PROBABILITY

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- Mathematical Definition of Probability

MATHEMATICAL DEFINITION OF PROBABILITY

Let there are n exhaustive, mutually exclusive and equally likely cases for an event A and m of those are favourable to it, then probability of happening of the event A is defined by the ratio m/n which is denoted by P(A). Thus

$$P(A) = \frac{m}{n}$$

$$= \frac{\text{No. of favourable cases to A}}{\text{No. of exhaustive cases to A}}$$

Note : It is obvious that $0 \le m \le n$. If an event A is certain to happen, then m = n thus P (A) = 1.

If A is impossible to happen then m = 0 and so P (A) = 0. Hence we conclude that

$0 \le P(A) \le 1$

Further, if $\overline{A}$ denotes negative of A i.e. event that A doesn't happen, then for above cases m, n ; we shall have

$$P(\overline{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$\therefore$ $P(A) + P(\overline{A}) = 1$

Playing Cards :

(i) Total : 52 (26 red, 26 black)

(ii) Four suits : Heart, Diamond, Spade, Club - 13 cards each

(iii) Court Cards : 12 (4 Kings, 4 queens, 4 jacks)

(iv) Honour Cards:16 (4 aces, 4 kings, 4 queens, 4 jacks)

❖ EXAMPLES ❖

Ex.1 Two dice are thrown at a time. Find the probability of the following -

(i) these numbers shown are equal;

(ii) the difference of numbers shown is 1.

Sol. The sample space in a throw of two dice

s = {1, 2, 3, 4, 5, 6} ×{1, 2, 3, 4, 5, 6}.

total no. of cases n (s) = 6 × 6 = 36.

(i) Here E_1 = the event of showing equal number on both dice

= {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6) }

$\therefore$ $n(E_1) = 6$

$\therefore$ $P(E_1) = \frac{n(E_1)}{n(s)} = \frac{6}{36} = \frac{1}{6}$

(ii) Here E_2 = the event of showing numbers whose difference is 1.

= {(1, 2) (2, 1) (2, 3) (3, 2) (3, 4) (4, 3) (4, 5) (5, 4) (5, 6) (6, 5)}

$\therefore$ $n(E_2) = 10$ $\therefore p(E_2) = \frac{n(E_2)}{n(s)} = \frac{10}{36} = \frac{5}{18}$

Ex.2 Three coins are tossed together -

(i) Find the probability of getting exactly two heads,

(ii) Find the probability of getting at least two tails.

Sol. The sample space in tossing three coins

S = (H, T) × (H, T) × (H, T)

$\therefore$ Total no. of cases n (s) = 2 × 2 × 2 = 8

(i) Here E_1 = the event of getting exactly two heads

= {HHT, HTH, THH}

$\therefore$ n $(E_1) = 3$ $\therefore$ P $(E_1) = \dfrac{n(E_1)}{n(s)} = \dfrac{3}{8}$

(ii) E_2 = {HTT, THT, TTH, TTT}

$\therefore$ n $(E_2) = 4$,

$\therefore$ $P(E_2) = \dfrac{n(E_2)}{n(s)} = \dfrac{4}{8} = \dfrac{1}{2}$

Ex.3 Find the probability of throwing (a) 3, (b) an even number with an ordinary six faced die.

Sol.(a) There are 6 possible ways in which the die can fall and there is only one way of throwing 3.

$\therefore$ The required probability

$= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \dfrac{1}{6}$

(b) Total number of outcomes of throwing a die = 6.

Number of outcomes of falling even number i.e. 2, 4, 6 = 3.

The required probability = $\dfrac{3}{6} = \dfrac{1}{2}$

Ex.4 A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that the card drawn is neither a red card nor a queen.

Sol. There are 26 red cards (including 2 red queens) and 2 more queens are there. Thus, we have to set aside 28 cards.

And, we have to draw 1 card out of the remaining (52 – 28) = 24 cards.

$\therefore$ Required probability = $\dfrac{24}{52} = \dfrac{6}{13}$.

Ex.5 Find the probability of getting a number less than 5 in a single throw of a die.

Sol. There are 4 numbers which are less than 5, i.e. 1, 2, 3 and 4.

Number of such favourable outcomes = 4.

$\because$ The number marked on all the faces of a die are 1, 2, 3, 4, 5 or 6

$\therefore$ Total number of possible outcomes = 6

$\therefore$ P(a number less than 5) = $\dfrac{4}{6} = \dfrac{2}{3}$

Ex.6 If the probability of winning a game is 0.3, what is the probability of lossing it ?

Sol. Probability of winning a game = 0.3.

Probability of losing it = q (say).

$\Rightarrow$ 0.3 + q = 1

$\Rightarrow$ q = 1 – 0.3

$\Rightarrow$ q = 0.7

Ex.7 Two coins are tossed simulataneously. Find the probability of getting

(i) two heads

(ii) at least one head

(iii) no head

Sol. Let H denotes head and T denotes tail.

$\therefore$ On tossing two coins simultaneously, all the possible outcomes are

(i) The probability of getting two heads = P(HH)

$= \dfrac{\text{Event of occurence of two heads}}{\text{Total number of possible outcomes}} = \dfrac{1}{4}$

(ii) The probability of getting at least one head = P(HT or TH or HH)

$= \dfrac{\text{Event of occurence of at least one head}}{\text{Total number of possible outcomes}} = \dfrac{3}{4}$

(iii) The probability of getting no head = P(TT)

$= \dfrac{\text{Event of occurence of no head}}{\text{Total number of possible outcomes}} = \dfrac{1}{4}$

Ex.8 On tossing three coins at a time, find -

(i) All possible outcomes.

(ii) events of occurence of 3 heads, 2 heads, 1 head and 0 head.

(iii) probabilty of getting 3 heads, 2 heads, 1 head and no head.

Sol. Let H denotes head and T denotes tail. On tossing three coins at a time,

(i) All possible outcomes = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}. These are the 8 possible outcomes.

(ii) An event of occurence of 3 heads

= (HHH) = 1

An event of occurence of 2 heads

= {HHT, HTH, THH} = 3

An event of occurence of 1 head

= {HTT, THT, TTH} = 3

An event of occurence of O head = {TTT} =1

(iii) Now, probability of getting 3 heads = P (HHH)

$$= \frac{\text{Event of occurence of 3 heads}}{\text{Total number of possible outcomes}} = \frac{1}{8}$$

Simultaneously, probability of getting 2 heads

= P(HHT or THH or HTH)

$$= \frac{\text{Event of occurence of 2 heads}}{\text{Total number of possible outcomes}} = \frac{3}{8}$$

Probability of getting one head

= P (HTT or THT or TTH)

$$= \frac{\text{Event of occurence of 1 head}}{\text{Total number of possible outcomes}} = \frac{3}{8}$$

Probability of getting no head = P (TTT)

$$= \frac{\text{Event of occurence of no head}}{\text{Total number of possible outcomes}} = \frac{1}{8}$$

Ex.9 One card is drawn from a well-shiffled deck of 52 cards. Find the probability of drawing:

(i) an ace

(ii) '2' of spades

(iii) '10' of black suit

Sol.(i) There are 4 aces in deck.

$\therefore$ Number of such favourable outcomes = 4

$\because$ Total number of cards in deck = 52.

$\therefore$ Total number of possible outcomes = 52.

$\therefore$ P(an ace) = $\frac{4}{52} = \frac{1}{13}$.

(ii) Number of '2' of spades = 1

Number of favourable outcomes = 1

Total number of possible outcomes = 52

$\therefore$ P('2' of spades) = $\frac{1}{52}$

(iii) There are 2 '10' of black suits (i.e. spade and club)

$\therefore$ Number of favourable outcomes = 2

Total number of possible outcomes = 52

$\therefore$ P('10' of a black suit) = $\frac{2}{52} = \frac{1}{26}$

Ex.10 A bag contains 12 balls out of which x are white,

(i) If one ball is drawn at random, what is the probability that it will be a white ball ?

(ii) If 6 more white balls are put in the bag, the probability of drawing a white ball will double than that in (i). Find x.

Sol. Random drawing of balls ensures equally likely outcomes

Total number of balls = 12

$\therefore$ Total number of possible outcomes = 12

Number of white balls = x

(i) Out of total 12 outcomes, favourable outcomes = x

$$P(\text{White ball}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{x}{12}$$

(ii) If 6 more white balls are put in the bag, then

Total number of white balls = x + 6

Total number of balls in the bag

= 12 + 6 = 18

$$P(\text{White ball}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{x+6}{12+6}$$

According to the question,

Probability of drawing white ball in second case

= 2 × probability drawing of white ball in first case

$$\Rightarrow \frac{x+6}{18} = 2\left(\frac{x}{12}\right) \quad \Rightarrow \frac{x+6}{18} = \frac{x}{6}$$

$\Rightarrow$ $6x + 36 - 18x$

$\Rightarrow$ $12x = 36$

$\Rightarrow$ $x = 3$

Hence, number of white balls = 3

Ex.11 What is the probability that a leap year, selected at random will contain 53 Sundays ?

Sol. Number of days in a leap year = 366 days

Now, 366 days = 52 weeks and 2 days

The remaining two days can be

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

For the leap year to contain 53 Sundays, last two days are either Sunday and Monday or Saturday and Sunday.

$\therefore$ Number of such favourable outcomes = 2

Total number of possible outcomes = 7

$\therefore$ P(a leap year contains 53 sundays) = $\frac{2}{7}$

Ex.12 Three unbiased coins are tossed together. Find the probability of getting :

(i) All heads, (ii) Two heads

(iii) One head (iv) At least two heads.

Sol. Elementary events associated to random experiment of tossing three coins are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$\therefore$ Total number of elementary events = 8.

(i) The event "Getting all heads" is said to occur, if the elementary event HHH occurs i.e. HHH is an outcome. Therefore,

$\therefore$ Favourable number of elementary events = 1

Hence, required probability = $\frac{1}{8}$

(ii) The event "Getting two heads" will occur, if one of the elementary events HHT, THH, HTH occurs.

$\therefore$ Favourable number of elementary events = 3

Hence, required probability = $\frac{3}{8}$

(iii) The events of getting one head, when three coins are tossed together, occurs if one of the elementary events HTT, THT, TTH happens.

$\therefore$ Favourable number of elementary events = 3

Hence, required probability = $\frac{3}{8}$

(iv) If any of the elementary events HHH, HHT, HTH and THH is an outcome, then we say that the event "Getting at least two heads" occurs.

$\therefore$ Favourable number of elementary events = 4

Hence, required probability = $\frac{4}{8} = \frac{1}{2}$.

Ex.13 17 Cards numbered 1, 2, 3 ... 17 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is

(i) Odd

(ii) A prime

(iii) Divisible by 3

(iv) Divisible by 3 and 2 both.

Sol. Out of 17 cards, in the box, one card can be drawn in 17 ways.

$\therefore$ Total number of elementary events = 17.

(i) There 9 odd numbered cards, namely, 1, 3, 5, 7, 9, 11, 13, 15, 17. Out of these 9 cards one card can be drawn in 9 ways.

$\therefore$ Favourable number of elementary events = 9.

Hence, required probability = $\frac{9}{17}$.

(ii) There are 7 prime numbered cards, namely, 2, 3, 5, 7, 11, 13, 17. Out of these 7 cards one card can be chosen in 7 ways.

$\therefore$ Favourable number of elementary events = 7.

Hence, P (Getting a prime number) = $\frac{7}{17}$.

(iii) Let A denote the event of getting a card bearing a number divisible by 3. Clearly, event A occurs if we get a card bearing one of the numbers 3, 6, 9, 12, 15.

$\therefore$ Favourable number of elementary events = 5.

Hence, P (Getting a card bearing a number divisible by 3) = $\frac{5}{17}$.

(iv) If a number is divisible by both 3 and 2, then it is a multiple of 6. In cards bearing number 1, 2, 3 ..., 17 there are only 2 cards which bear a number divisible by 3 and 2 both i.e. by 6. These cards bear numbers 6 and 12

$\therefore$ Favourable number of elementary events = 2

Hence, P (Getting a card bearing a number divisible by 3 and 2) = $\frac{2}{17}$.

Ex.14 A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. If one ball is drawn at random, find the probability that it is

(i) Black (ii) Red (iii) Not green.

Sol. Total number of balls in the bag

= 5 + 8 + 4 + 7 = 24

$\therefore$ Total number of elementary events = 24

(i) There are 7 black balls in the bag.

$\therefore$ Favourable number of elementary events = 7

Hence, P (Getting a black ball) = $\frac{7}{24}$.

(ii) There are 5 red balls in the bag.

$\therefore$ Favourable number of elementary events = 5

Hence, P (Getting a red ball) = $\frac{5}{24}$

(iii) There are 5 + 8 + 7 = 20 balls which are not green.

$\therefore$ Favourable number of elementary events = 20

Hence, P (No getting a green ball) = $\frac{20}{24} = \frac{5}{6}$

Ex.15 Find the probability that a number selected at random from the numbers 1 to 25 is not a prime number when each of the given number is equally likely to be selected.

Sol. Total number (1, 2, 3, 4, ... 25) = 25.

Out of 25 numbers prime numbers = 2, 3, 5, 7, 11, 13, 17, 19, 23.

So, remaining not a prime number are 25 – 9 = 16

Total number of possible outcomes = 25

and number of favourable outcomes = 16

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

P (not a prime) = $\frac{16}{25}$

Ex.16 A piggy bank contains hundred 50 p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) will be a 50 p coin ?

(ii) will not be a Rs. 5 coin ?

Sol. Number of 50 Rs coins = 100

Number of 1 Rs coins = 50

Number of 2 Rs coins = 20

Number of 5 Rs coins = 10

(i) The number of favourable outcomes of 50 p coin to fall = 100

Total number of coins = 100 + 50 + 20 + 10 = 180

Total number of possible outcomes = 180

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P\ (50\ p) = \frac{100}{180} = \frac{5}{9}$$

(ii) Number of favourable outcomes of 5 Rs coin to not fall = 180 – 10 = 170

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P\ (\text{not Rs. } 5) = \frac{170}{180} = \frac{17}{18}$$

Ex.17 (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective ?

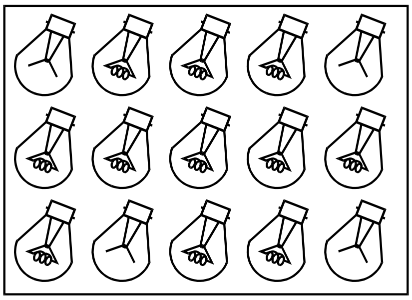
(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective ?

Sol. (i) The total number of bulbs = 20

Total number of possible outcomes = 20

Number of favourable outcomes of defective bulbs = 4

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$



$$P\ (\text{defective bulb}) = \frac{4}{20} = \frac{1}{5}$$

(ii) The bulb drawn is not defective

Total number of bulbs without replacement = 19

Number of defective bulbs = 4

Number of non defective bulbs = 19 – 4 = 15

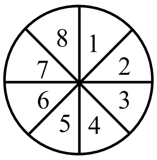
Number of favourable outcomes of non defective bulbs = 15

Total number of possible outcomes = 19

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P\ (\text{non defective}) = \frac{15}{19}$$

Ex.18 A game of chance consists of spinning an arrow which comes to rest pointing at one of the number 1, 2, 3, 4, 5, 6, 7, 8 (see fig), and these are equally likely outcomes. What is the probability that it will point at



(i) 8

(ii) an odd number ?

(iii) a number greater than 2 ?

(iv) a number less than 9 ?

Sol. Total number of possible outcomes in the game = 8

(i) Number of rest of arrow on 8 = 1

Number of favourable outcomes of 8 = 1

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P\ (8) = \frac{1}{8}$$

(ii) In the game the number of odd number

1, 3, 5, 7 = 4

Number of favourable outcomes of odd number = 4

$$P = (\text{odd number}) = \frac{4}{8} = \frac{1}{2}$$

(iii) Numbers greater than 2 = 6

Number of favourable outcomes of greater than 2 = 6

$$P = (\text{greater than } 2) = \frac{6}{8} = \frac{3}{4}$$

(iv) Number less than 9 = 8

Number of favourable outcome of less than 9 = 8

$$P\ (\text{less than } 9) = \frac{8}{8} = 1$$

Ex.19 It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday ?

Sol. Probability of 2 students from a group of 3 students not having the same birthday = 0.992

Probability of 2 students from a group of 3 students having the same birthday

$[\therefore p + q = 1] = 1 - 0.992 = 0.008$

Ex.20 A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that the card drawn is neither a red card nor a queen.

Sol. Number of red cards including 2 red queens = 26

Number of black queens = 2

Therefore, number of red cards including 2 red queens and 2 black queens = 26 + 2 = 28

Number of cards neither a red card nor a queen = 52 – 28 = 24

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P\text{ (neither a red nor a queen card)} = \frac{24}{52} = \frac{6}{13}$$

Ex.21 A card is drawn from a well-shuffled deck of playing cards. Find the probability of drawing

(i) a face card (ii) a red face card.

Sol. Random drawing of cards ensures equally likely outcomes

(i) Number of face cards (King, queen and Jack of each suits) = 3 × 4 = 12

Total number of cards in a deck = 52

∴ Total number of possible outcomes = 52

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P\text{ (drawing a face card)} = \frac{12}{52} = \frac{3}{13}$$

(ii) Number of red face cards 2 × 3 = 6

Number of favourable outcomes of drawing red face card = 6

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P\text{ (drawing of red face card)} = \frac{6}{52} = \frac{3}{26}$$

Ex.22 Two dice are thrown simultaneously. Fill up the table for number of events of sum on two dice.

Events: 'sum on 2 dice'	Probability
2	$\frac{1}{36}$
3	
4	
5	
6	
7	
8	$\frac{5}{36}$
9	
10	
11	
12	$\frac{1}{36}$

Sol. Total number of possible outcomes = 6 × 6 = 36

(i) Number of favourable outcomes of sum

(2) = (1,1) = 1

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$\Rightarrow P\text{ (sum, 2)} = \frac{1}{36}$$

(ii) Number of favourable outcomes of sum (3) is (1, 2), (2, 1) = 2

$$P\text{ (sum, 3)} = \frac{2}{36}$$

(iii) Favourable outcomes of sum (4) are

{2, 2), (1, 3), (3, 1)}

Number of favourable outcomes of sum

(4) = 3

$$P\text{ (sum, 4)} = \frac{3}{36}$$

(iv) Favourable outcomes of sum (5) are

{(1, 4), (4, 1), (2, 3), (3, 2)}

Number of favourable outcomes of sum

(5) = 4

$$P\text{ (sum, 5)} = \frac{4}{36}$$

(v) Favourable outcomes of sum (6) are

{(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)}

Number of favourable outcomes of sum (6) = 5

P (sum, 6) = $\frac{5}{36}$

(vi) Favourable outcomes of sum (7) are

{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}

Number of favourable outcomes of sum (7) = 6

P (sum, 7) = $\frac{6}{36}$

(vii) Favourable outcomes of sum (8) are

{(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)}

Number of favourable outcomes of sum (8) = 5

P (sum, 8) = $\frac{5}{36}$

(viii) Favourable outcomes of sum (9) are

{(3, 6), (6, 3), (4, 5), (5, 4)}

Number of favourable outcomes of sum (9) = 4 ; P (sum, 9) = $\frac{4}{36}$

(ix) Favourable outcomes of sum (10) are

{(4, 6), (6, 4), (5, 5)}

Number of favourable outcomes of sum (10) = 3 ; P (sum, 10) = $\frac{3}{36}$

(x) Favourable outcomes of sum (11) are

{(6, 5), (5, 6)}

Number of favourable outcomes of sum (11) = 2 ; P (sum, 11) = $\frac{2}{36}$

(xi) Favourable outcomes of sum (12) are (6, 6)

Number of favourable outcomes of sum (12) = 1 ; P (sum, 12) = $\frac{1}{36}$

Events: 'sum on 2 dice'	Probability
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

Ex.23 Two customers Abbas and Shehla are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any one day as on another. What is the probability that both will visit the shop on

(i) the same day

(ii) different days

(iii) consecutive days ?

Sol. Two customers Abbas and Shehla visiting a shop Tuesday to Saturday.

Total possible ways of visiting shop by them

= 5 × 5

= 25

(i) They can visit the shop on all week days Tuesday to Saturday.

Favourable outcomes of visiting shop by them on the same day = 5

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

P (visiting shop same day)

$$= \frac{5}{25} = \frac{1}{5}$$

(ii) Favourable outcomes of visiting shop on the different days by them

= 25 – 5

= 20 days

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

P (visiting shop different days)

$$= \frac{20}{25} = \frac{4}{5}$$

(iii) Favourable outcomes of visiting shop by them on consecutive days are

Abbas	T	W	Th	F
Shehla	W	Th	F	S

Shehla	T	W	Th	F
Abbas	W	Th	F	S

Total favourable outcomes = 4 + 4 = 8 days

Number of favourable outcomes = 8

P (visiting shop on consecutive days)

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{8}{25}$$

Ex.24 A box contains 12 balls out of which x are black.

(i) If one ball is drawn at random from the box, what is the probability that it will be a black ball ?

(ii) If 6 more white balls are put in the bag, the probability of drawing a black ball will double than that in (i). Find x.

Sol. Random drawing of balls ensures equally likely outcomes

Total number of balls = 12

$\therefore$ Total number of possible outcomes = 12

Number of black balls = x

(i) Out of total 12 outcomes, favourable outcomes = x

P (black ball)

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{x}{12}$$

(ii) If 6 more black balls are put in the bag, then

Total number of black balls = x + 6

Total number of balls in the bag = 12 + 6 = 18

$$\text{P (black ball)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{x+6}{12+6}$$

According to the question,

Probability of drawing black ball in second case

= 2 × Probability drawing of black ball in first case

$$\Rightarrow \frac{x+6}{18} = 2\left(\frac{x}{12}\right)$$

$$\Rightarrow \frac{x+6}{18} = \frac{x}{6}$$

$\Rightarrow$ 6x + 36 = 18 x

$\Rightarrow$ 12x = 36

$\Rightarrow$ x = 3

Hence, number of black balls = 3

Ex.25 A box contains 20 balls bearing numbers, 1, 2, 3, 4, ... 20. A ball is drawn at random from the box. What is the probability that the number on the balls is

(i) An odd number

(ii) Divisible by 2 or 3

(iii) Prime number

(iv) Not divisible by 10

Sol. Total number of possible outcomes = 20

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

(i) Number of odds out of first 20 numbers = 10

Favourable outcomes by odd = 10

$$\text{P(odds)} = \frac{\text{Favourable outcomes of odd}}{\text{Total number of possible outcomes}}$$

$$= \frac{10}{20}$$

$$= \frac{1}{2}$$

(ii) The numbers divisible by 2 or 3 are 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

Favourable outcomes of numbers divisible by 2 or 3 = 13

P (numbers divisible by 2 or 3)

$$= \frac{\text{Favourable outcomes of divisible by 2 or 3}}{\text{Total number of possible outcomes}}$$

$$= \frac{13}{20}$$

(iii) Prime numbers out of first 20 numbers are 2, 3, 5, 7, 11, 13, 17, 19

Favourable outcomes of primes = 8

P(primes)

$$= \frac{\text{Favourable outcomes of primes}}{\text{Total number of possible outcomes}}$$

$$= \frac{8}{20} = \frac{2}{5}$$

(iv) Numbers not divisible by 10 are 1, 2, ... 9, 11, ...19

Favourable outcomes of not divisible by 10

= 18

P(not divisible by 10)

$$= \frac{\text{Favourable outcomes of not divisible by 10}}{\text{Total number of possible outcomes}}$$

$$= \frac{18}{20} = \frac{9}{10}$$