LINEAR PROGRAMMING

3.1 BASIC CONCEPTS

1. Linear inequations the inequalities of the form $ax + by \leq c$, $ax + by < c$, $ax + by \geq c$ and $ax + by > c$ are called linear inequations.

2. Linear programming It is the method used in decision making in business for obtaining the least cost and maximum profits through linear expressions subject to certain given conditions called linear constraints.

3.2 OBJECTIVE FUNCTION : A linear function of the involved variables which we want to minimize or maximize subject to the given linear constraints is known as objective function.

1. Optimal value The maximum (minimum) value of an objective function is known as its optimal value.

2. Feasible solution A set of values of the variables satisfying all constraints is known as a feasible solution.

3. Optimal solution A feasible solution which leads to the optimal value (maximum or minimum) of an objective function is called its optimal solution.

4. Optimization techniques The process of obtaining the optimal values are called optimization techniques.

5. Feasible region The common region determined by all the constraints and non-negativity restriction of the problem is called feasible region.

6. Linear Programming.Problem(LPP) A L.P.P. consists of maximizing or minimizing an objective function subject to certain constraints.

7. If the feasible region is unbounded then a maximum or minimum may not exist. However if it exists, it must occur at corner point (vertex) of R.

3.3 FUNDAMENTAL THEOREMS : Theorem 1. Let R be the feasible region (convex polygon) for a linear programming problem and let $z = ax + by$ be the objective function. When z has an optimal value (maximum or minimum) where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a vertex (corner point) of the feasible region.

Theorem 2. Let R be the feasible region for a linear programming problem and let $z = ax + by$ be the objective function. If R is bounded, then the objective function z has both maximum and minium value of R and each of these occurs at a vertex (corner point) of R.

3.4 CORNER POINT METHOD : This method comprises of the following steps :

(i) Identify the unknowns (decision variables) and denote them by x and y.

(ii) Formulate the objective function in terms of x and y

(iii) Transform all constraints to linear inequations

(vi) Solve these inequations and find the feasible region.

(v) Determine the corner points of the feasible region. Let M and m be the largest and smallest values at these points

(vi) If the feasible region is bounded, M and m are respectively maximum and minimum values of the objective function.

If the feasible region is unbounded, then

(i) M is the maximum value of the objective function, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise the objective function has no maximum value.

(ii) m is the minimum value of the objective function, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise the objective function has no minium value.

SOLVED PROBLEMS

Ex.1 ***An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table :***

Distance (in km)

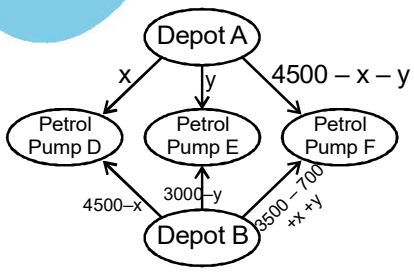
From/To	*A*	*B*
D	*7*	*3*
E	*6*	*4*
F	*3*	*2*

Assuming that the transportation cost of 10 litres of oil is Rs 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum ? What is the minimum cost ?

Sol. Let x litres and y litres of oil be supplied from depot A to petrol pumps D and E respectively. Then (7,000 – x – y) litres of oil is supplied to petrol pump F from depot A. Hence,

$$x \geq 0,\ y \geq 0 \text{ and } x + y \leq 7000$$

Since the requirement of petrol pumps D, E and F are 4500 litres, 3000 litres and 3500 litres respectively, the depot B will supply (4500 – x) litres, (3000 – y) and (3500 – 7000 + x + y) litres to the petrol pumps D, E and F respectively.



Also, (4500 – x) + (3000 – y) + (3500 – 7000 + x + y) = 4000, which is equal to the capacity of the depot B.

Hence, $4500 - x \geq 0,\ 3000 - y \geq 0$ and $x + y - 3500 \geq 0$

i.e., $x \leq 4500,\ y \leq 3000$ and $x + y \geq 3500$.

Thus, the LPP is

$$\text{Minimise : } C = \frac{7x}{10} + \frac{6y}{10} + \frac{3(700 - x - y)}{10}$$

$$+ \frac{3(4500 - x)}{10} + \frac{4(3000 - y)}{10} + \frac{2(x + y - 3500)}{10}$$

$$= 3x + y + 39{,}500$$

subject to the constrainst :

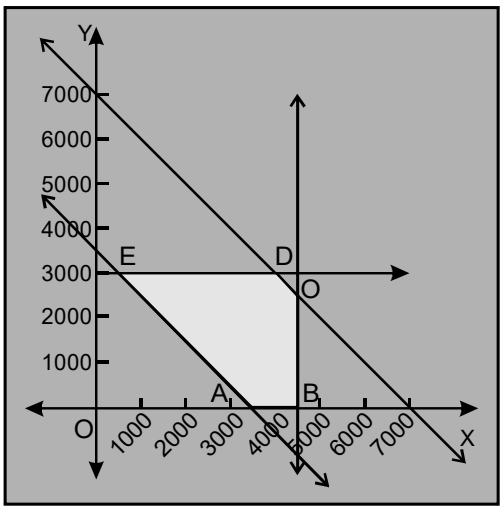
$$x + y \leq 7{,}000$$

$$x \leq 4{,}500$$

$$y \leq 3{,}000$$

$$x + y \geq 3{,}500$$

and $$x \geq 0,\ y \geq 0$$



The shaded region in Graph 12.28 is the feasible region with corner points A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000) and E (500, 3000)

The value of C at these five points are :

C at A (3500, 0) = 50,000

C at B (4500, 0) = 53,000

C at C (4500, 2500) = 55,500

C at D (4000, 35000) = 54,000

C at E (500, 3000) = 44,000

Hence, the minimum of transportation is Rs 4400 at (500, 3000), since the transportation cost of 10 litres is Re 1 per km.

Thus, 500 litres, 3000 litres and 3500 litres to D, E and F respectively were supplied from depot A; and 4000 litres were supplied to D from depot B.

Ex.2 ***A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of each brand are given in the table below. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.***

If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used ? What is the minimum amount of nitrogen added in the garden ?

(kg per bag)

	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric Acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Sol. Let the fruit grower use x bags of brand P and y bags of brand Q. Then the Linear Programming formulated from the given problem is :

Minimise : $N = 3x + 3.5y$

subject to the contraints :

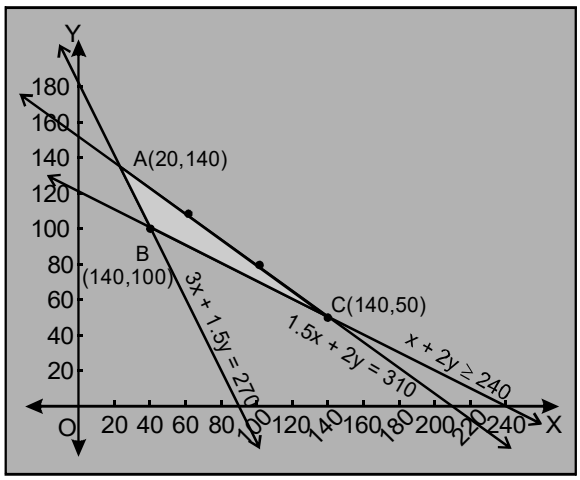
$x + 2y \geq 240$

$3x + 1.5y \geq 270$

$1.5x + 2y \leq 310$

and $x \geq 0, y \geq 0$

The shaded region ABC is the feasible region which is bounded. The coordinates of the corner points are A (20, 140), B(40, 100) and C(140, 50). The values of N at these points are 550, 470 and 475.



Hence, N is minimum at B (40, 100). The minimum amount of nitrogen is 470 kg.

Hence, 40 bags of brand P and 100 bags of brand Q can be used by the fruit grower.

Ex.3 ***A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat take 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 32 hours of machine time and 24 hours of craftman's time.***

(i) What number of rackets and bats must be made if the factory is the work at full capacity ?

(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

Sol. Let the factory make x tennis rackets and y cricket bats.

Then, the LPP corresponding to this problem is

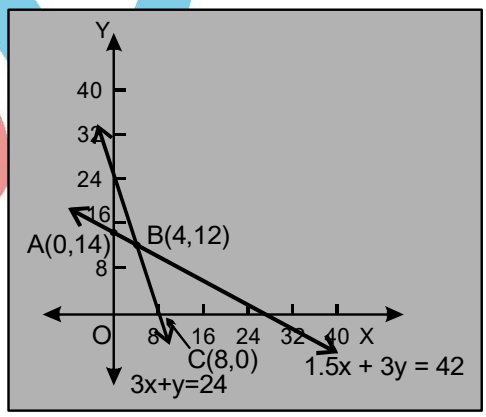
Maximise P = 20x + 10y

Subject to the constraints :

$1.5x + 3y \le 42$

$3x + y \le 24y$

and $x \ge 0, y \ge 0$



Drawing the graph of the system of inequations, we get ABC as the feasible region (shaded region). The coordinates of the corner points A, B and C are (0, 14), B (4, 12) and C (8, 0) respectively. The values of P at A, B, C are 140, 200 and 160 respectively.

Hence, P is maximum when x = 4 and y = 12.

Thus, (i) 4 tennis rackets and 12 cricket bats should be made.

(ii) Maximum Profit = Rs 200

Ex.4 ***A factory manufactures tow types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can shell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit ? Determine the maximum profit.***

Sol. Suppose the factory manufactures x packages of screw A and y packages of screw B. Then, the LPP is

Maximise : P = 7x + 10y

subject to the constraints :

$4x + 6y \le 240$ i.e., $2x + 3y \le 120$

$6x + 3y \le 240$ i.e., $2x + y \le 80$

and $x \ge 0, y \ge 0$

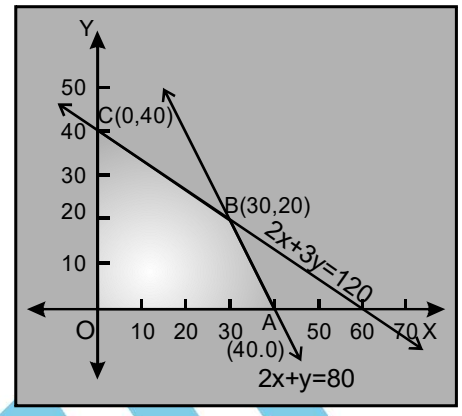
The graph of this system of inequations gives the shaded region OABC as the feasible region.

The coordinates of the corner points O, A, B and C (0, 0), (40, 0), (30, 20) and (0, 40) respectively.

The values of P at these points are 0, 280, 410 and 400.

Hence, P is maximum at B (30, 20).

Thus, to earn maximum profit of Rs 410, the factory should manufacture 30 packages of screw A and 20 packages of screw B.



Ex.5 ***A manufacturer has three machines I,II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine II must be operated for at least 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table :***

Items	*Numbers of hours required on machines*		
	I	*II*	*III*
M	*1*	*2*	*1*
N	*2*	*1*	*1.25*

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each them should she produce so as the maximise her profit assuming that she can shell all the items that she produced ? What will be the maximum profit ?

Sol. Let x and y be the number of items M and N respectively.

Total profit on the production = Rs (600x + 400y)

Mathematical formulation of the given problem is as follows :

Maximise Z = 600x + 400y

subject to the constraints :

$x + 2y \le 12$ (constraint on Machine I) .....(1)

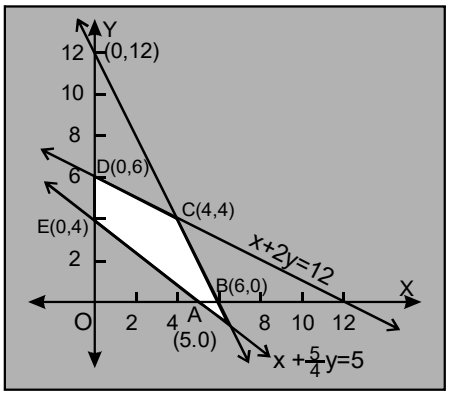
$2x + y \le 12$ (constraint on Machine II) .....(2)

$x + \frac{5}{4}y \ge 5$ (constraint on Machine III) .....(3)

$x \ge 0, y \ge 0$ .....(4)

Let us draw the graph of constraints (1) to (4). ABCDE is the feasible region (shaded) as shown in Graph 12.32 determined by the constraints (1) to (4). Observe that the feasible region is bounded, coordinates of the corner points A, B, C, D and E are (5, 0), (6, 0), (4, 4), (0, 6) and (0, 4) respectively.

Let us evaluate Z = 600x + 400y at these corner points. We. see that the point (4, 4) is giving the maximum value of Z.



Corner Point	Z = 600x + 400y
(5, 0)	3,000
(6, 0)	3,600
(4, 4)	4000 Maximum
(0, 6)	2,400
(0, 4)	1,600

Hence, the manufacturer has to produce 4 units of each item to get the maximum profit of Rs 4,000.

EXERCISE – I UNSOLVED PROBLEMS

Q.1 Maximize $z = 10x + 15y$ under the conditions

$$5x + 2y \geq 10,$$
$$3x + 7y \geq 21,$$
$$7x + 5y \leq 35,$$
$$y \leq 4,$$
$$x, y \geq 0.$$

Q.2 A firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 30. It takes 2 hours to make a ring and one hour for a chain. The maximum number of hours available is 40 per day. If the profit on a ring is Rs 250 and on a chain is Rs 150, how many of each should be manufactured daily so as to maximize the profit ?

Q.3 A producer has 30 and 17 units of labour and capital respectively, which he can use to produce two types of goods, X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and one unit of capital is required to produce one unit of Y. If X and Y are priced at Rs 100 and Rs 120 per unit respectively, how should the producer use his resources to maximize the total revenue. Solve the problem graphically ?

Q.4 A firm manufactures two types of products, A and B, and sells them at a profit of Rs 5 per unit of type A and Rs 3 per unit of type B. Each product is produced on two machines, M_1 and M_2. One unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2, whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2. Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

Q.5 A dietician wishes her clients to have a minimum of 30 units of proteins and a maximum of 18 and 14 units of carbohydrates and fats respectively. Food X costs Rs 15 per kg and provides 3 units of proteins, 2 units of carbohydrates and 1 unit of fat. Food Y costs Rs 40 per kg and provides 2 units of proteins, 1 unit of carbohydrates and 1 unit of fat. Find out the least cost combinations of the foods meeting the requirements.

Q.6 Suppose one gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates and the corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs 5 and rice costs Rs 20 per kg. The minimum daily requirements of proteins and carbohydrates for an average man are 50 g and 200 g respectively. In what quantities should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of proteins and carbohydrates at minimum cost, assuming that both wheat and rice are to be taken in the diet ?

Q.7 A gardener has a supply of fertilizer of type I which consists of 10% nitrogen and 6% phosphoric acid, and type II which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, be finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type-I fertilizer costs 60 paise per kg and the type-II fertilizer costs 40 paise per kg, determine how many kilograms of each fertilizer should be used so that the nutrient requirements are met at a minimum cost. What is the minimum cost ?

Q.8 A man owns a field of area 1000 sq m. He wants to plant fruit trees in it. He has a sum of Rs 1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 sq m of ground per tree and costs Rs 20 per tree and type B requires 20 sq. m of ground per tree and costs Rs 25 per tree. When fully grown, a tree of type A produces an average of 20 kg of fruit which can be sold at a profit of Rs 2 per kg, and type B produces an average of 40 kg of fruit which can be sold at a profit of Rs 1.50 per kg. How many of each type should be planted to achieve maximum profit when the trees are fully grown? What is the maximum profit ?

Q.9 An aeroplane can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first class ticket and a profit of Rs 300 is made on each economy class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passengers prefer to travel by economy class to the first class. Determine how many of each type of tickets must be sold in order to maximize the profit for the airline. What is the maximum profit ?

Q.10 A factory owner has to purchase two types of machines, A and B, for his factory. The requirements and limitations for the machines are as follows :

	Area occupied by the machine	Labour required for each machine	Daily output in units
Machine A	1000 sq m	12 men	60
Machine B	1200 sq m	8 men	40

He has an area of 9000 sq m available and 72 skilled men to operate the machines. How many machines of each type should be buy to maximize the daily output ?

Q.11 A manufacture produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work for at most 18 hours and 15 hours per day respectively. He earns a profit of Rs 15 and Rs 25 per trunk on the first type and second type respectively. How many trunks of each type must he make each day to make maximum profit ?

Q.12 A toy company manufactures two types of dolls A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed 3 times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per. doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit ?

Q.13 A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used ? What is the minimum amount of nitrogen added to the garden ?

Brand	Kg per bag			
	Nitrogen	Phosphoric acid	Potash	Chlorine
P	3	1	3	1.5
Q	3.5	2	1.5	2

Q.14 A cooperative society of farmers has 50 hectares of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10500 and Rs 9000 respectively. To control weeds, a liquid herbicide has to be used fro crops X and Y at the rate of 20 litres and 10 litres per hectare. Further no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society ?

Q.15 A medical company has factories at two places, A and B. From these places supplies are made to each of its three agencies situated at P, Q and R. The monthly requirements of the agencies are respectively 40, 40 and 50 packets of the medicines, while the production capacity of the factories, A and B, are 60 and 70 packets respectively. The transportation cost per packet from the factories to the agencies are given below:

Transportation cost per packet (in Rs)		
From / To	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each factory should be transported to each agency so that the cost of transportation is minimum ? Also, find the minimum cost.

Q.16 An oil company has two depots, A and B, with capacities of 7000 litres and 4000 litres respectively. The company has to supply oil to three petrol pumps, D, E and F, whose requirements are 4500 litres, 3000 litres and 3500 litres respectively. The distance between the depots and the petrol pumps is given in the following table :

Distance (in km)		
From / To	A	B
D	7	3
E	6	4
F	5	2

Assuming that the transportation cost per km is Rs 2 per litre, how should the delivery be scheduled in order that the transportation cost is minimum ?

Q.17 A company makes two kinds of leather belts, A and B. The respectively profits are Rs 4 and Rs 3 per belt. Each belt of type A requires twice as much time as a belt of type B. If all the belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both types of belts A and B require the same amount of leather). 400 buckles are available for belts of type A, and 700 buckles are available for type B per day. What should be the daily production of each type of belt so as to maximize the profit ?

Q.18 A toy manufacture produces two types of dolls, A and B. Each doll of type B takes twice as long to produce as one doll of type A. The company has time to make a maximum of 2000 dolls of type A per day. The supply of plastic is sufficient to produce 1500 dolls per day and each type requires an equal amount of it. For type B, only 600 dresses are available per day. If the company makes a profit of Rs 3 and Rs 5 per doll respectively on dolls of type A and type B, how many of each should be produced per day in order to maximize the profit ?

Q.19 If a young man rides his motorcycle at 25 km/h, he has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 40 km/h, the petrol cost increases to Rs 5 per km. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express it as a linear programming problem and solve it graphically.

EXERCISE – II BOARD PROBLEMS

Q.1 A producer has 30 and 17 units of labour and capital respectively which he can use to produce two type of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of capital is required to produce one unit of Y. If X and Y are priced at Rs 100 and 120 per unit respectively, how should the producer use his resources to maximise the total revenue ? Solve the problem graphically.

Q.2 A company manufactures two types of toys A and B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There as 3 hours available for cutting and 4 hours available for assembling in a day. The profit is Rs 50 each on type A and Rs 60 each on type B. How many toys of each type should the company manufacture in a day to maximise the profit ?

Q.3 A company sells two different products A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows, that the maximum number of units of A that can be sold is 70 and that for B is 125. Profit on each unit of A is Rs 20 and that on B is Rs 15. How many units of A and B should be produced to aximise the profit ? Solve it graphically.

Q.4 A manufacturer makes two types of cups, A and B. Three machines are required to manufacture the cups and the time in minutes required by each is as given below :

Type of Cup	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise, and on B it is 50 paise, show that 15 cups of type A and 30 cups of type B should be manufactured per day to get the maximum profit.

Q.5 Kellogg is a new cereal formed of a mixture of bran and rice that contains atleast 88 grams of protein and atleast 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing the new cereal if bran costs Rs 4 per kilogram and rice costs Rs 4 per kilogram.

Q.6 A man has Rs 1500 for purchase of rice and wheat. A bag of rice and a bag of wheat cost Rs 180 and Rs 120 respectively. He has a storage capacity of 10 bags only. He earns a profit of Rs 11 and Rs 9 respectively per bag of rice and wheat. Formulate it as a linear programming problem and solve it graphically for maximum profit.

Q.7 Anil wants to invest atmost Rs 12000 in Bonds A and B. According to rules, he has to invest atleast Rs 2000 in Bond A and atleast Rs 4000 in Bond B. If the rate of interest on Bond A is 8% p.a. and on Bond B is 10% p.a., how should he invest his money for maximum interest ? Solve the problem graphically.

Q.8 A firm deals with the two kinds of fruit juices–pine apple and orange juice. These are mixed and two mixtures are sold are soft drinks A and B. One tin of A requires 4 litres of pine apple and 1 litre of orange juice. One tin of B requires 2 litres of pine apple and 3 litres of orange juice. The firm has only 46 litres of pine apple juice and 24 litres of orange juice. Each tin of A and B are sold at a profit of Rs 4 and Rs 3 respectively. How many tins of each type should the firm produce to maximise the profit ? Solve the problem graphically.

Q.9 A manufacturer makes two products, A and B. Product A sells at Rs 200 per unit and takes 30 minutes to make. Product B sells at Rs 300 per unit and takes 1 hour to make. There is a permanent order of 14 units of product A and 16 units of product B. A working week consists of 40 hours of production and the weekly turn over must not be less than Rs 10000. If the profit on each of product A is Rs 20 and on product B is Rs 30, then how many of each should be produced so that the profit is maximum ? Also, find the maximum profit. Solve the problem graphically.

Q.10 Two tailors A and B earn Rs 15 and Rs 20 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce atleast 60 shirts and 32 pants at a minimum labour cost ? Solve the problem graphically.

Q.11 A farmer has a supply of chemical fertilizers of type A which contains 10% nitrogen and 6% phosphoric acid and of type B which contains 5% nitrogen and 10% of phosphoric acid. After soil testing it is found that atleast 7 kg of nitrogen and the same quantity of phosphoric acid is required for a good crop. The fertilizer of type A costs Rs 5 per kg and the type B costs Rs 8 per kg. Using linear programming find how many kilograms of each type of the fertilizer should be bought to meet the requirement and the cost be minimum. Solve the problem graphically.

Q.12 A furniture dealer deals only in two items – tables and chairs. He has Rs 10000 to invest and a space to store atmost 60 pieces. A table costs him Rs 500 and a chair Rs 200. He can sell a table at a profit of Rs 50 and a chair at a profit of Rs 15. Assume that he can sell all items that he buys. Using linear programming formulate the problem for maximum profit and solve it graphically.

Q.13 A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5760 to invest and has space for atmost 20 items. A fan and sewing machine cost Rs 360 and Rs 240 respectively. He can sell a fan at a profit of Rs 22 and sewing machine at a profit of Rs 18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximise his profit ? Translate the problem into L.P.P and solve it graphically.

Q.14 If a young man rides his motorcycle at 25 km/h, he had to spend Rs 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increase at Rs 5 per km. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as on L.P.P and solve it graphically.

Q.15 An aeroplane can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first class ticket and a profit of Rs 300 is made on each second class ticket. The airline reserves atleast 20 seats for first-class. However, tickets of each type must be sold to maximise profit for the airline. Form an LPP and solve it graphically.

Q.16 A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 5% phosphoric acid, and type B which contains 6% nitrogen and 10% phosphoric acid. After testing the soil conditions of the field, it was found that at least 14 kg of nitrogen and 14 kg of phosphoric acid is required for producing a good crop. The fertilizer of type of A costs Rs 5 per kg and the type B costs Rs 3 per kg. How many kg of each type of the fertilizer should be used to meet the requirement at the minimum possible cost ? Using LPP, solve the above problem graphically.

Q.17 A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs 5 and Rs 4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of the food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost but it must satisfy the requirements of the sick person. Form the question as LPP and solve it graphically.

Q.18 One kind of cake requires 200 g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as a linear programming problem and solve graphically.

Q.19 A diet is to contain 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs 4 per unit and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meet the minimal nutritional requirements.

Q.20 A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day 7 is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number hours available per day is 16. If the profit on a ring is Rs 300 and that on a chain is Rs 190, find the number of rings and chains that should be manufactured per day, so as the earn the maximum profit. Make it as an L.P.P. and solve it graphically.

Q.21 A merchant plans to sell two types of personal computers – a desktop model and a protable model that will cost Rs. 25,000 and Rs. 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if the does not want to invest more than Rs. 70 lakhs and his profit on the desktop model is Rs. 4,500 and on the portable model is Rs. 5,000. Make an L.P.P. and solve it graphically.

Q.22 A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food II contains 1 units/kg of vitamin A and 2 units /kg of vitamin C. It costs Rs. 5 per kg to purchase Food I and Rs. 7 per kg to purchase Food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically.

Q.23 A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at Rs. 100 and Rs. 120 per unit respectively, how should he use his resources to maximise the total revenue ? Form the above as an LPP and solve graphically.
Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate ?

Answers

EXERCISE – 1 (UNSOLVED PROBLEMS)

1. The maximum value of $z = \frac{570}{7}$, when $x = \frac{15}{7}$ and $y = 4$

2. 10 rings and 20 chains ; maximum profit = Rs 5500

3. Good X = 3 units, good Y = 8 units ; maximum revenue = Rs 1260

4. Product A = 60 units, product B = 240 units ; maximum profit = Rs 1020

5. Food X = 6 units, Food Y = 6 units ; minimum cost = Rs 300

6. No minimum value

7. Fertilizer type 1 = 100 kg, fertilizer type II = 80 kg ; minimum cost = Rs 92

8. Type A = 20 trees, type B = 40 trees ; maximum profit = Rs 3200

9. First class = 40 tickets, economy class = 160 tickets ; maximum profit = Rs 64,000

10. Type A = 6 machines, type B = 0 machines ; maximum output = 360 units

11. Type I = 0 trunks, type II = 6 trunks; maximum profit = Rs 150.

12. Type A =800 dolls, type B = 400 dolls; maximum profit, Rs 16000

13. Brand P = 40 bags, brand Q = 100 bags ; minimum amount of nitrogen = 470 kg

14. Crop X = 30 hectares, Crop Y = 20 hectare ; maximum profit = Rs. 4,95,000

15. Minimum cost = Rs 400

	No. of backets transported		
	Agency P	Agency Q	Agency R
Factory A	10	0	50
Factory B	30	40	0

EXERCISE – 2 (BOARD PROBLEMS)

1. 1260, (3, 8) **2.** 1500, (12, 15) **3.** A : 25 units, B : 125 units **5.** Rs 4.6, (.6,.4)
6. 100, (5, 5) **7.** (2000, 10000) **8.** A : 9 tin, B : 5 tin **9.** 1440, (48, 16) **10.** A : 5 days, B : 3 days

11. 570, (50, 40) **12.** 1000, (20, 0) **13.** 392, (8, 12) **14.** 30, $\left(\frac{50}{3}, \frac{40}{3}\right)$ **16.** A : 80 kg, B : 100 Kg

17. A : 5 unit, B : 30 unit **18.** 30 **19.** 104 **20.** 5440, (8, 16) **21.** 1150000, (200, 50) **22.** 38, (2, 4)