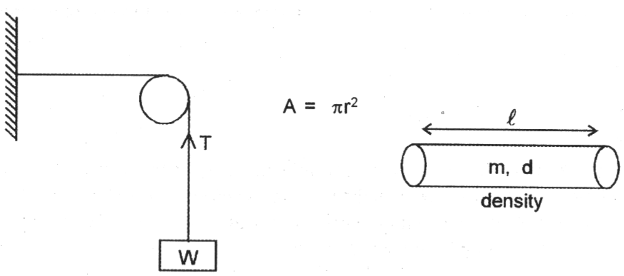
Mass of per unit length $m = \frac{\pi r^2 \ell \times d}{\ell}$, $m = \pi r^2 d$,

where d = density of mater

Velocity of transverse wave in any wire $v = \sqrt{\frac{T}{m}}$

giving by following Method

or $V = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\pi r^2 d}} = \sqrt{\frac{T}{Ad}}$ $\because$ $\pi r^2 = A$



(1) If m is constant then, $v \alpha \sqrt{T}$ it is called tension law.

(2) If tension is T then $v \alpha \sqrt{\frac{1}{m}}$ ← it is called law of mass.

(3) If T is constant & take wire of different radius for same material then $v \alpha \frac{1}{r}$ ← it is called law of radius.

(4) If T is constant & take wire of same radius for different material. Then $V \propto \sqrt{\frac{1}{d}}$ ← law of density

Solved Examples

Ex.7 A string of mass 2.50 kg is under a tension of 200N. The length of the stretched string is 20.0m. If a transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end.

Sol. Given that

tension is the string, T = 200 N, length of the string, ℓ = 20 m, mass of the string = 2.50 kg

$\therefore$ (mass/ length) of the string,

$$m = \frac{2.50\ kg}{20m} = 0.125 kg\ m^{-1}$$

Speed of transverse waves generated in the string is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{200}{0.125}} = 40 ms^{-1}$$

Hence, time taken by the disturbance to reach the other end after travelling a distance of 20 m.

$$t = \frac{distance}{velocity} = \frac{20}{40} = 0.5 sec$$

Ex.8 Calculate the velocity of transverse waves in a copper wire 1mm^2 in cross-section, under the tension produced by 1kg wt. The relative density of copper = 8.93

Sol. The velocity of transverse waves in the strings is

$$v = \sqrt{\frac{T}{m}}$$

Now, T = 1 kg wt = 9.8 N, m = a × 1× ρ

where a is the cross-sectional area and ρ is the density of material of the wire.

ρ = 8.93 × 1000 kg m^{-3}, a = 1 mm^2 = 10^{-6} m^2

m = 10^{-6} × 8.93 × 1000 = 8.93 × 10^{-3} kg/m

$$\therefore\ v = \sqrt{\frac{T}{m}} = \sqrt{\frac{9.8}{8.93\times10^{-3}}} = 33.12\ ms^{-1}$$

Ex.9 A steel wire has length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on wire equals the speed of sound in dry air at 20ºC (=343 ms^{-1})

Sol. Given that length of the wire, ℓ = 12.0 m

mass/length of the wire, $m = \frac{2.10\text{kg}}{12\text{m}}$

= 0.175 kg m^{-1}

speed of sound in dry air, at 20ºC = 343 ms^{-1}

Let T be tension in the string so that speed of transverse wave on wire equals v, Hence

$$v = \sqrt{\frac{T}{m}} \quad \text{or} \quad T = v^2 m$$

or, T = $(343)^2$ × 0.175 = 2.06 × 10^4 N.

SUPER POSITION OF WAVES

Principle of superposition

Two, or more, progressive waves can travel simultaneously in medium without affecting the motion of one another. Therefore the resultant displacement of each particle of the medium m any instant is equal to the vector sum of the displacements produced by the two waves separately. This principle is called 'principle of superposition'.

$y = y_1 \pm y_2 \pm$ ....

$\vec{y} = \vec{y}_1 + \vec{y}_2 +$ ....

Special point -

(i) It holds for all types of waves, provided the waves are not of very large amplitude. If wave are of very large amplitude, as laser waves, then this principle does not hold.

(ii) When we listen to an orchestra, we receive a complex sound due to the superposition of sound waves of different characteristics produced by different musical instruments. Still we can recoginize separately the sounds of different instruments.

Similarly, our radio antenna is open to the wave of different frequencies transmitted simultaneously by different radio stations. But when we tune the radio of particular station, we receive the programm of that station only as if the other stations were silent. Thus, then principle of superposition holds not only for the mechanical wave but also for the electromagnetic waves.

INTERFERENCE

When two waves of same frequency travel in medium simultaneously in the same direction then, due to their superposition, the resultant intensity at any point of the medium is different from the sum of intensities of the two waves. At some point the intensity of the resultant waves is very large while at some other points it is very small or zero. This phenomenon is called the 'interference' of waves.

Mathematically,

Ist wave $\Rightarrow Y_1 = a_1 \sin(\omega t)$

IInd wave $\Rightarrow Y_2 = a_2 \sin(\omega t + \phi)$

By principle of superposition

$y = y_1 + y_2$

$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$

$y = a_1 \sin \omega t + [a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi]$

$y = \sin \omega t\ (a_1 + a_2 \cos\phi) + a_2 \cos \omega t \sin \phi$

Suppose that

$a_1 + a_2 \cos \phi = A\cos \phi$ .....(1)

$a_2 \sin \phi = A \sin \phi$ .....(2)

sum the square of eq. (1) & (2)

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$

This is amplitude of resultant wave

Divide Eq. (2) by Eq. (1)

$$\tan\theta = \frac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$$

ϕ = phase difference between two waves

Eq. of resultant wave

$y = A \sin(\omega t + \theta)$

θ = Initial phase of resultant wave

For interference $I \propto a^2$

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos\phi$$

(1) Constructive interference - When waves are meat in same phase

phase difference : $\Delta\phi = 0, 2\pi, 4\pi$ ....$2n\pi$

Path difference : $\Delta x = 0, \lambda, 2\lambda$ ....$n\lambda$

$n = 0, 1, 2$ ....

$\cos\phi = +1$

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$A_{max} = a_1 + a_2$

if $a_1 = a_2 = a$

$I_1 = I_2 = I$

$I_{max} = 4I \quad A_{max} = 2a$

(2) Destructive interference -

phase difference : $\Delta\phi = \pi, 3\pi,$ ....$(2n+1)\pi$

Path difference : $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}$ ....$(2n+1)\frac{\lambda}{2}$

$n = 0, 1, 2$ ....

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$$

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

$\cos\phi = -1$

$A_{min} = a_1 - a_2$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

if $a_1 = a_2 = a$

$I_1 = I_2 = I$

$A_{min} = 0$

$I_{min} = 0$

Special results about interference pattern

Results

(1) Maximum and minimum intensities in any interference wave form

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

(2) Average intensity of interference wave form -

$$< I > \text{ or } I_{av} = \frac{I_{max} + I_{min}}{2}$$

Put the value of I_{max} & I_{min}

or $I_{av} = I_1 + I_2$

if $a = a_1 = a_2$ and $I_1 = I_2 = I$

then $I_{max} = 4I$, $I_{min} = 0$ and $I_{AV} = 2I$

(3) Degree of interference pattern (f)

Sound wave
↓
Degree of hearing

$$f = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100$$

Light wave
↓
Degree of visiblity

Special note : In condition of perfect interference degree of interference pattern is maximum

$f_{max} = 1$ or 100%

(4) Condition of maximum contract in interference wave form

$a_1 = a_2$ and $I_1 = I_2$

then $I_{max} = 4I \quad I_{min} = 0$

For perfect destructive interference we have a maximum contrast in interference wave form.

For problems

$\frac{A}{B} = \frac{C}{D}$ by addition substraction ratio

$$\frac{A+B}{A-B} = \frac{C+D}{C-D}$$

we can solve following problems by addition substraction ratio

Solved Examples

Ex.10 Two waves of the same frequency but of amplitude in the ratio 1 : 3 are superposed. What is the ratio of maximum to minimum intensity.

Sol. Let the two amplitudes be a and 3a.

Maximum amplitude = a + 3a = a

$I_{max} = (4a)^2 = 16a^2$

Minimum amplitude = 3a – a = 2a

$I_{min} = (2a)^2 = 4a^2$

$\therefore \frac{I_{max}}{I_{min}} = \frac{16a^2}{4a^2} = \frac{4}{1}$

Ex.11 Two waves of the same frequency and amplitudes 2a and 3a are super imposed on each other. (i) For what values of the phase difference ϕ the amplitude A of the resultant wave will be maximum and for what values minimum? (ii) Calculate the maximum and minimum intensities of the resultant wave.

Sol. (i) The intensity is maximum for $\phi = 0, 2\pi, 4\pi$.......

The intensity is maximum for $\phi = \pi, 3\pi, 5\pi$.......

(ii) The maximum intensity is

$I_{max} = (a_1 + a_2)^2 = 25a^2$ and

minimum intensity is $I_{min} = (a_1 - a_2)^2 = a^2$

Ex.12 Consider interference between waves from two sources of intensities I and 4I. Find the intensities at points where the phase difference is (i) $\pi/2$ (ii) π

Sol. Resultant intensity

$I_R = A^2 = a_1^2 + a_2^2 \cos \delta = I + 4I + 4I \cos\delta$

$\therefore I_R = 5I + 4I \cos (\pi/2) = 5I$

$I_R = 5I + 4I \cos \pi = I$

$\therefore$ (i) $\phi = \pi/2, \ I_R = 5I$

(ii) $\phi = \pi, I_R = I$

Ex.13 Two sources of intensities I and 4I are used in interference experiment. Find the intensity at points where the waves from the sources super-impose with a phase difference of

(a) zero (b) $\pi/2$ (c) π

Sol. Resultant intensity is

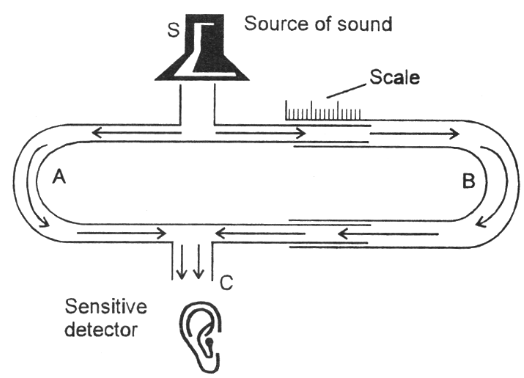
(a) $I_R = I_1 + I_2 + 2\ 2\sqrt{(I_1 I_2)} \cos \delta = I + 4I + 4I \cos 0 = 9I$

(b) $I_R = I + 4I + 4I \cos (\pi/2) = 5I$

(c) $I_R = I + 4I + 4I \cos \pi = I$

1. Qunke's tube : Experiment demonstration of interference of sound -

(a)



(b) There are two U-shaped tubes SAC and SBC. The tube SBC is movable and can be move back and forth and its displacement can be measured on a scale whereas the tube SAC is fixed.

(c) Sound is produced by a source S at the open end of the tube. These waves propagates along the paths SAC and SBC and superimpose at open mouth C as a result of which the interference phenomenon of sound is produced. The resultant amplitude at C depends upon the path differ once. (SBC - SAC)

(d) If $\Delta = (SBC - SAC) = (2n)\frac{\lambda}{2}$ then constructive interference of sound takes place at C and the flame flickers and becomes small.

(e) If $\Delta = (2n \pm 1)\frac{\lambda}{2}$ then destructive interference of sound takes place at C and the sensitive flame is diminished.

(f) Thus when the SBC tube is moved in and out then alternately maximum and minimum sounds are heard

(g) The path difference between two consecutive maxima and minima is λ and the corresponding reading of movable tube B will be $\frac{\lambda}{2}$.

(h) By doubling the reading of the tube B, λ and consequently the velocity of sound can be determined.

(i) In this tube if the movable tube is displaced by distance d in order to listen two consecutive maxima or minima then $d = \frac{n\lambda}{2}$.

(j) Due to absorption of energy inside the tube, the amplitude and intensity of sound waves change.

2. Beats -

When two waves of nearly equal frequencies superpose then the intensity of the resultant waves decreases and increases alternately with time. This phenomenon of periodic waxing (process, of increasing) and waning (process of decreasing) of intensity of the resultant wave is known as beats.

SPECIAL POINTS :

1. The frequency difference of two waves must not be more than 10, because the effect of sound persists on human ear drums only for 1/10 second.
2. The direction of propagation of two waves must be same.
3. Period of Beats (T) -

   (i) The time taken for one beat to be heard is defind as period of beats.

   (ii) The time interval between two consecutive maxima (waxing of sound) of minima (waning of sound) is known as the period of beats.

   (iii) The time interval between two consecutive beats is known as the period of beats.

   (iv) It is equal to the reciprocal of beat frequency.
4. Beat frequency (Δn) -

   (i) The number of beats heard per second in defined as beat frequency.

   (ii) The number of waxing and waning taking place per second is defined as beat frequency.

   (iii) It is equal to reciprocal of the period of beats.

   (iv) Its value i.e. the number of beats heard per second is equal to the difference of frequencies of two sound nodes, i.e.

   $\Delta n = n_1 - n_2$

WAVES INTERFERENCE ON THE BASIS OF BEATS -

Condition : Two equal frequency wave travel in same direction.

Mathematical analysis

If displacement of first wave

$y_1 = a \sin \omega_1 t \rightarrow (N_1, a) \quad I \propto N^2a^2$

displacement of second wave

$y_2 = a \sin \omega_2 t \rightarrow (N_2, a)$

by superposition $y = y_1 + y_2$

Eq. of resulting wave or

$y = a \{\sin 2\pi N_1 t + \sin 2\pi N_2 t\}$

$$Y = a\left\{2\sin 2\pi t\frac{(N_1+N_2)}{2}\cos 2\pi t\frac{(N_1-N_2)}{2}\right\}$$

or $$Y = \left\{2a\sin 2\pi t\frac{(N_1-N_2)}{2}\right\}\sin 2\pi t\frac{(N_1+N_2)}{2}$$

or $y = A\sin 2\pi N' t$

Amplitude $A = 2a \cos \pi t \left(\frac{N_1 - N_2}{2}\right)$

$A = 2a \cos \pi t\,(N_1 - N_2)$

Frequency $N' = \frac{N_1 + N_2}{2}$

SOME SPECIAL FORMULA OF TRIGONOMETRY

$$\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) . \cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) . \sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) . \cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) . \sin\left(\frac{D-C}{2}\right)$$

(1) For max intensity -

$A = \pm 2a$

If $\cos \pi (N_1 - N_2) t = \pm 1$

$\cos \pi (N_1 - N_2) t = \cos n\pi$

$n = 0, 1, 2$ ....

$\pi (N_1 - N_2) t = n\pi$

$$t = \frac{n}{N_1 - N_2}$$

$$t = 0, \frac{1}{\Delta N}, \frac{2}{\Delta N}, \frac{3}{\Delta N} ....$$

(2) For minimum intensity -

$A = 0$

$\cos \pi (N_1 - N_2) t = 0$

$\cos \pi (N_1 - N_2) t = \cos (2n + 1) \frac{\pi}{2}$

$n = 0, 1, 2,$ ....

$\pi (N_1 - N_2) t = (2n + 1) \frac{\pi}{2}$

$$t = \frac{2n+1}{2(N_1 - N_2)}$$

$$t = \frac{1}{2\Delta N}, \frac{3}{2\Delta N}, \frac{5}{2\Delta N} ....$$

PRACTICAL APPLICATIONS OF BEATS

(1) Determination of frequency - If we know the frequency n_1 of a tuning fork, then we can determine the exact frequency of another fork of nearly equal frequency by the phenomenon of beats. For this, both the tuning forks are sounded together and the beats are heard. Suppose, x beats are heard in 1 second. Then the frequency of the second fork will be either $(n_1 + x)$ or $(n_1 - x)$. Now one prong of this fork is loaded with a small wax so that its frequency is slightly lowered. Again, the two forks are sounded together and beats are heard. If the number of beats per second decreases then it means that the new (lowered) frequency of the second tuning forks is move nearer to the frequency of the first tuning fork. This would happen it the second tuning fork is higher than the frequency of the first fork. Hence the frequency of the second fork is $(n_1 + x)$. On the other hand, if on loading with wax, the number of beats per second increases, then the frequency of the second fork is $(n_1 - x)$

(2) Tuning of musical instruments - The musicians make use of beats for tuning their instruments. They sound two instruments, one by one, and adjust the frequency of one in such a way that its sound appears to them similar to the sound of the other instrument. Thus they make the frequencies of the two instruments nearly equal. Then they sound the two instruments simultaneously and hear beats. Now they adjust the frequency of one in such a way that the number of beats per second goes on decreasing slowly until the beats disappear. Now the frequency of the two instruments are exactly equal.

STATIONARY WAVES

Condition -

1. Formation of stationary wave is possible only and only in bounded medium.
2. The wave propagation in such a medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves will give rise to a stationary ave. Transmission of energy from stationary wave is not possible.

CHARACTERISTIC OF THE STATIONARY WAVES

The characteristics of properties of the stationary waves are as below :

1. In the stationary waves, the disturbance (crests and troughs or compressions and rarefaction) does not move forward or backward. The disturbance or the energy is not transferred from particle to particle.
2. The time period of periodic motions of all the particles of the medium is same, except those at the nodes.
3. The amplitude of vibration of different particles is different. It is maximum at the antinodes and minimum or zero at the nodes. The nodes are permanently at rest and the velocity of particle is zero.
4. Nodes and antinodes occur alternately and the separation between any two consecutive nodes or antinodes is half the wavelength.
5. Twice during one vibration, all the particles simultaneously pass through their mean positions with their maximum velocities. Also, twice during one vibration, all the particles simultaneously reach their extremes with maximum displacement and zero velocity.
6. The direction of motion of the particles is reversed after half a vibration of half time period.
7. The wavelength and frequency or period of stationary wave is the same as that of the component waves (direct and reflected waves).
8. The pressure variation is maximum at the nodes and minimum at the antinodes.

REFLECTION FROM RIGID END

When the pulse reaches the right end which is clamped at the wall. The element at the right end exerts a force on the clamp and the clamp exerts equal and opposite force on the element. The element at the right end is thus acted upon by the force from the clamp. As this end remains fixed, the two forces are opposite to each other. The force from the left part of the string transits the forward wave pulse and hence, the force exerted by the clamp sends a return pulse on the string whose shape is similar to a return pulse but is inverted. The original pulse tries to pull the element at the fixed end up and the return pulse sent by the clamp tries to pull it down. The resultant displacement is zero. Thus, the wave is reflected from the fixed end and the reflected wave is inverted with respect to the original wave. The shape of the string at any time, while the pulse is being reflected, can be found by adding an inverted image pulse to the incident pulse.

equation of wave propagating in +ve x-axis

$y_1 = a \sin(\omega t - kx)$

After reflection from rigid end

$y_2 = a \sin(\omega t + kx + \pi)$

$y_2 = -a \sin(\omega t + kx)$

By principle of super position, $y = y_1 + y_2$

$y = a \sin(\omega t - kx) - a \sin(\omega t + kx)$

$y = -2a \sin kx \cos \omega t$

This is equation of stationary wave reflected from rigid end

Amplitude $= 2a \sin kx$ .....(1)

velocity of particle

$V_{pa} = \frac{dy}{dt} = 2a\,\omega \sin kx \sin \omega t$ .....(2)

strain $\frac{dy}{dx} = -\,2ak \cos kx \cos \omega t$ .....(3)

$\text{Elasticity } E = \frac{\text{stress}}{\text{strain}} = \frac{dp}{\frac{dy}{dx}}$

change in pressure $dp = E\frac{dy}{dx}$ .....(4)

(1) Node $x = 0, \frac{\lambda}{2}, \lambda$ ....

$A = 0 \qquad V_{pa} = 0$

strain $\rightarrow$ max, change in pressure $\rightarrow$ max

This point is known as node,

position of nodes $\rightarrow 0, \frac{\lambda}{2}, \lambda$ ....

(2) Antinode $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$ ....

$A \rightarrow$ max , $V_{pa} \rightarrow$ max, strain = 0

change in pressure = 0

This point is known as antinode.

Position of antinodes $\rightarrow \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.......

REFLECTION FROM FREE END

The right end of the string is attached to a light frictionless ring which can freely move on a vertical rod. A wave pulse is sent on the string from left. When the wave reaches the right end, the element at this end is acted on by the force from the left to go up, However, there is no corresponding restoring force from the right as the rod does not exert a vertical force on the ring. As a result, the right end is displaced in upward direction more than the height of the pulley i.e., it overshoots the normal maximum displacement. The lack of restoring force from right can be equivalent described in the following way. An extra force acts from right which sends a wave from right to left with its shape identical to the original one. The element at the end is acted upon by both the incident and the reflected wave and the displacements add. Thus, a wave is reflected by the free end without inversion.

$y_1 = a \sin(\omega t - kx)$

$y_2 = a \sin(\omega t + kx)$

By principle of superposition,

$y = y_1 + y_2$

$y = a \sin(\omega t - kx) + a \sin(\omega t + kx)$

$y = 2\ a \sin \omega t \cos kx$

Amplitude = 2a cos kx

velocity of particle = $V_{pa} = \dfrac{dy}{dt}$

$= 2a\ \omega \cos \omega t \cos kx$

strain $\dfrac{dy}{dx} = -2ak \sin \omega t \sin kx$

change in pressure $dp = E \dfrac{dy}{dx}$

(1) Antinode :

$x = 0, \dfrac{\lambda}{2}, \lambda$ ....

$A \rightarrow$ max, $V_{pa} \rightarrow \dfrac{dy}{dt} \rightarrow$ max

strain = 0, dp = 0

This point is known as antinode.

Position of antinode $\rightarrow 0, \dfrac{\lambda}{2}, \lambda$ ....

(2) Node :

$x = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}$ ....

A = 0

$\dfrac{dy}{dt} = 0$

strain $\rightarrow$ max

dp $\rightarrow$ max

This point is known as node.

Position of node $\dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}$ ....

(i) Reflection from rigid end -

$y_1 = a \cos(\omega t - kx)$

$y_2 = a \cos(\omega t + kx + \pi)$

$y_2 = -a \cos(\omega t + kx)$

By principle of superposition

$y = y_1 + y_2$

$y = a \cos(\omega t - kx) - a \cos(\omega t + kx)$

$y = 2a \sin kx \sin \omega t$

This is equation of stationary wave reflected from rigid end.

(ii) Refraction from free end -

$y_1 = a \cos(\omega t - kx)$

$y_2 = a \cos(\omega t + kx)$

By principle of superposition

$y = y_1 + y_2$

$y = a \cos(\omega t - kx) + a \cos(\omega t + kx)$

$y = 2a \cos \omega t \cos kx$

This is equation of stationary wave rejected from free end.

Stationary wave are of two types

(1) Longitudinal st. wave

(2) Transverse st. wave

(a) Longitudinal stationary wave

it is produced in

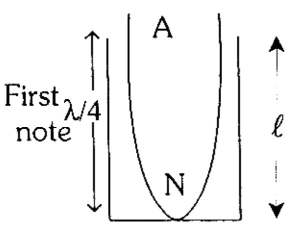
(i) Organ pipe

(ii) Resonance tube

APPLICATION OF STATIONARY WAVES

- Longitudinal stationary waves
  - Organ Pipe
    - **Closed organ pipe**
    - **Open organ pipe**
  - Resonance tube
- Transverse stationary waves
  - Vibration in stretched string
  - Sonometer

Closed organ pipe

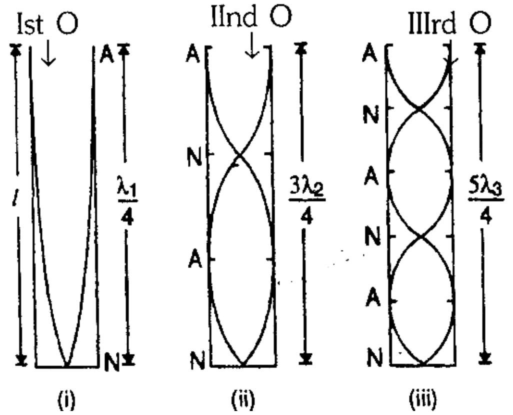


ℓ = Length of air coloum

$n = \dfrac{V}{4\ell}$ Minimum Frequency

O : Overtone

$n_1 : n_2 : n_3 : n_4$ 1 : 3 : 5 : 7



m[th] Overtone (2m+1)[th] harmone frequency

Only Odd Harmones

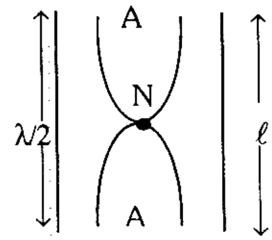
(A,N) ⇒ (1, 1) Fundmantal Note, First Harmonics

(2, 2) F.O., Third Harmonics

(3, 3) S.O., Fifth Harmonics

Anti Nodes = m+1
Nodes = m+1

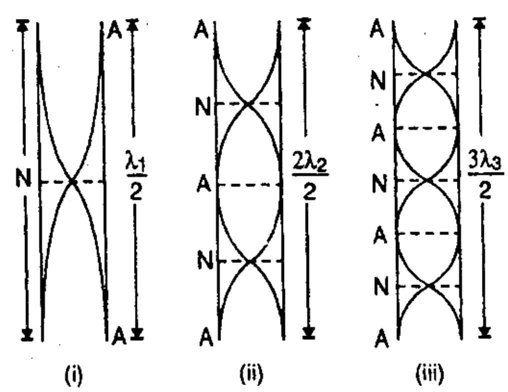
Open organ pipe



$n = \dfrac{V}{2\ell}$ Minimum Frequency

FO SO TO

$n_1 : n_2 : n_3 : n_4$: : 1 : 2 : 3 : 4



m[th] Overtone → (m+1)[th] Harmone frequency

All Harmones

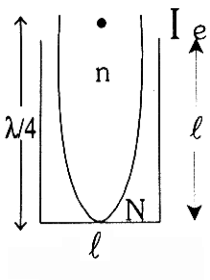
(A,N) ⇒ (2, 1) Fundamental Note, First Harmonics

(3, 2) F.O., Second Harmonics

(4, 3) S.O., Third Harmonics

m + 2 ⇒ Anti Nodes
m + 1 ⇒ Nodes

Organ pipe with end correction $\rightarrow e = 0.6R = 0.3D$



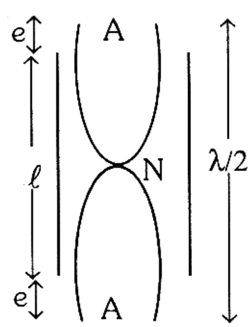
$$\frac{\lambda}{4} = \ell + e$$

or $$\frac{\lambda}{4} = \ell + 0.6r$$

$$\lambda = 4(\ell + 0.6r)$$

$$\because n_C = \frac{V}{\lambda}$$

$$\boxed{n_C = \frac{V}{4(\ell + 0.6r)}}$$



OOP मे दोनों ओर end correction

अत: कुल = $2 \times .6r = 1.2r$

$$\frac{\lambda}{2} = \ell + 1.2r$$

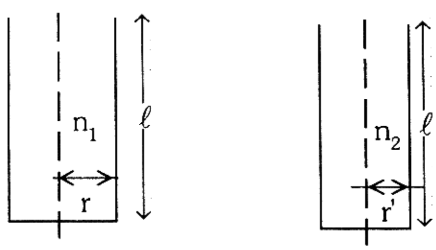
$$\lambda = 2(\ell + 1.2r)$$

or $$\boxed{n_O = \frac{V}{2(\ell + 1.2r)}}$$

More wider is the tube, more will be the loss the sound energy.

Frequency increases as tube in narrowed.

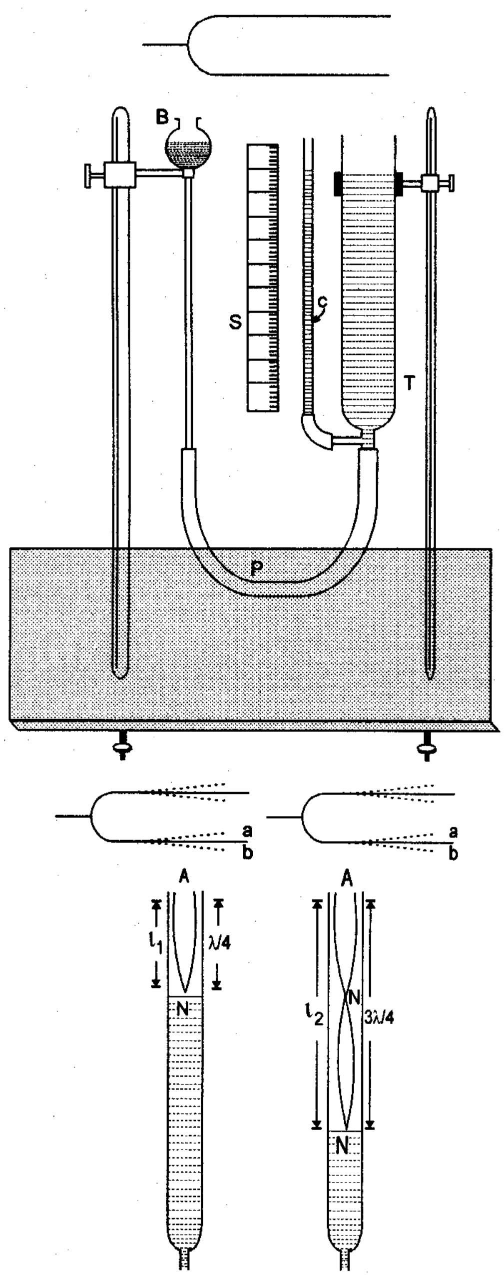
If end correction is taken n_0 is slightly greater than half of n_0



If r > r' than $n_1 < n_2$

RESONANCE TUBE -

Construction : The resonance tube is T (fig.) made of brass or glass, about 1 meter long and 5 cm in diameter and fixed on a vertical stand. Its lower end is connected to a water reservoir B by means of a flexible rubber tube. The rubber tube carries a pinch-cock P. The level of water in T can be raised or lowered by water adjusting the height of the reservoir B and controlling the flow of water from B to T or from T to B by means of the pinch-cock P. Thus the length of the air-column in T can be changed. The position of the water level in T can be read by means of a side tube C and a scale S.



Determination of the speed of sound in air by resonance tube -

First of all the water reservoir B is raised until the water level in the tube T rises almost to the top of the tube. Then the pinch-cock P is tightened and the reservoir B is lowered. The water level in T stays at the top. Now a tuning fork is sounded and held over the mouth of tube. The pinch-cock P is opened slowly so that the water level in T falls and the length of the air-column increases. At a particular length of air-column in T, a loud sound is heard. This is the first state of resonance. In this position the following phenomenon takes place inside the tube.

(i) For fist resonance $I_1 = \lambda/4$ ......(1)

(ii) For second resonance $I_2 = 3\lambda/4$ ......(2)

Substract eq. (2) from eq. (1)

$I_2 - I_1 = \pi/2$

$\lambda = 2(I_2 - I_1)$

If the frequency of the fork be n and the temperature of the air-column be t°C, then the speed of sound at t°C is given by

$v_1 = n\lambda = 2n(I_2 - I_1)$

The speed of sound wave at 0°C

$v_0 = (v_t - 0.61t)$ m/s.

End correction - In the resonance tube, the antinode is not formed exactly at the open end but slightly outside at a distance x. Hence the length of the air-column in the first and second states of resonance are $(I_1 + x)$ and $(I_2 + x)$ then

(i) For first resonance $I_1 + x = \lambda/4$ .....(1)

(ii) For second resonance $I_2 + x = 3\lambda/4$ .....(2)

Substract eq. (2) from eq. (1)

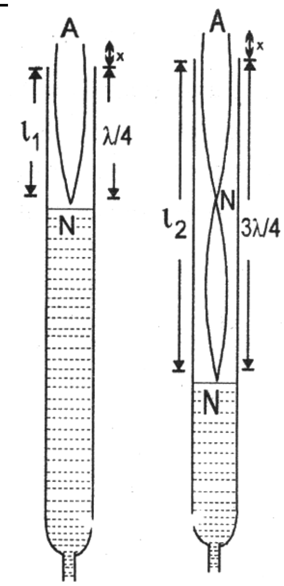
$I_2 - I_1 = \lambda/2$

$\lambda = 2\ (I_2 - I_1)$

Put the value of λ in eq. (1) $I_1 + x = \frac{2(I_2 - I_1)}{4}$

$\Rightarrow I_1 + x = \frac{I_2 - I_1}{2}$

$x = \frac{I_2 - 3I_1}{2}$



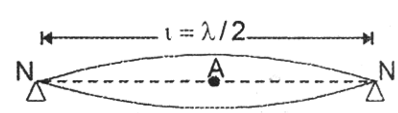
(ii) Transverse stationary wave - It is produced in (i) stretched string and (ii) sono meter

(i) Vibration in stretched string - When a wire clamped to rigid supports at its ends is plucked in the middle, transverse progressive waves travel towards each end of the wire. The speed of these waves is

$$v = \sqrt{\left(\frac{T}{m}\right)}$$

where T is the tension in the wire and m is the mass per unit length of the wire. These waves are reflected at the ends of the wire. By the superposition of the incident and the reflected wave, transverse stationary waves are set up in the wire. Since the ends of the wire are clamped, there is a node N at each end and an antinode A in the middle (Fig.)

We know that the distance between two consecutive nodes is $\lambda/2$, where λ is wavelength. Hence if l be the length of the wire between the clamped, ends, then



$$l = \frac{\lambda}{2}$$

or $\lambda = 2l$

If n be the frequency of vibration of the wire, then

$$n = \frac{V}{\lambda} = \frac{V}{2l}$$

Substituting the value of v from eq. (i), we have

$$n = \frac{l}{2l}\sqrt{\left(\frac{T}{m}\right)}$$

This is the frequency of the node emitted by the wire.

Fundamental and overtones of a string - When a stretched wire is plucked in the middle, the wire usually vibrates in a single segement (fig.) At the ends of the wire are nodes (N) and in the middle an antinode (A). In this condition, the tone emitted from the wrie is called the 'fundamental tone'.

If l be the length of the wire, and λ_1 be the wavelength is thin are, then

$$l = \lambda_1/2$$

or $l_1 = 2l$

If n_1 be the frequency of vibration of the wire and v the speed of the wave in the wire, then

$$n_1 = \frac{v}{\lambda_1} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)}$$

This is the fundamental frequency of the wire.

We can make the wire vibrate in more than one segment. If we touch the middle-point of the wire by a feather, and pluck is at one-fourth of its length for an end, then the wire vibrates in two segment (fig.) In this case, in adition at the ends of the wire, there will be a node (N) at the middle-point also, and in between three nodes there will be two antinodes (A) Therefore, if λ_2 be the wavelength in this case, then

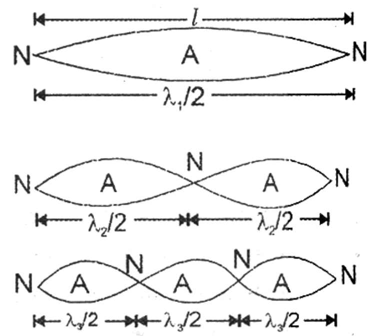
$$\ell = \frac{\lambda_2}{2} + \frac{\lambda}{2} = \frac{2\lambda_2}{2}$$

or $l_2 = 2\ell/2$

If the frequency of the wire be one n_2, then

$$n_2 = \frac{v}{\lambda_2} = \frac{2v}{2\ell} = \frac{2}{2\ell}\sqrt{\left(\frac{T}{m}\right)} = 2n_1$$

that is, in this case the frequency of the tone emitted from the wire is twice the frequency of the fundamental tone. This tone is called the 'fist overtone'.



Similarly, if the wire vibrates in three segments (fig.) and the wavelength in this case be λ_3, then

$$\ell = n = \frac{p}{2\ell}\sqrt{\frac{T}{m}} \qquad \lambda_3 = 2\ell/3$$

If the frequency of the wire be now n_3, then

$$n_3 = \frac{v}{\lambda_3} = \frac{3v}{2\ell} = \frac{3v}{2\ell} = \frac{3}{2\ell}\sqrt{\left(\frac{T}{m}\right)} = 3n_1$$

that is, in this case the frequency of the emitted tone is three times the frequency of the fundamental tone. This tone is called 'second overtone'.

Similarly, if the wire is made to vibrate in four, five, ...segments then still higher overdoes can be produced. If the wire vibrates in p segments, then its frequency is given by

$$n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$$

Thus, the frequencies of the fundamental tone and the overtones of a stretched string have the following relationship:

$$n_1 : n_2 : n_3 : n_4 ..... = 1 : 2 : 3 : 4 ....$$

These frequencies are in a harmonic series. Hence these tones are also called 'harmonics'. The fundamental tone (n_1) is the first harmonic, the first overtone (n_2) is the second harmonic, the second overtone (n_3) is the third harmonic, etc. The tonic of frequencies n_1, n_2, n_3 ..... are the 'odd harmonics' and the tones of frequencies n_2, n_4, n_6....... are the 'even harmonics' clearly, a stretched string gives both even and odd harmonics.

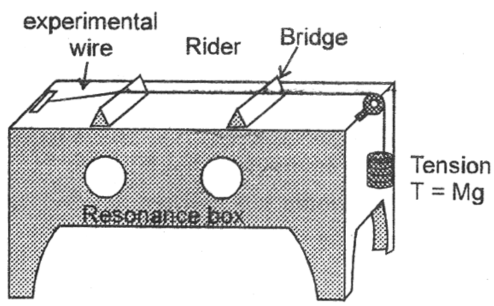
SONOMETER

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley at the other end of the box. The wire is stretched by a tension T.

The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length are wire between the two bridges is ℓ, then the frequency of vibration is

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$$

To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridge is property adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire.



Comment :

$$m = \frac{\text{mass of wire}}{\text{length of wire}} = \frac{\pi r^2 d\ell}{\ell} = \pi r^2 d$$

where r is the radius of the wire and d is the density of the material of the wire. Thus the frequency of vibration of a given string under tension is

$$n \propto \frac{1}{\sqrt{r^2 d}}$$

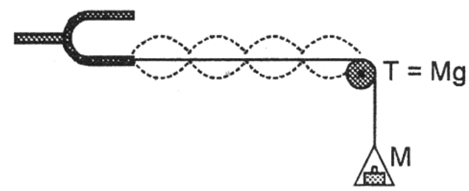
Thus $n \propto \frac{1}{r}$ (for same material value)

and $n \propto \frac{1}{\sqrt{d}}$

(for different material waves of same radius)

MISCELLANEOUS TOPICS

(1) Melde's experiment : In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be suitable loaded.

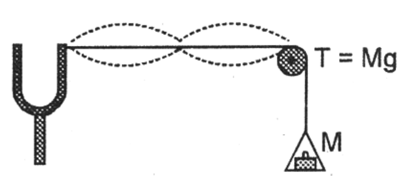


Case 1. In a vibrating string of fixed length, the product of number of loops in a vibrating string and square root of tension is a constant or

$$p\sqrt{T} = \text{constant}$$

Case 2. When the tuning fork is set vibrating as shown in fig. then the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread string is equal to frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal mode of the string matched with the frequency of the tuning fork). Thin, if p loops are formed in the thread, then the frequency of the tuning fork is given by $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

Case 3. If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread sage only when the prong moves towards the pulley i.e only once in a vibration.



The thread performs sustained oscillation when the natural frequency of the given length of the thread under tension is half that of the fork. Thus if p loops are formed in the thread, then the frequency of the tuning fork is

$$n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$$

Solved Examples

Ex.14 The length of an organ pipe open at both ends is 0.5 meter. Calculate the fundamental frequency of the pipe, if the velocity of sound in air be 350 m/sec. If one end of the pipe is closed, then the fundamental frequency will be -

Sol. Speed of sound v = 350 m/sec

length of pipe ℓ = 0.5 m

The frequency of the fundamental tone of a pipe open at both ends is given by

$$n = \frac{v}{2\ell} = \frac{350}{2 \times 0.5} = 350\ \text{sec}^{-1}$$

The frequency of the fundamental tone of pipe open at one end is given by

$$n = \frac{v}{4\ell} = \frac{350}{4 \times 0.5} = 175\ \text{sec}^{-1}$$

Ex.15 The length of a pipe one at both ends is 48 cm and its fundamental frequency is 320 Hz. If the speed of sound be 320m/sec, then determine the diameter of the pipe. If one end of the pipe be closed, then what will be the fundamental frequency?

Sol. Fundamental frequency of the pipe of diameter D, open at both ends, is

$$n = \frac{v}{2(\ell + 2e)} = \frac{v}{2(\ell + 2 \times 0.3D)}$$

$$\Rightarrow 320 = \frac{32000}{2(48 + 2 \times 0.3D)} \quad \Rightarrow D = 3.3 \text{ cm}$$

For a pipe closed at one end

$$n = \frac{v}{4(\ell + e)} = \frac{v}{4(1 + 0.3D)}$$

$$n = \frac{32000}{4(48 + 0.3 \times 3.33)} \quad \Rightarrow n = 163.3 \text{ Hz}$$

Ex.16 When a closed pipe is suddenly opened then the second overtone of closed pipe and first overtone of open pipe differ by 100 Hz. The fundamental frequency of closed pipe will be -

Sol. Second overtone of closed pipe = $\frac{5V}{4L}$

First overtone of open pipe = $\frac{V}{L}$

Given that $\frac{5V}{4L} - \frac{V}{L} = \frac{V}{4L} = 100$ Fundamental frequency of closed pipe.

Ex.17 Two closed pipes, one filled with O_2 and the other with H_2, have the same fundamental frequency. Find the ratio of their lengths.

Sol. $n = \frac{v_1}{4\ell_1} = \frac{v_2}{4\ell_2}$

$$\therefore \frac{\ell_1}{\ell_2} = \frac{v_1}{v_2} = \sqrt{\left(\frac{d_2}{d_1}\right)} = \sqrt{\left(\frac{1}{16}\right)} = \frac{1}{4}$$

$\therefore \ell_1 : \ell_2 = 1 : 4$

Ex.18 The speed of sound in gas in which two waves of wavelengths 1.00 m and 1.01 m produce 10 beats in 3 second is

Sol. Let v be the speed of sound in the gas n_1 and n_2 be the frequencies of the two waves. Then,

$$n_1 = \frac{v}{\lambda_1} = \frac{v}{1.00} \text{ and } n_2 = \frac{v}{\lambda_2} = \frac{v}{1.01}$$

number of beats per sec. $n_1 - n_2 = 10/3$

$\therefore n_1 - n_2 = v/1.00 - v/1.01 = 10/3$

$\Rightarrow v = 336.7$ m/sec

Ex.19 A column of air and a tuning fork produce 4 beats per second when sounded together. The tuning fork gives the lower note. The temperature of air is 15ºC. When the temperature falls to 10ºC, the two produce 3 beats per second. Find the frequency of the fork.

Sol. Let the frequency of the tuning fork be n Hz.

Then frequency of air column at 15ºC = n + 4

Frequency of air column at 10ºC = n + 3

According to v = nλ, we have

$v_{15} = (n + 4)\lambda$ and $v_{10} = (n + 3)\lambda$

$$\therefore \frac{v_{15}}{v_{10}} = \frac{n+4}{n+3}$$

The speed of sound in directly proportional to the square-root of the absolute temperate.

$$\therefore \frac{v_{15}}{v_{10}} = \sqrt{\frac{15+273}{10+273}} = \sqrt{\frac{288}{283}}$$

$$\therefore \frac{n+4}{n+3} = \sqrt{\frac{288}{283}} = \left(1+\frac{5}{283}\right)^{1/2}$$

$$\Rightarrow 1 + \frac{1}{n+3} = 1 + 1/2 \times \frac{5}{283} = 1 + \frac{5}{566}$$

$$\Rightarrow \frac{1}{n+3} = \frac{5}{566}$$

$$\Rightarrow n + 3 = 113$$

$$\Rightarrow n = 110 \text{ Hz}$$

Ex.20 A 70 cm long sonometer wire is in unison with a tuning fork. If the length of the wire is decreased by 1.0cm, it produces 4 beats per sec with the same tuning fork. Find the frequency of the tuning fork.

Sol. Let the frequency of the fork be n.

In the first case, the length of the wire is 70 cm (0.70 cm)

Therefore,

$$n = \frac{1}{2 \times 0.70}\sqrt{\frac{T}{m}} \quad ....(1)$$

On decreasing the length of the wire, its frequency will increases. Hence in the second case when the length is 69 cm = 0.69 m, then

$$n + 4 = \frac{1}{2 \times 0.69}\sqrt{\frac{T}{m}} \quad ....(2)$$

Dividing equation (1) by (2), we get

$$\Rightarrow \frac{n}{n+4} = \frac{0.69}{0.70}$$

$$\Rightarrow n \times 0.70 = (n + 4) \times 0.69$$

$$\Rightarrow n = \frac{4 \times 0.69}{0.01} = 276 \text{ sec}^{-1}$$

Ex.21 Two identical sonometer wires have fundamental frequencies of 500 vibration/sec, when kept under the same tension. What fractional increases in the tension of one wire would cause an occurrence of 5 beats per sec. What both wires vibrates together ?

Sol. $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

$$n + 5 = \frac{1}{2\ell}\sqrt{\frac{T+\Delta T}{m}}$$

$$\therefore \frac{n+5}{n} = \sqrt{\frac{T+\Delta T}{T}}$$

$$\Rightarrow 1 + \frac{\Delta T}{T} = \left(1+\frac{5}{n}\right)^2 = 1 + \frac{2\times 5}{n} = 1 + \frac{10}{500}$$

$$\Rightarrow \frac{\Delta T}{T} = \frac{10}{500} = 0.020$$

MUSICAL SCALE

A series of notes arranged such that their fundamental frequencies have definite ratios is called a musical scale.

It is mainly two types

(i) Diatonic scale

(ii) Tempered scale

1.	**Diatonic Scale** -	1	2	3	4	5	6	7	8
	Indian Name	Sa	Re	Ga	Ma	Pa	Da	Ni	Sa
	Simbol	C	D	E	F	G	A	B	C
	English	Doh	Re	Me	Fah	So	Lah	Tee	Doh
		8 :	9 :	10	8 :	9 :	10	15 :	16
				15	16		8	9	
If Sa = 24 Hz then other tone is		24	27	30	32	36	40	45	48 Hz

2. **Tempered Scale -**

In such time of scale there are 11 note b/w fundamental and its octave and they are arranged in geometrical progression (G. P.)

so n, nx, nx^2 .... nx^{12}

$nx^{12} = 2n$, $x = 2^{1/12} = 1.059$

FEATURES OF MUSICAL SOUND

(i) **Loudness -** The quality of sound on the basis of which, sound is said to be high or low, it depends on the

(1) Shape of the source

(2) Intensity of sound

$L \propto \log I$ (unit of L is 'phone' when I is measured in decibel)

⇒ According to weber – Fechner law - the loudness of a sound of intensity I is given

by : $L = 10 \log_{10} (I / I_0)$ db

where I_0 represents the threshold of hearing at 0 db loudness level.

If $I = 10\ I_0$, then $L = 10 \log_{10} (10\ I_0 / I_0) = 10$ db. 1 bel.

Thus the loudness of a sound is said to be 1 bel. If its intensity is 10 times that of the threshold of hearing.

A practical and smaller unit of loudness is decibel (bB) 1 dB = 1/10 bel.

In decibets the loudness of a sound of intensity I is given by $L = 10 \log_{10} (I / I_0)$

The amplitude of a roaring lion is more than sound produced by a mosquito.

(ii) **Pitch -** On the basis of this characteristic said to be sharp or dull. Pitch increases with the frequency of sound. For example pitch of roaring of lion is less than that of the sound of a mosquito.

(iii) **Quality -** On the basis of this property the sound of same loudness and pitch can be differentiated. The basis of this differentiation is number of harmonics present in the sound, relative intensity etc. On increasing the number of overtones, sweetness of the sound increases. Example - Whistle's sound (closed pipe) is less sweet than that of flute (open pipe).

Important information about waves -

(i) **Shock waves -** A body moving with a supersonic velocity leaves behind itself a conical disturbance region. Disturbance of this kind is called a shock wave. These have a large amount of energy which can damage the buildings.

(ii) Mach number $= \dfrac{\text{speed of any vehicle or body}}{\text{speed of sound}}$

(iii) Vibration of tuning fork : When tuning fork is sounded by striking its one end on rubber pad, then

(a) The ends of prongs vibrate in and out while the stem vibrates up and down or vibration of the prongs are transverse and that of the stream is longitudinal. Generally turning fork produces fundamental tone.

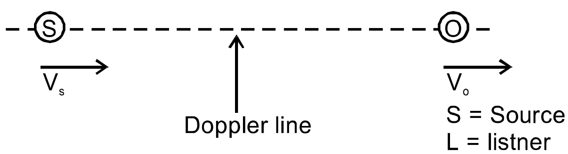
(b) At the free end of a fork antinodes are formed. At the place where stream is fixed antinode is formed. In between these antinodes, nodes are formed.

(c) Frequency of tuning fork decrease with increase in temperature.

(d) Increasing the weight, the frequency of a tuning fork decreases while on filing the prongs near stem, the frequency decreases.

DOPPLER EFFECT

medium at rest



$$N = \left(\frac{\text{Listner effect}}{\text{Source effect}}\right) N = \left(\frac{V \pm V_0}{V \mp V_s}\right) N_0$$

"Effect of motion of medium on sound velocity"

V_M = velocity of medium

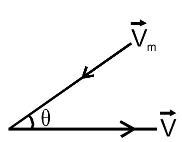
V = velocity of sound

(1) $\vec{V}_m$ & $\vec{V}$ (Parallel to each other) $V_{eff} = V + V_m$

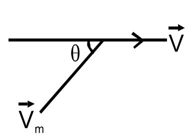
(2) $\vec{V}_m$ & $\vec{V}$ (antiparallel to each other)

$V_{eff} = V - V_m$

(3) $V_{eff} = V + V_m \cos\theta$



(4) $V_{eff} = V - V_m \cos\theta$



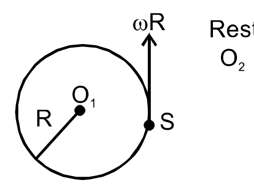
Doppler effect will not be observed

(i) If both S & L are at rest & only medium is flowing then no DE.

(ii) For Super sonic sound source (waves produced by super sonic source is super sonic wave)

(iii) If either source or listener moving to the line joining them then Doppler effect is not observed.

Imp.



O_1 do not observed DE but O_2 will observe

$$N_{max} = \left(\frac{V}{V - \omega R}\right) N$$

N → Original frequency $\quad N_{mim} = \left(\frac{V}{V + \omega R}\right) N$

R → Radius of circular path $\quad \Rightarrow \frac{N_{max}}{N_{min}} = \frac{V + \omega R}{V - \omega R}$

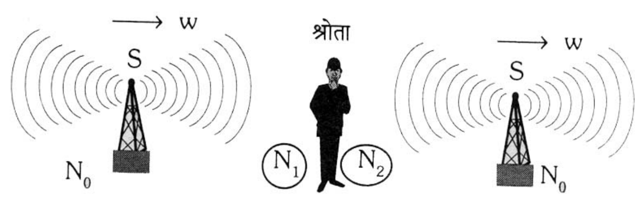
(vi) If source & listener are moving with common velocity DE is not seen becouse there is no relative motion.

$$N' = N\left(\frac{V \pm V_0}{V \pm V_s}\right)$$

V = Velocity of light, V_0 = Velocity of observer, V_s = Velocity of source

Sp. Problems

1. If a observer move towards a source with velocity w then its frequency increases & If observer moves from the source with velocity w then its frequency dcreases.



Espeical Note :-

(1) Doppler shift does not depends on the distance between sound source and listner

(i) $N_1 = \left(\frac{V}{V - V_s}\right) N_0$ ......(i)

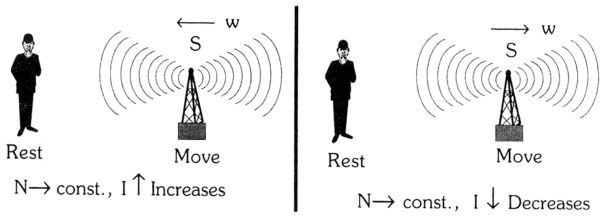
$N_2 = \left(\frac{V}{V + V_s}\right) N_0$ ......(ii)

Imp. Change in frequency $N_1 - N_2$ $\Delta N \cong \frac{2V_s}{V} N_0$

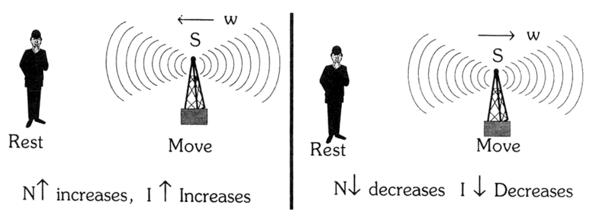
When source crosses observer the frequency becames 5/6. Then find velocity of source.

Two Important Cases :

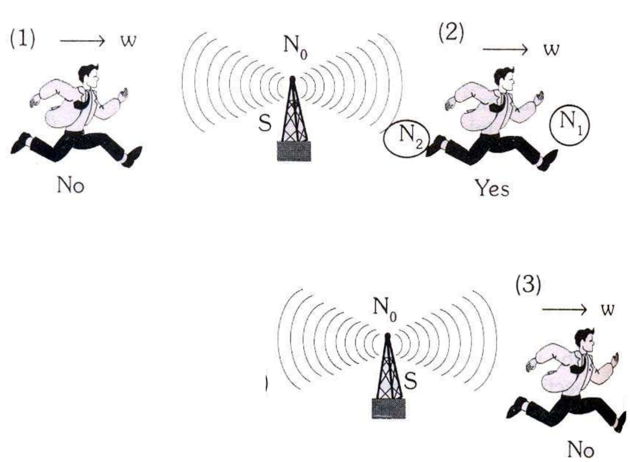
(A) When source move with constant velocity.



(B) When source move with constnat accleration.



Beats Phenomenon :



First condition – No beats phenomenon occur.

Second Condition – Beats phenomenon occur and Beat frequency, $b = N_1 - N_2$

$N_1 = \left(\frac{V + w}{V}\right) N_0$

$b = \frac{2w}{V} N_0$

$N_2 = \left(\frac{V - w}{V}\right) N_0$

Third Condition – No beats because listener is moving far from both sources.

Specital Notes :

If any listner moving in between two stationary sound sources of identical frequency, beats phenomenon will be observed by listner.

The wave length of sound wave produced by souce only & only depends on motion of source. First λ change then N change if source is moving.

If source is at rest sλ is constant & directly N' changes because its velocity of motion which is given by.

extra waves per sec. $= \pm \frac{V_L}{\lambda_0}$ where $\lambda_0 = \frac{V}{N_0}$

Optical Doppler effect.

(1) Eintein's theory of relativity is used to work out Dopper shift.

According to Einstein if distance between (S) & (O) w.r.t. time is

(a) $\downarrow \Rightarrow \nu > \nu_0 \Rightarrow \lambda < \lambda_0 \Rightarrow$ $\Delta\nu$ (+ve), $\Delta\lambda$ (-ve)

(decrease) = Special lines shifted towards violet end $\Rightarrow$ voilet shift

Apparent optical frequency

$$\gamma' = \sqrt{\frac{1 + \frac{v}{C}}{1 - \frac{v}{C}}}\ \gamma$$

(b) $\uparrow \Rightarrow \nu > \nu_0 \Rightarrow \lambda > \lambda_0 \Rightarrow$ $\Delta\nu$ (-ive), $\Delta\lambda$ (-ive) (inreases)

$\nu' = \sqrt{\dfrac{1-\frac{v}{C}}{1+\frac{v}{C}}}\,\nu$ = Special lines towards Red end = Red shift or Blood shift

If V = Relative velocity of source & observer

(a) Wave length shift in optical DE $\Delta\lambda = \dfrac{V}{C}\lambda_0$

(b) Frequency shift $\Delta\nu = \dfrac{V}{C}\nu_0$

Applications of Doppler effect :

(1) To find out velocity of any light source w.r.t. to us or earth.

(2) Spin Rotational speed of Sun.

[Sun rotates on its own axis from East to West with 2 km/s]

In solar spectrum both types of Doppler shifts (Voilet & Red) are obtained from the Eastern & western part of Sun because one part is moving towards observer & other is away

from the observer

(3) To find out width of spectral line :

$\lambda = \dfrac{V}{C}\lambda_0$

Spectral width = $\lambda_{max} - \lambda_{min} = (V + \Delta\lambda) - (V - \Delta\lambda)$

$= 2\lambda\Delta\lambda$ $\quad S_w = \dfrac{2V}{C}\lambda_0$

(4) Invention of Saturn Ring

It is a group of satelite revolving around saturn planer.

Super sonic → Produced by suspersonic sound sources

Mach Number = $\dfrac{\text{Vel. of Body}}{\text{Vel. of sound}}$

For Supersonic sound source

Mach Number > 1

MELDE'S EXPERIMENT

(i) In a vibrating string of fixed length, the product of no. of loops in a vibrating string and square root of tension is a constant or $P\sqrt{T}$ = const.

(ii) In a transverse vibtation system the frequency of the tuning fork = frequency of vibration of string

$n = \dfrac{P}{2L}\sqrt{\dfrac{T}{m}}$

(iii) In longitudinal vibration system the frequency of tuning fork = 2 × (vibration frequency of string),

$n = \dfrac{P}{L}\sqrt{\dfrac{T}{m}}$

(iv) In this experiment vibration of sting are always transverse, but in longitudinal vibration system the vibration of the arms of the turning fork are along the direction of string. This experiment is also based on the stationary (transverse) waves.

DOPPLER'S EFFECT

Accoustic Doppler Effect — Optical Doppler Effect

Sound wave — Light wave

1. Medium dependent wave as effect of motion of medium	No effect
2. If source or listner is moving to the doppler line then no doppler effect.	There is doppler shift which is visualised or either source or observer moving in $\perp$ direction to doppler line
3. Velocity addition orsubtraction can be applied in accoustic doppler effect.	Light is an absolute motion so addition and subtraction of velocities can not apply here and to find out doppler shift Einstein theroy of relativity can be used.
4. Iimitation - If Vs > V or Vo > V then doppler shift not visulised because in this cases shock waves are produced for which wave front is distorted & in this case Hygen's wave theroy cannot be applied. Applications SONAR System To find out velocity position of submarines If Vs = vel. of submarine w.r.t. SONAR system V = Vel. of sound ΔN = change in frequency then $\boxed{\Delta N = \frac{2V_s}{V}(No)}$ (ii) Frequency of ultrasonic wave is more than 20,000 Hz. (iii) Infrasonic waves's frequency is less than 20 Hz. $\sin\theta = \frac{V}{V_s} = \frac{1}{MNo}$ 2 θ = cone angle.	There is no such limitation Application RADAR Vs = vel of fighter plane w.r.t. RADAR system ν_0 = Material fre. of Radio wave transimitted from RADAR g = Chnage in frequency in light $\boxed{\Delta\nu = \frac{2V_s}{C}(\nu_0)}$