WAVE MOTION

INTRODUCTION OF WAVES

Wave motion is the phenomenon that can be observed almost everywhere around us, as well it appears in almost every branch of physics. Surface waves on bodies of matter are commonly observed. Sound waves and light waves are essential to our perception of the environment. All waves have a similar mathematical description, which makes the study of one kind of wave useful for the study of other kinds of waves. In this chapter, we will concentrate on string waves, which are type of a mechanical waves. Mechanical waves require a medium to travel through. Sound waves, water waves are other examples of mechanical waves. Light waves are not mechanical waves, these are electromagnetic waves which do not require medium to propagate.

Mechanical waves originate from a disturbance in the medium (such as a stone dropping in a pond) and the disturbance propagates through the medium. The forces between the atoms in the medium are responsible for the propagation of mechanical waves. Each atom exerts a force on the atoms near it, and through this force the motion of the atom is transmitted to the others. The atoms in the medium do not, however, experience any net displacement. As the wave passes, the atoms simply move back and forth. Again for simplicity, we concentrate on the study of harmonic waves (that is those that can be represented by sine and cosine functions).

Note : A wave is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter. e.g. The ripples on a pond (water waves), the sound we hear, visible light, radio and TV signals etc.

CLASSIFICATION OF WAVES

Wave classification According to

- Necessity of medium
  - (i) Elastic or mechanical wave
  - (ii) Electro magnetic wave (E.M. wave) or non-mech.
- Propagation of energy
  - (i) Progressive
  - (ii) Sationary
- Dimension
  - (i) One dimensional
  - (ii) Two dimensional
  - (iii) Three dimentional
- Vibration of particle
  - (i) Transverse
  - (ii) Longitudinal

1. **Based on medium necessity** :- A wave may **or** may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as **elastic waves**. e.g. Sound waves in water, seismic waves in earth's crust.

2. **Based on energy propagation :-** Waves can be divided into two parts on the basis of energy propagation **(i)** Progressive wave **(ii)** Stationary waves. The progressive wave propagates with constant velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.

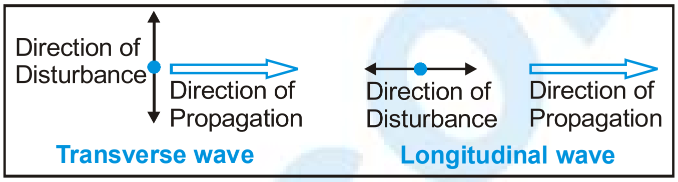
3. **Based on direction of propagation** :- Waves can be one, two **or** three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves **or** ripples on water are two dimensional , while sound **or** light waves from a point source are three dimensional.

4. **Based on the motion of particles of medium** :

Waves are of two types on the basis of motion of particles of the medium.

(i) Longitudinal waves

(ii) Transverse waves



In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

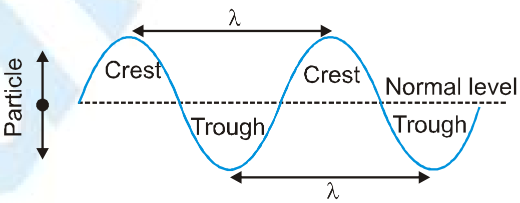
TRANSVERSE WAVE MOTION

Mechanical transverse waves produce in such type of medium which have shearing property, so they are known as shear wave or S-wave

Note :- Shearing is the property of a body by which it changes its shape on application of force.

⇒ Mechanical transverse waves are generated only in solids & surface of liquid.

In this individual particles of the medium execute SHM about their mean position in direction $\perp^r$ to the direction of propagation of wave motion.

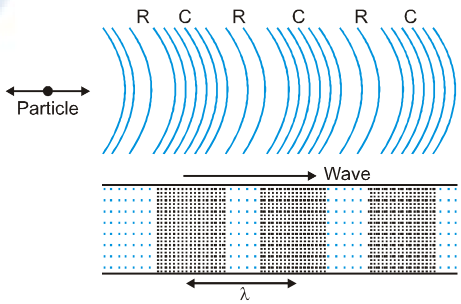


A crest is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.

A trough is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

LONGITUDINAL WAVE MOTION

In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).



Note : The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

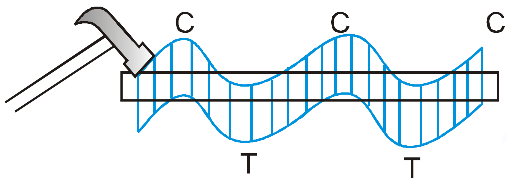
The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

Mechanical Waves in Different Media

(i) A mechanical wave will be transverse **or** longitudinal depends on the nature of medium and mode of excitation.

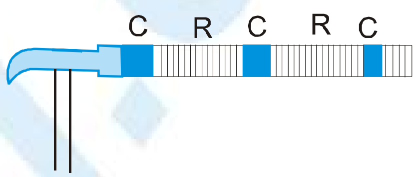
(ii) In strings mechanical waves are always transverse when string is under a tension. In gases and liquids mechanical waves are always longitudinal e.g. sound waves in air **or** water. This is because fluids cannot sustain shear.

(iii) In solids, mechanical waves (may be sound) can be either transverse **or** longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle as shown in fig. **(A)** the waves in the rod will be transverse while if the rod is struck at the side as shown in fig. **(B)** or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal.



Transverse-wave

(A)



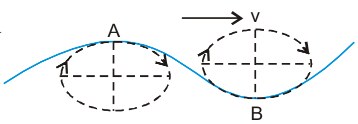
Longitudinal-wave

(B)

Further more in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds

[$v_S \cong 5$ km/s while $v_P \cong 9$ km/s] S–waves are transverse while P–waves longitudinal.

Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane.



Ripple

CHARACTERISTICS OF WAVE MOTION

Some of the important characteristics of wave motion are as follows :

(i) In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.

(ii) The energy is transferred from place to another without any actual transfer of the particles of the medium.

(iii) Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.

(iv) The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.

(v) The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.

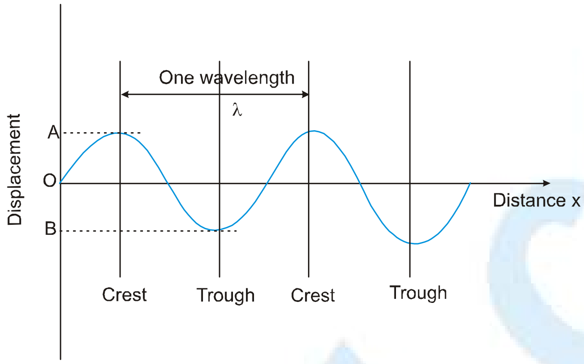
(vi) For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

DESCRIBING WAVES :

Two kinds of graph may be drawn - displacement-distance and displacement-time.

A displacement - distance graph for a transverse mechanical wave shows the displacement y of the vibrating particles of the transmitting medium at different distance x from the source at a certain instant i.e. it is like a photograph showing shape of the wave at that particular instant.

The maximum displacement of each particle from its undisturbed position is the amplitude of the wave. In the figure, it is OA or OB.



The wavelength λ of a wave is generally taken as the distance between two successive crests or two successive trough. To be more specific, it is the distance between two consecutive points on the wave which have same phase.

A displacement-time graph may also be drawn for a wave motion, showing how the displacement of one particle at a particular distance from the source varies with time. If this is simple harmonic variation then the graph is a sine curve.

Some important terms connected with wave motion

(i) Wavelength (λ) **[length of one wave]**

Distance travelled by the wave during the time, any one particle of the medium completes one vibration about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.

(ii) **Frequency (n)** :Number of vibrations (Number of complete wavelengths) complete by a particle in one second.

(iii) **Time period (T)** : Time taken by wave to travel a distance equal to one wavelength.

(iv) **Amplitude (A)** : Maximum displacement of vibrating particle from its equilibrium position.

(v) **Angular frequency (ω) :** It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$

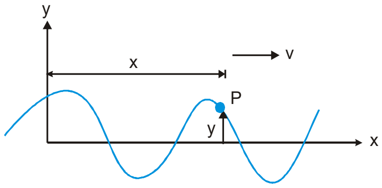
(vi) **Phase :** Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation $y = A\sin(\omega t - kx)$; $(\omega t - kx)$ = phase.

(vii) **Angular wave number (k) :** It is defined as $k = \frac{2\pi}{\lambda}$

(viii) **Wave number ($\vec{\nu}$) :** It is defined as $\vec{\nu} = \frac{1}{\lambda} = \frac{k}{2\pi}$ = number of waves in a unit length of the wave pattern.

Wave Function

The disturbance created by a wave, given as a function of time and distance is called wave function. It gives idea about the characteristics of a wave. It may be particle displacement in case of wave in string and sound wave or it may be variation in pressure and density for sound waves.



Imagine a horizontal string stretched in the x direction. Let y measure the displacement of any particle of the string from its equilibrium position, perpendicular to the string. Let the string is plucked on the left end such that a wave travels to the positive x - direction. The vertical displacement y of the particle P is given by   y = f (x, t)

The disturbance which is at P now (i.e. at time t) was at x = 0, time $\frac{x}{v}$ earlier i.e. at time $\left(t-\frac{x}{v}\right)$

i.e. $y = f(x,t) = f\left(x=0, t-\frac{x}{v}\right) \Rightarrow y = f\left(t-\frac{x}{v}\right)$

Also of the wave moves towards nagative x - direction

then $y = f(x_0 t) = f\left(x=0, t+\frac{x}{v}\right)$

i.e. $y = f\left(t+\frac{x}{v}\right) \quad \therefore \; y = f\left(t \pm \frac{x}{v}\right)$

Travelling sine wave in one dimension

The wave equation $y = f\left(t-\frac{x}{v}\right)$ is quite general. If holds for ability wave shapes, and for transverse as well as for longitudinal waves.

When, under the effect of a wave, the motion of the particle about its mean position is SHM the wave is called sine wave or harmonic wave.

The displacement of the particle at x at time t will be for a sine wave (like wave in a string) will be

$$y = A\sin\omega\left(t-\frac{x}{v}\right) \qquad y = A\sin(\omega t - kx)$$

where term ω is called angular frequence, $\omega = \frac{2\pi}{T} = 2\pi f$

and $k = \frac{\omega}{v} = \frac{2\pi}{\lambda}$ is called angular wave number.

The wave equation y = A sin(ωt – kx)says that at x = 0 and t = 0, y = 0. This is not necessarily the case. For the some conditions, y may not equal to zero. Therefore, the most general expression would involve a phase constant ϕ, which allows for other possibilities,

$$Y = A\sin(\omega t - kx + \phi)$$

The term (ωt – kx + ϕ) is called the phase angle or simply phase of the wave. Two waves with the same phase (or phase differing by a multiple of 2π) are said to be "in phase". They execute the same motion at the same time.

Phase Angle

(ωt – kx + ϕ) which is called the phase of wave is actually the phase angle at time t of a particle mean position of which is at a distance x from reference point.

For the particle, mean position of which is at x = 0, phase angle = ωt + ϕ . For this particle only, the phase angle at t = 0 i.e. angle at t = 0 i.e. initial phase angle = ϕ.

Thus ϕ is the initial (at t = 0) phase angle of the particle which is at x = 0.

For example if the particle at x = 0 was at

(i) its mean position and was moving in positive direction then ϕ = 0

(ii) positive extreme position, $\phi = \frac{\pi}{2}$

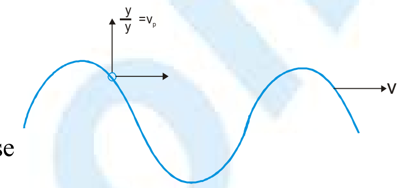
Wave Velocity and Particle Velocity

Wave velocity is the velocity of the disturbance which propagates through a medium. It only depends on the properties of the medium and is independent of time and position.

Particle velocity (v_p) is the rate at which particle's displacement vary as function of time.

If Y = A sin(ωt ± kx + ϕ)

$$v_p = \frac{\delta y}{\delta t} = A\ \omega \cos(\omega t \pm kx + \phi)$$



The difference between the wave velocity v and the velocity of a particle of the medium $\left(\frac{\delta y}{\delta t}\right)$ can be understood with the help of a transverse progressive in shown in the figure.

The velocity of particle on a string given by $\frac{\delta y}{\delta t}$ is perpendicular to wave velocity v for the transverse wave.

The acceleration of the particle is obtained by differentiating particle velocity w.r.t. time

$$\frac{\delta^2 y}{\delta t^2} = \omega^2 y$$

The above equation shows that acceleration of particle is directly proportional to its displacement from equilibrium position, i.e. the particles of the medium execute simple harmonic motion.

Ex. The equation of a transverse wave in a stretched wire is given by :

$$y = 2\sin\left[2\pi\left(\frac{t}{0.01} - \frac{x}{30}\right)\right]$$

where t is in sec. and x & y are in cm.

Find :

(a) amplitude **(b)** frequency **(c)** wavelength and **(d)** speed of the wave

Sol. Comparing the given equation with the standard equation

$$y = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

(a) Amplitude, A = 2 cm **(b)** The period, T = 0.01 s ∴ Frequency, $f = \frac{1}{T} = 100 Hz$

(c) Wavelength λ = 30 cm **(d)** Velocity of the wave v = λf = 30 × 100 = 3000 cm/s = 3 m/s

The Wave Equation

By using wave function $y = \sin(\omega t - kx + \phi)$, we can describe the motion of any point on the string.

$$v = \left[\frac{dy}{dt}\right]_{x-constant} \Rightarrow \frac{\partial y}{\partial t} = \omega A\cos(\omega t - kx + \phi) \quad ......(1)$$

$$a = \left[\frac{dv}{dt}\right]_{x-constant} \Rightarrow \frac{\partial v}{\partial t} = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(\omega t - kx + \phi) \quad ......(2)$$

Further, slope of y vs x graph is

$$\left[\frac{dy}{dx}\right]_{t-constant} \Rightarrow \frac{\partial y}{\partial x} = -kA\cos(\omega t - kx + \phi) \quad ......(3)$$

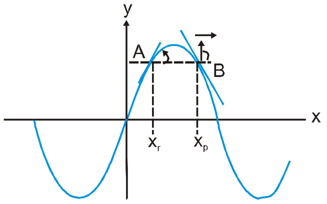
$$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(\omega t - kx + \phi) \quad ......(4)$$

From **(1)** and **(3)** $\quad \frac{\partial y}{\partial t} = -\left(\frac{\omega}{k}\right)\frac{\partial y}{\partial x}$

$\Rightarrow$ **vp =–vw × slope** ......**(A)**

i.e. if the slope at any point is negative, particle velocity is positive and vice-versa, for a wave moving along positive x and i.e. v_w is slope.

For example, consider two points A and B on the y-x curve for a wave as shown. The wave is moving along positive x-axis.



Slope at A is positive therefore at the given moment, its velocity is negative. That means it is coming downward. Reverse is the situation for particle at point B.

Now using equation **(2)** and **(4)**

$$\frac{\partial^2 y}{\partial x^2} = \frac{k^2\partial^2 y}{\partial x^2} = \frac{1\partial^2 y}{v^2\partial t^2} \quad ......(B)$$

$$\frac{dy}{dt} = -vx\frac{dy}{dx} \qquad \frac{d^2y}{dt^2} = -v^2\left(\frac{d^2y}{dx^2}\right)$$

The equation (B) known as the linear wave equation of differential equation representation of the travelling wave model. We have developed the linear wave equation from a sinusoidal mechanical wave travelling through a medium, but all the travelling wave satisfy this equation. The linear wave equation successfully describes wave on string, sound wave and also electromagnetic waves.

Ex. Verify that wave equation $y = e^{2x-5t}$ is a solution to linear equation of the wave (here x an y are can & t is in second.)

Sol. $y = e^{2x-5t}$

$$\frac{\delta y}{\delta t} = -5e^{(2x-5t)} \quad \Rightarrow \quad \frac{\delta^2 y}{\delta t^2} = 25e^{2x-5t} \quad ..... (i)$$

Also $\quad \frac{\delta y}{\delta x} = 2e^{2x-5t} \quad \Rightarrow \frac{\delta^2 y}{\delta x^2} = 4e^{2x-5t} \quad ..... (ii)$

$\therefore$ from **(i)** and **(ii)**

$$\frac{\delta^2 y}{\delta x^2} = \frac{4}{25}\left(\frac{\delta^2 y}{\delta t^2}\right)$$

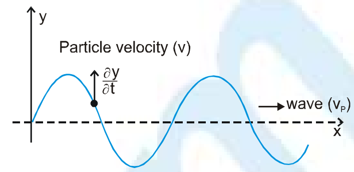
Comparing with linear wave equation, we can observe the above function represents a wave travelling with a speed of $\frac{5}{2}$ cm/s.

Particle velocity, wave velocity and particle's acceleration :

In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae what we have read in SHM apply to the particles here also. For example, maximum particle velocity is ± Aω at mean position and it is zero at extreme positions etc. Similarly maximum particle acceleration is $\pm\omega^2 A$ at extreme positions and zero at mean position. However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between + Aω and – Aω) the wave velocity is constant for given characteristics of the medium.

Particle Velocity in Wave Motion :

The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "Particle velocity".



Wave Velocity :

The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.

Relation Between Particle Velocity and Wave Velocity :

Wave equation :- $y = A\sin(\omega t - kx)$, Particle velocity $v = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$.

Wave velocity $= v_P = \frac{\lambda}{T} = \lambda\frac{\omega}{2\pi} = \frac{\omega}{k}$, $\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) = -\frac{A}{\omega}\omega k\cos(\omega t - kx) = -\frac{1}{v_p}\frac{\partial y}{\partial t} \Rightarrow \frac{\partial y}{\partial x} = -\frac{1}{v_P}\frac{\partial y}{\partial t}$

Note : $\frac{\partial y}{\partial x}$ represent the slope of the string (wave) at the point x.

Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.

Differential Equation of Harmonic Progressive Waves :

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v_P^2}\frac{\partial^2 y}{\partial t^2}$$

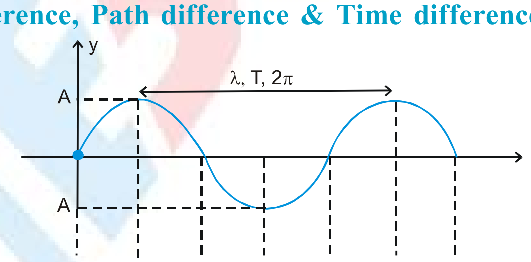
Particle Velocity (V_p) and Acceleration (A_p) in a Sinusoidal Wave :

The acceleration of the particle is the second particle is the second partial derivative of y (x, t) with respect to t,

$$\therefore\ a_P = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\sin(kx - \omega t) = -\omega^2 y(x,t)$$

i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the result we obtained for SHM. Thus, $a_p = -\omega^2$ (displacement)

Relation between Phase difference, Path difference & Time difference



Phase (ϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Wave length (λ)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3}{2}\lambda$
Time-period (T)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

$\Rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \Rightarrow$ Path difference $= \left(\frac{\lambda}{2\pi}\right)$ Phase difference

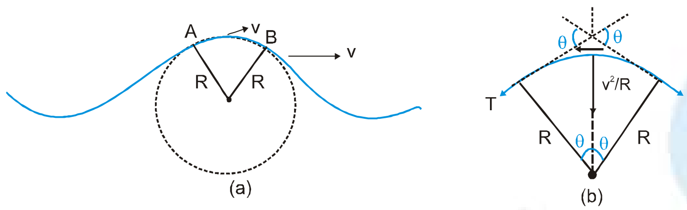
Ex. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase.

Sol. We know that for a wave v = f λ So $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

Phase difference Δϕ = 60° = (π/180) x 60 = (π/3) rad, so path difference $\Delta x = \frac{\lambda}{2\pi}(\Delta\phi) = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

Velocity of a Transverse Wave on a String

A transverse wave is produced in a taut string as shown in figure. Let us observe a small segment, such as AB, on the string from the frame that moves with the wave. In this frame, the waves is stationary while the string moves to the left at speed v.



(a) A wave moves to the right on string with respect to ground.

(b) In the frame moving with wave, the string moves toward the left.

The segment AB may be treated as a circular are some radius R, as shown in the figure (b).

Length of AB = R(2θ) 2Rθ

If μ is the linear mass density of the string, then

m = μ(2Rθ) = 2μRθ

The vertical component of tension in the string must provide the centripetal acceleration, therefore,

$$2T\sin\theta = \frac{mv^2}{R}$$

Since the angle θ is small, $\sin\theta \approx \theta$

∴ $$2T\theta = (2\mu R\theta)\frac{v^2}{R} \quad \text{or} \quad v = \sqrt{\frac{T}{\mu}}$$

Note that the velocity is measured with respect to the string.

Energy transferred by a progressive wave

As a wave propagates along string, it transfer energy consider a small element of length dx which is stretched to the length *dl* under the influence of the wave.

$$dK = \frac{1}{2}(dm)\left(\frac{\delta y}{\delta t}\right)^2 \quad \text{......... (i)}$$

If y = Asin (ωt – kx) ⇒ $\frac{\delta y}{\delta t} = A\omega\cos(\omega t - kx)$

Also dm = μdx

∴ $$dk = \frac{1}{2}\mu A^2\omega^2\cos^2(\omega t - kx).dx \quad \text{............ (ii)}$$

The potential energy of the element is equal to the work done to stretch it from dx to *dl*. Taking the tension remaining uniform

$$dU = T(dl - dx) = T\left[\frac{dl}{dx} - 1\right]dx = T[\sec\alpha - 1]dx = T\left[(1+\tan^2\alpha)^{1/2} - 1\right]dx$$

For small amplitude waves α is very small

$\therefore$ $$(1+\tan^2\alpha)^{1/2} \simeq 1 + \frac{1}{2}\tan^2\alpha = 1 + \frac{1}{2}\left(\frac{\delta y}{\delta x}\right)^2$$

$\therefore$ $$dU = \frac{1}{2}T\left(\frac{\delta y}{\delta x}\right)^2 dx \quad \text{.............(3)}$$

but $$\frac{\delta y}{\delta x} = -Ak\cos(\omega t - kx)$$

$\therefore$ $$dU = \frac{1}{2}TA^2k^2\cos^2(\omega t - kx)dx \quad \text{.............(4)}$$

Total mechanical energy of the element is

$dE = dK + dU$

$\therefore$ from equation (2) and (4)

$$dE = \mu A^2\omega^2\cos^2(\omega t - kx)\,dx \quad \text{.............(5)}$$

Now, for an point, such as at x = 0, the average value of $\cos^2\omega t$ for one time period is $\frac{1}{2}$

$\therefore$ $$dE_{av} = \frac{1}{2}(\mu A^2\omega^2)dx$$

$\therefore$ Average energy per unit length

$$\frac{dE_{av}}{dx} = \frac{1}{2}\mu A^2\omega^2$$

from equation (1) $dK \propto \left(\frac{\delta y}{\delta t}\right)^2$

$\therefore$ K.E. of element is maximum for the particles of element A, since particle velocity is maximum at A.

From equation (3) ; $dU \propto \left(\frac{\delta y}{\delta x}\right)^2$

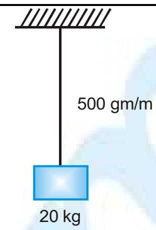
$\therefore$ P.E. is maximum where slope of y vs x graph is maximum i.e. at A only. Thus K.E. and P.E. is maximum for same point such as A, similarly minimum for same point such as B.

Power transmitted by a progressive wave

The average power transmitted along the string by the wave is

$$P_{av} = \frac{dE_{av}}{dt} = \frac{dE_{av}}{dx}\cdot\frac{dx}{dt} \quad \Rightarrow \quad P_{av} = \frac{1}{2}\mu A^2\omega^2 v$$

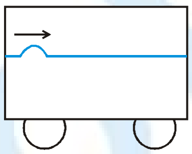
Ex. Find speed of the wave generated in the string as in the situation shown. Assume that the tension is not affected by the mass of the cord.



Sol. $T = 20 \times 10 = 200$ N

$$v = \sqrt{\frac{200}{0.5}} = 20 m/s$$

Ex. A taut string having tension 100 N and linear mass density 0.25 kg/m is used inside a cart to generate a wave pulse starting at the left end, as shown. What should be the velocity of the cart so that pulse remains stationary w.r.t. ground.



Sol. Velocity of pulse $= \sqrt{\frac{T}{\mu}} = 20 m/s$

Now $\vec{v}_{PG} = \vec{v}_{PC} + \vec{v}_{CG}$

$0 = 20\,i + \vec{v}_{CG}$

$\vec{v}_{CG} = -20i$ m/s

Intensity

Intensity is the energy transmitted per second through unit area of the medium.

$$I = \frac{P}{Area}$$

for string, $I = \frac{1}{2}\frac{\mu A^2\omega^2 v}{Area}$

Now, $dm = \int dV = \rho(Area)dx$

also $dm = \mu dx \Rightarrow \mu = \rho Area \Rightarrow \frac{\mu}{Area} = \rho$

$\therefore$ $I = \frac{1}{2}\rho A^2\omega^2 v$

Although the above formula is derived for wave on string but is applicable for other mechanical wave like sound, as well.

Ex. A string with linear mass density m = 5.00×10^{-2} kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm ?

Sol. The wave speed on the string is

$$v = \sqrt{\frac{T}{\mu}} = \left(\frac{80.0N}{5.00 \times 10^{-2} kg/m}\right)^{1/2} = 40.0 m/s$$

Because f = 60Hz, the angular frequency ω of the sinusoidal wave on the string has the value

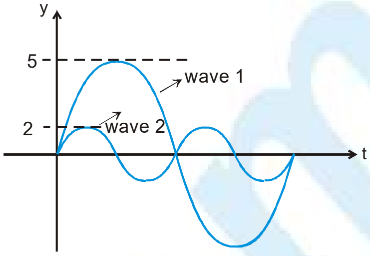
$$\omega = 2\pi f = 2\pi(60.0Hz) = 377\ s^{-1}$$

using these values in following equation for the power, with A = 6.00×10^{-2} m, gives

$$p = \frac{1}{2}\mu\omega^2 A^2 v$$

$$= \frac{1}{2}(5.00 \times 10^{-2} kg/m)\ (377\ s^{-1}) \times (6.00 \times 10^{-2}\ m)^2\ (40.0\ m/s) = 512\ W$$

Ex. Two waves in the same medium are represented by y-t curves in the figure. Find ratio of their average intensities?



Sol. : $\frac{I_1}{I_2} = \frac{\omega_1^2 A_1^2}{\omega_2^2 A_2^2} = \frac{f_1^2 \cdot A_1^2}{f_2^2 \cdot A_2^2} . = \frac{1 \times 25}{4 \times 4} = \frac{25}{16}$

Superposition Principle

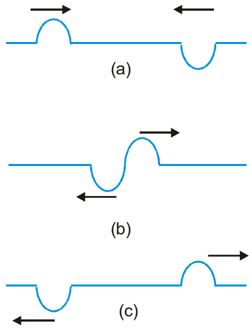
Two or more wave can propagate in the some medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacement produced by individual waves. The phenomenon of intermixing of two or mores to produce a new wave is called superposition of waves. Therefore according to superposition principle "The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacements caused to the particle by the individual waves.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3$ .... are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3$ ....

Principle of superposition holds for all types of waves, i.e. mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

If we have two or more waves moving in the medium the resultant waveform is the sum of wave functions of individual waves.

Fig: a sequence of pictures showing two pulses travelling in opposite directions along a stretched string. When the two disturbances overlap they give a complicated pattern as shown in **(b)**. In **(c)**, they have passed each other and proceed unchanged.



An Illustrative examples of this principle is phenomena of interference and reflection of waves.

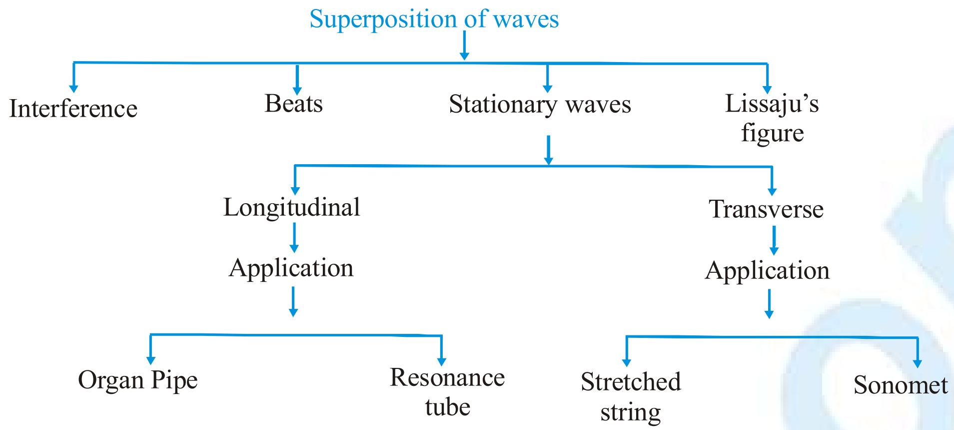
Due to superposition of waves the following phenomenon can be seen.

(i) **Interference :** Superposition of rwo waves having equal frequency and nearly equal amplitude.

(ii) **Beats :** Superposition of two waves of nearly equal frequency in same direction.

(iii) **Stationary waves :** Superposition of equal wave from opposite direction.

(iv) **Lissajous figure :** Superposition of perpendicular waves.



Interference of waves differing in phase

Suppose two identical sources send sinusoidal waves of same angular frequency ω in positive x-direction. Also, the wave velocity and hence, the wave number k is same for the two waves. One source may be situated at different points. The two waves arriving at a point then differ in phase. Let the amplitudes of the two waves be A_1 and A_2 and the two waves differ in phase by an angle ϕ. Their equations may be written as

$$y_1 = A_1 \sin(\omega t - kx)$$

and $$y_2 = A_2 \sin(\omega t - kx + \phi).$$

According to the principle of superposition, the resultant wave is represented by

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx + \phi).$$

on expanding and solving as we did in S.H.M., we get

$$y = A \sin(\omega t - kx + \alpha)$$

where amplitude of the resultant wave is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Also, $$\tan\alpha = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$ (α is phase difference of the resultant wave with the first wave)

Since intensity is proportional to the square of the amplitude, therefore

$$I = I_1 + I_2 + 2\left(\sqrt{I_1 + I_2}\right)\cos\phi$$

Constructive Interference :

when $\cos\phi = +1$

i.e. when $\phi = 2n\pi$ (n = 0,1,2, ...)

i.e. when position of crests and through of one wave coincide with the positions of crests and thorough respectively of another wave then

$A = A_1 + A_2$ i.e. maximum

Also I is maximum

and $$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

This is called constructive interference.

Destructive interference :

When $\cos\phi = -1$

i.e. when $\phi = (2n-1)\pi$ $(n = 1, 2, ...)$

i.e. when positive crests of one wave matches with troughs of another wave.

A is minimum and $A_{min} = |A_1 - A_2|$

and $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

(i) Maximum and minimum intensities in any interference wave form. $\frac{I_{Max}}{I_{Min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{A_1+A_2}{A_1-A_2}\right)^2$

(ii) Average intensity of interference wave form % & $< I >$ or $I_{av} = \frac{I_{max}+I_{min}}{2} = I_1 + I_2$

if $A = A_1 = A_2$ and $I_1 = I_2 = I$ then $I_{max} = 4I, I_{min} = 0$ and $I_{AV} = 2I$

(iii) Degree of interference Pattern (f) : Degree of hearing (Sound Wave) or

Degree of visibility (Light Wave) $f = \frac{I_{max}-I_{min}}{I_{max}+I_{min}} \times 100$

In condition of perfect interference degree of interference pattern is maximum $f_{max} = 1$ or 100%

(iv) Condition of maximum contrast in interference wave form $a_1 = a_2$ and $I_1 = I_2$ then $I_{max} = 4I$ and $I_{min} = 0$

For perfect destructive interference we have a maximum contrast in interference wave form.

Ex. If ratio of maximum to minimum resultants intensities of two intensities wave travelling in same direction is 9 : 4 then find ratio of individual intensities.

Sol. $\frac{I_{Max}}{I_{Min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \frac{9}{4}$

$$\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \frac{3}{2} \Rightarrow \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{3+2}{3-2} \Rightarrow \frac{I_1}{I_2} = 25:1$$

Ex. Two waves each of intensity I_0 are disturbing the same medium. Find resultant intensities at the points where the phase difference f is **(i)** 2π **(ii)** π **(iii)** $\frac{\pi}{2}$ **(iv)** $\frac{2\pi}{3}$

Sol. **(i)** $I = I_0 + I_0 + 2\sqrt{I_0 \times I_0} = 4\,I_0$

(ii) $I = I_0 + I_0 - 2\sqrt{I_0 \times I_0} = 0$

(iii) $I = I_0 + I_0 + 2\sqrt{I_0 I_0} \times 0 = 2\,I_0$

(iv) $I = I_0 + I_0 + 2\sqrt{I_0 \times I_0} \times \left(-\frac{1}{2}\right) = I_0$

Reflection and Transmission of Waves

A travelling wave, at a rigid or denser boundary, is reflected with a phase reversal but the reflection at an open boundary (rarer medium) takes place without any phase change. The transmitted wave is never inverted, but propagation constant k is changed.

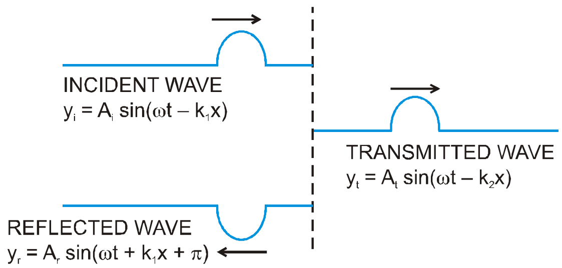


Fig. : Reflection at denser boundary

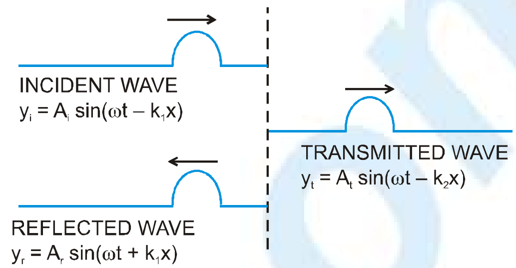
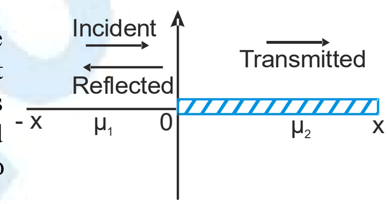


Fig. : Reflection at rarer boundary

Let two string 1 and 2 or linear density μ_1 and μ_2 respectively lie along x-axis and be joined at x = 0 in their equilibrium position. The tension in both the strings is identical. Let the wave velocity in the strings v_1 & v_2 respectively let the incident wave travelling from left to right arrive at the boundary. This wave is party reflected and partly transmitted. Both transmitted and reflected wave will have the same frequency v and also the same angular frequency ω as the incident wave.



Let the equation of wave incident from the left hand side be given by,

$$y_i = A_i \sin(\omega t - kx)$$

At O this equation becomes,

$$y_i = A_i \sin(\omega t)$$

Let the equation of the wave reflected from the boundary be given as,

$$y_r = A_r \sin(\omega t - kx)$$

At O this equation becomes

$$y_r = A_r \sin(\omega t)$$

Let the equation of the wave transmitted be given as,

$$y_t = A_t \sin(\omega t)$$

Now, since the vertical displacement of the two strings must be the same at all times at x = 0, we have,

$$y_i + y_r = y_t$$

$$A_i \sin(\omega t) + A_r \sin(\omega t) = A_t \sin(\omega t)$$

$$A_i + A_r = A_t \qquad \text{....... (i)}$$

And for the continuity of the strain at the boundary at O, we have,

$$\left(\frac{\delta y_i}{\delta x} + \frac{\delta y_r}{\delta x}\right)_{x=0} = \left(\frac{\delta y_t}{\delta x}\right)_{x=0}$$

$$-A_i k \cos(\omega t - kx) + A_r k \cos(\omega t + kx) = -k' A k \cos(\omega t - k'x)$$

So, $$-\frac{A_i}{\lambda_i} + \frac{A_r}{\lambda_r} = \frac{-A_t}{\lambda_t} \quad \Rightarrow \quad \frac{\omega A_i}{v_i} - \frac{\omega A_r}{v_r} = \frac{\omega A_t}{v_t}$$

$$\frac{A_i}{v_i} - \frac{A_r}{v_r} = \frac{A_t}{v_t} \quad \Rightarrow \quad A_i - A_r = A_t \frac{v_1}{v_2} \qquad \text{....... (ii)}$$

Eliminating A from equation **(i)** and **(ii)** we have

$$\frac{A_i - A_r}{A_i + A_r} = \frac{v_1}{v_2} \quad \Rightarrow \quad \frac{A_r}{A_i} = \frac{v_2 - v_1}{v_2 + v_1} \qquad \text{....... (iii)}$$

Now the velocities of the 2 string are,

$$v_1 = \sqrt{\frac{T_1}{\mu_1}} \quad \text{and} \quad v_2 = \sqrt{\frac{T_2}{\mu_2}}$$

Putting these values in equation **(iii)**, we get

$$\frac{A_r}{A_i} = \frac{\sqrt{\mu_1} - \sqrt{\mu_2}}{\sqrt{\mu_1} + \sqrt{\mu_2}} \qquad \text{....... (iv)}$$

similarly the ratio of amplitude of transmitted pulse to incident pulse is given by

$$\frac{A_t}{A_i} = \frac{2\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}} \qquad \text{....... (v)}$$

Note :

1. It is clear from equation **(iv)** that if $\mu_1 < \mu_2$, Ar is negative w.r.t. A_i i.e. the pulse is inverted i.e. 180° out of phase with incident pulse,

$A_r \approx A_i$ when $\mu_1 << \mu_2$.

Such is the case when a rope is tied to a wall or a sound wave is reflected from the closed end of an organ pipe. At the close end, the displacement of the air molecules in the incident and reflected wave are in the opposite direction. Consequently a compression is reflected as a compression and a rarefaction as a rarefaction.

2. If on the other hand $\mu_2 > \mu_1$, then $\frac{A_r}{A_i}$ is positive. In this case A_r the same sing A_i and there is no change of phase at the boundary. In acoustics, this corresponds to reflection at the open end of the organ pipe. These the compression is reflected as rarefaction and vice versa.

3. It is clear from equation **(v)** that the $\frac{A_t}{A_i}$ is always positive indicating that there is no phase change in the transmitted wave w.r.t. the incident wave.

Ex. A harmonic wave is travelling on string 1. At a junction with string 2 it is partly reflected and partly transmitted. The linear mass density of the second string is four times that of the first string, and that the boundary between the two strings is at x = 0. If the expression for the incident wave is, $y_i = A_i \cos(k_1 x - \omega_1 t)$

What are the expressions for the transmitted and the reflected waves in terms of A_i, k_1 and ω_1?

Sol. : Since $v = \sqrt{T/\mu}$, $T_2 = T_1$ and $\mu_2 = 4\mu_1$

we have, $$v_2 = \frac{v_1}{2} \qquad \text{....... (i)}$$

The frequency does not change, that is,

$$\omega_1 = \omega_2 \qquad \text{....... (ii)}$$

Also, because $k = \omega/v$, the wave numbers of the harmonic waves in the two strings are related by,

$$k_2 = \frac{\omega_2}{v_2} = \frac{\omega_1}{v_1/2} = 2\frac{\omega_1}{v_1} = 2k_1 \qquad \text{....... (iii)}$$

The amplitudes are,

$$A_t = \left(\frac{2v_2}{v_1+v_2}\right) A_i = \left[\frac{2(v_1/2)}{v_1+(v_1/2)}\right] A_i = \frac{2}{3} A_i \qquad \text{....... (iv)}$$

and

$$A_r = \left(\frac{v_2-v_1}{v_1+v_2}\right) A_i = \left[\frac{(v_1/2)-v_1}{v_1+(v_1/2)}\right] A_i = \frac{A_i}{3} \qquad \text{....... (v)}$$

Now with equation **(ii)**, **(iii)** and **(iv)**, the transmitted wave can be written as,

$$y_t = \frac{2}{3} A_i \cos(2k_1 x - \omega_1 t) \qquad \textbf{Ans.}$$

Similarly the reflected wave can be expressed as,

$$= \frac{A_i}{3} \cos(k_1 x + \omega_1 t + \pi) \qquad \textbf{Ans.}$$

Standing Waves :

Standing wave is obtained due to superposition of two waves travelling with same speed in opposite directions along the same line.

Suppose two sine waves of equal amplitude and frequency propagate on a long string in opposite directions. The equations of the two waves are given by

$y_1 = A\sin(\omega t - kx)$ and $y_2 = A\sin(\omega t + kx + \phi)$.

These waves interfere to produce what we call standing waves. To understand these waves, let us discuss the special case when $\phi = 0$.

The resultant displacements of the particles of the string are given by the principle of superposition as

$y = y_1 + y_2 \quad = A[\sin(\omega t - kx) + \sin(\omega t + kx)] = 2A\sin\omega t\cos kx$

or, $y = (2A\cos kx)\sin\omega t$.

we may write, $y = A_0 \sin(\omega t)$ [where $A_0 = 2A\cos kx$]

The result obtained from the above equation are :

1. As this equation satisfies the wave equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

it represents a wave. However, as it is not of the form $f(ax \pm bt)$, the wave is not travelling and so is called standing or stationary wave.

2. The amplitude of the wave

$A_c = 2A\cos kx$

is not same for all prints of the medium but varies periodically with position (and not with time as in beats).

3. The points for which amplitude is minimum are called nodes.

For nodes $\cos kx = 0$, i.e., $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

i.e., $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$ $\left[\text{as } k = \frac{2\pi}{\lambda}\right]$

In a stationary wave, consecutive nodes are equally spaced and their separation is $\frac{\lambda}{2}$.

4. The points for which amplitude is maximum are called antinodes

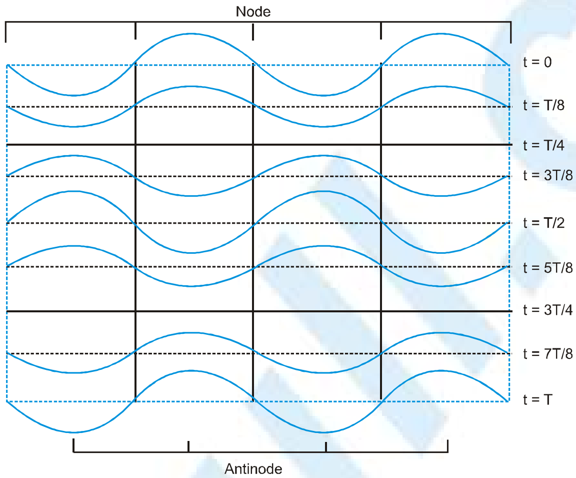
For antinodes, $\cos kx = \pm 1$, i.e., $kx = 0, \pi, 2\pi, 3\pi, ...$

i.e., $x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, ...$ $\left[\text{as } k = \frac{2\pi}{\lambda}\right]$

i.e., like nodes, antinodes are also equally spaced with spacing $(\lambda/2)$ and $A_{max} = \pm 2A$.

Furthermore, nodes and antinodes are alternate with spacing $(\lambda/4)$.

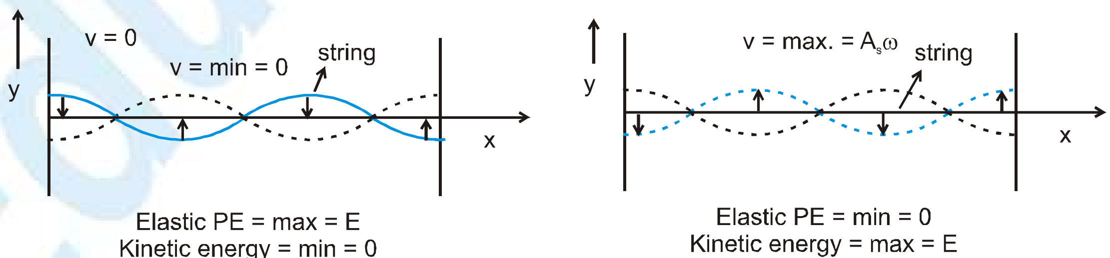
5. The nodes divide the medium into segments (or loops). All the particles in a segment vibrate in same phase, but in opposite phase with the particles in the adjacent segment. Twice in one period all the particles pass through their mean position simultaneously with maximum velocity $(A\omega)$, the direction of motion being reversed after each half cycle.



(a)

6. Standing waves can be transverse or longitudinal, e.g., in strings (under tension) if reflected wave exists, the waves are transverse-stationary, while in organ pipes waves are longitudinal-stationary.

7. As in stationary waves nodes are permanently at rest, so no energy can be transmitted across them, i.e., energy of one region (segment) is confined in that region. However, this energy oscillates between elastic potential energy and kinetic energy of the particles of the medium. When all the particles are at their extreme positions KE is minimum while elastic PE is maximum (as shown in figure A), and when all the particles (simultaneously) pass through their mean position KE will be maximum while elastic PE minimum (Figure B). The total energy confined in a segment (elastic PE + KE), always remains the same.



(A) (B)

Ex. Two waves travelling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = (4.0\text{ cm})\sin(3.0x - 2.0t)$$
$$y_2 = (4.0\text{ cm})\sin(3.0x + 2.0t)$$

where x and y are in centimeter.

(a) Find the maximum displacement of a particle of the medium at x = 2.3 cm.

(b) Find the position of the nodes and antinodes.

Sol. **(a)** When the two waves are summed, the result is a standing wave whose mathematical representation is given by Equation, with A = 4.0 cm and k = 3.0 rad/cm;

$$y = (2A\sin kx)\cos\omega t = [(8.0\text{ cm})\sin 3.0x]\cos 2.0t$$

Thus, the maximum displacement of a particle at the position x = 2.3 cm is

$$y_{max} = [(8.0\text{ cm})\sin 3.0x]_{x = 2.3\text{ cm}}$$
$$= (8.0\text{ m})\sin(6.9\text{ rad}) = 4.6\text{ cm}$$

(b) Because $k = 2\pi/\lambda = 3.0$ rad/cm, we see that $\lambda = 2\pi/3$cm. Therefore, the antinodes are located at

$$x = n\left(\frac{\pi}{6.0}\right)\text{ cm} \quad (n = 1, 3, 5, ...)$$

and the nodes are located at

$$x = n\frac{\lambda}{2}\left(\frac{\pi}{6.0}\right)\text{ cm} \qquad (n = 1, 2, 3, ...)$$

Ex. Two travelling waves of equal amplitudes and equal frequencies move in opposite direction along a string. They interfere to produce a standing wave having the equation.

$$y = A\cos kx\sin\omega t$$

in which A = 1.0 mm, k = 1.57 cm^{-1} and $\omega = 78.5\ s^{-1}$. **(a)** Find the velocity and amplitude of the component travelling waves. **(b)** Find the node closest to the origin in the region x > 0. **(c)** Find the antinode closest to the origin in the region x > 0. **(d)** Find the amplitude of the particle at x = 2.33 cm.

Sol. **(a)** The standing wave is formed by the superposition of the waves

$$y_1 = \frac{A}{2}\sin(\omega t - kx) \qquad \text{and} \qquad y_2 = \frac{A}{2}\sin(\omega t + kx).$$

The wave velocity (magnitude) of either of the waves is

$$v = \frac{\omega}{k} = \frac{78.5\,s^{-1}}{1.57\,cm^{-1}} = 50\text{ cm/s; Amplitude} = 0.5\text{ mm.}$$

(b) For a node, cos kx = 0.

The smallest positive x satisfying this relation is given by

$$kx = \pi/2 \quad \text{or,} \quad x = \frac{\pi}{2k} = \frac{3.14}{2\times 1.57\text{ cm}^{-1}} = 1\text{ cm}$$

(c) For an antinode, |cos kx| = 1.

The smallest positive x satisfying this relation is given by

$$kx = \pi \quad \text{or,} \quad x = \frac{\pi}{k} = 2\text{ cm}$$

(d) The amplitude of vibration of the particle at x is given by $|A \cos kx|$. For the given point,

$$kx = (1.57\ cm^{-1})(2.33\ cm) = \frac{7}{6}\pi = \pi + \frac{\pi}{6}.$$

Thus, the amplitude will be

$$(1.0\ mm)\,|\cos(\pi + \pi/6)| = \frac{\sqrt{3}}{3}\ mm = 0.86\ mm.$$

STANDING WAVES IN STRINGS :

(a) Fixed at both ends :

Suppose a string of length L is kept fixed at the ends x = 0 and x = L. In such a system suppose we send a continuous sinusoidal wave of a certain frequency, say, toward the right. When the wave reaches the right end. It gets reflected and begins to travel back. The left-going wave then overlaps the wave, which is still travelling to the right. When the left-going wave reaches the left end, it gets reflected again and the newly reflected wave begins to travel to the right. Overlapping the left-going wave. This process will continue and, therefore, very soon we have many overlapping waves, which interfere with one another. In such a system, at any point x and at any time t, there are always two waves, one moving to the left and another to the right. We, therefore, have

$y_1(x, t) = y_m \sin(kx - \omega t)$ (wave travelling in the positive direction of x-axis)

and $y_2(x, t) = y_m \sin(kx + \omega t)$ (wave travelling in the negative direction of x-axis).

The principle of superposition gives, for the combined wave

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$

$$= y_m \sin(kx - wt) + y_m \sin(kx + \omega t)$$

$$= (2y_m \sin kx) \cos \omega t$$

It is seen that the points of maximum or minimum amplitude stay at one position.

Nodes : The amplitude is zero for values of kx that give sin kx = 0 i.e. for,

$$kx = n\pi, \text{ for } n = 0, 1, 2, 3, ...$$

Substituting $k = 2\pi/\lambda$ in this equation, we get

$$x = n\frac{\lambda}{2}, \text{ for } n = 0, 1, 2, 3, ...$$

The positions of zero amplitude are called the nodes. Note that a distance of $\frac{\lambda}{2}$ or half a wavelength separates two consecutive nodes.

Antinodes :

The amplitude has a maximum value of $2y_m$, which occurs for the values of kx that give $|\sin kx| = 1$. Those values are

$$kx = (n + 1/2)\pi \text{ for } n = 0, 1, 2, 3, ...$$

Substituting $k = 2\pi/\lambda$ in this equation, we get.

$$x = (n + 1/2)\frac{\lambda}{2} \text{ for } n = 0, 1, 2, 3, ...$$

as the positions of maximum amplitude. These are called the **antinodes**. The antinodes are separated by $\lambda/2$ and are located half way between pairs of nodes.

For a stretched string of length L, fixed at both ends, the two ends of the ends is chosen as position x = 0, then the other end is x = L. In order that this end is a node; the length L must satisfy the condition

$$L = n\frac{\lambda}{2}, \text{ for } n = 1, 2, 3, ...$$

This condition shows that standing waves on a string of length L have restricted wavelength given by

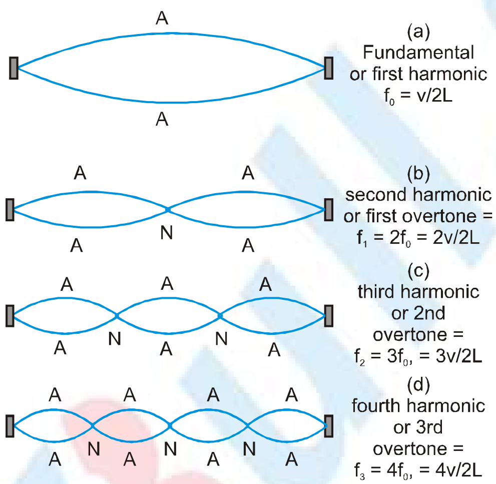
$$\lambda = \frac{2L}{n}, \text{ for } n = 1, 2, 3,.....$$

The frequencies corresponding to these wavelengths follow from Eq. as

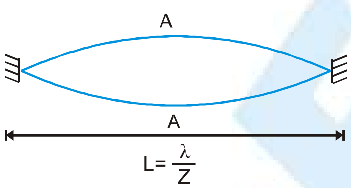
$$f = n\frac{v}{2L}, \text{ for } n = 1, 2, 3,.....$$

where v is the speed of travelling waves on the string. The set of frequencies given by equation are called the natural frequencies or modes of oscillation of the system. This equation tells us that the natural frequencies of a string are integral multiples of the lowest frequency $f = \frac{v}{2L}$, which corresponds to n = 1. The oscillation mode with that lowest frequency is called the fundamental mode or the first harmonic. The second harmonic or first overtone is the oscillation mode with n = 2. The third harmonic and second overtone corresponds to n = 3 and so on. The frequencies associated with these modes are often labeled as ν_1, ν_2, ν_3 and so on. The collection of all possible modes is called the harmonic series and n is called the harmonic number.

Some of the harmonic of a stretched string fixed at both the ends are shown in figure.



(a)

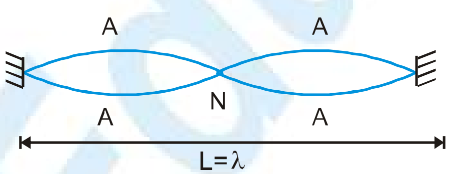


When n = 1 i.e. when only one loop is formed λ = 2L and $f = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

This is called fundamental frequency or (f_0) first harmonic.

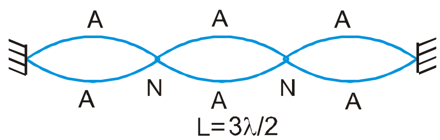
When n = 2

(b)



i.e. formed, λ = L

& $f = \frac{v}{2L} = 2f_0$ This is called second harmonic or first overtone.

(c) 

When n = 3, $\lambda = \frac{2L}{3}$ and $f = 3\left(\frac{v}{2L}\right) = 3f_0$

This is called third harmonic or second overtone.

Thus n^{th} harmonic or $(n-1)^{th}$ overtone is given by

$$f = nf_0 = n\left(\frac{v}{2L}\right) \quad \Rightarrow \quad \lambda = \left(\frac{2L}{n}\right)$$

Fundamental frequency of a string fixed at both ends is given

$$f_0 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Where T is tension and μ is mass per unit length

Also μ = density x Area of cross-section = δA

$$\Rightarrow \quad f_0 = \frac{1}{2L}\sqrt{\frac{T}{\delta A}}$$

Ex. Two string of same material are stretched to same tension between rigid supports. It diameter of second wire is double than that of first wire and fundamental frequency of first is equal to third harmonic of second wire, find ratio of their lengths.

Sol.
$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\rho \times \left(\pi \frac{d^2}{4}\right)}} = \frac{1}{L}\sqrt{\frac{T}{\rho\pi}}$$

$$(f_0)_{1st} = 3(f_0)_{2nd} \quad \Rightarrow \quad \frac{1}{L_1 d_1} = \frac{3}{L_3 d_2} \quad \Rightarrow \quad \frac{L_1}{L_2} = \frac{d_2}{3d_1} = \frac{2}{3}$$

Ex. A wire having a linear mass density 5.0×10^{-3} kg/m is stretched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 240 Hz. The next higher frequency at which the same wire resonates is 480 Hz. Find the length of the wire.

Sol. Suppose the wire vibrates at 420 Hz in its nth harmonic and at 490 Hz in its (n + 1)th harmonic.

$$420 = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \quad \text{....... (i)}$$

and
$$490 = \frac{n+1}{2L}\sqrt{\frac{T}{\mu}} \quad \text{....... (ii)}$$

Subtracting equation (i) from (ii)

$$\frac{1}{2L}\sqrt{\frac{T}{M}} = 60 \quad \Rightarrow \quad L = \frac{1}{2\times 60}\sqrt{\frac{T}{\mu}}$$

$$\therefore \quad L = \frac{1}{2\times 60}\sqrt{\frac{450}{5\times 10^{-3}}} = 2.5m$$

(b) Fixed at One End :

Standing waves can be produced on a string which is fixed at one end and whose other end is free to move in a transverse direction. Such a free end can be nearly achieved by connecting the string to a very light thread.

If the vibrations are produced by a source of "correct" frequency, standing waves are produced. If the end x = 0 is fixed and x = L is free, the equation is again given by

$$y = 2A \sin kx \cos \omega t$$

with the boundary condition that x = L is an antinode. The boundary condition that x = 0 is a node is automatically satisfied by the above equation. For x = L to be an antinode,

$$\sin kL = \pm 1$$

or, $kL = \left(n + \frac{1}{2}\right)\pi$ or, $\frac{2\pi L}{\lambda} = \left(n + \frac{1}{2}\right)\pi$

or, $\frac{2Lf}{v} = n + \frac{1}{2}$ or, $f = \left(n + \frac{1}{2}\right)\frac{v}{2L} = \frac{n + \frac{1}{2}}{2L}\sqrt{T/\mu}$ .....

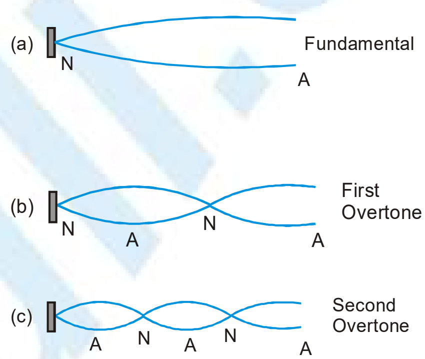
These are the normal frequencies of vibration. The fundamental frequency is obtained when n = 0, i.e.,

$$f_0 = v/4L$$

The overtone frequencies are

$$f_1 = \frac{3v}{4L} = 3f_0$$

$$f_2 = \frac{5v}{4L} = 5f_0$$



We see that all the harmonic of the fundamental are not the allowed frequencies for the standing waves. Only the odd harmonics are the overtones. Figure shows shapes of the string for some of the normal modes.

Laws of Transverse Vibration of a String

(i) Law of length : For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string, i.e, $n \propto \frac{1}{l}$ (T and m are constant)

(ii) Law of tension : The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant. $n \propto \sqrt{T}$ (l and m are constant)

(iii) Law of mass : The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant, i.e., $n \propto \frac{1}{\sqrt{m}}$ (l and T are constant)

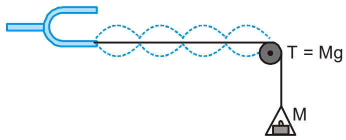
(iv) Melde's experiment : In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be loaded. There are two arrangements to vibrate the tied fork with thread.

Transverse Arrangement :

Case I

In a vibrating string of fixed length, the product of number of loops and square root of tension are constant or

$p\sqrt{T}$ = constant.



Case II

When the tuning fork is set vibrating as shown in fig. then the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread (string) is equal to the frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal modes of the string matched with the frequency of the tuning fork).

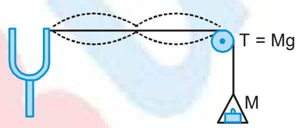
Then, if p loops are formed in the thread, then the frequency of the tuning fork is given by $n = \frac{p}{2l}\sqrt{\frac{T}{m}}$

Case III

If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread only makes node at the midpoint when the prong moves towards the pulley i.e. only once in a vibration.

Longitudinal Arrangement :

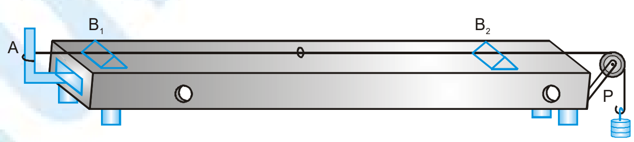
The thread performs sustained oscillations when the natural frequency of the given length of the thread under tension is half that of the fork.



Thus if p loops are formed in the thread, then the frequency of the tuning fork is $n = \frac{2p}{2l}\sqrt{\frac{T}{m}}$

SONOMETER :

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley P at the other end of the box. The wire is stretched by a tension T.



The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length the wire between the two bridges is ●, then the frequency of vibration is $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire.

Comparison of progressive and stationary waves

S.No.	Progressive waves	Stationary waves
1	These waves travels in a medium with definite velocity.	These waves do not travel and remain confined between two boundaries in the medium.
2	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.
3	The phase of vibration varies continuously from particle to particle.	The phase of all the particles in between two nodes is always same. But particles of two Adjacent nodes differ in phase by 180°
4	No particle of medium is Permanently at rest.	Particles at nodes are permanently at rest.
5	All particles of the medium vibrate and amplitude of vibration is same.	The amplitude of vibration changes from particle to particle. The amplitude is zero for all at nodes and maximum at antinodes.
6	All the particles do not attain the maximum displacement position simultaneously.	All the particles attain the maximum displacement.

SOUND WAVE

Sound waves are the most common example of longitudinal waves.

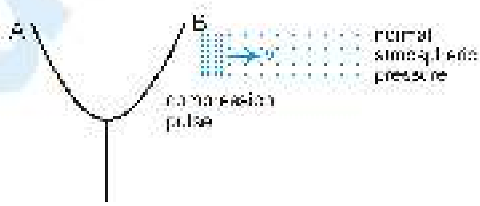
Sound waves can be created by a vibrating source such as a guitar string, the human vocal cords, the prongs of va tuning fork or the diaphragm of a loudspeaker.

They travel through any material medium with a speed that depends on the properties of the medium. As the waves travel through air, the elements of air vibrate to produce changes in density and pressure along the waves travel through air, the elements of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. The mathematical description of sinusoidally, the pressure variations are also sinusoidal sound waves is very similar to that of sinusoidal string waves.

1. Propagation of Sound Waves :

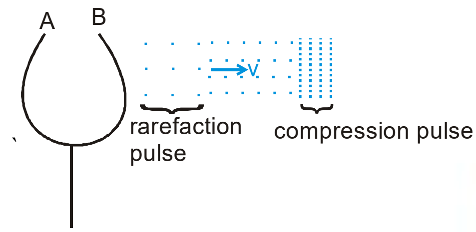
Sound is a mechanical three dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker. Being a mechanical wave, sound needs a medium having properties of inertia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

Consider a tuning fork producing sound waves.



When Prong B moves outward towards right it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a compression pulse and it travels away from the prong with the speed of sound.

After producing the compression pulse, the prong B reverses its motion and moves inward. This drags away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called a rarefaction pulse. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.



If the prongs vibrate in SHM, the pressure variations in the layer close to the prong also varies simple harmonically and hence increase in pressure above normal value can be written as

$$\delta P = \delta P_0 \sin \omega t$$

where δP_0 is the maximum increase in pressure above normal value.

As this disturbance travel towards right with wave velocity v, the excess pressure at any position x at time t will be given by

$$\delta P = \delta P_0 \sin \omega(t - x/v) \qquad \text{............(1)}$$

Using $p = \delta P$, $p_0 = \delta P_0$, the above equation of sound wave can b written as :

$$p = p_0 \sin \omega(t - x/v) \qquad \text{............(2)}$$

Equation of Sound Waves

As the piston oscillates sinusoidally, regions of compression and rarefaction are continuously set up. The distance between the centres of two successive compressions (or two successive rare factions) equals the wavelength λ. As these regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave. If y(x,t) is the position of a small element relative to its equilibrium position, we can express this harmonic position function as

$$y(x, t) = A \cos(\omega t - kx)$$

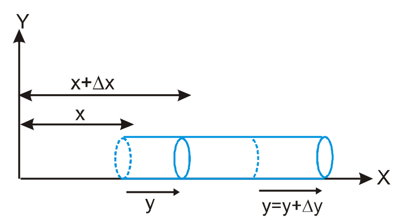
Where A is the maximum position of the element relative to equilibrium called the displacement amplitude of the wave. The parameter k is the wave number and ω is the angular frequency of the piston.

Note that the displacement of the elements is along y, in the direction of propagation of the sound wave, which means we are describing a longitudinal wave.

Consider a thin disk-shaped element of gas whose circular cross section is parallel to the piston in figure. This element will undergo changes in position, pressure, and density as a sound wave propagates through the gas. From the definition of bulk modulus, the pressure variation in the gas is

$$B = \frac{-\Delta P V}{\Delta V}$$

or $$\Delta P = -B\frac{\Delta V}{V_i}$$



The element has a thickness Δx in the horizontal direction and a cross-sectional area A, so its volume is $V_i = A\Delta x$. The change in volume ΔV accompanying the pressure change is equal to AΔy, where Δy is the difference between the value of y at x + Δx and the value of y at x respectively. Hence, we can express ΔP as

or $$\Delta P = -B\frac{\Delta V}{V_1} = -\frac{B[y'-y]}{A(\Delta x)}\frac{A\Delta y}{A\Delta x} = -B\frac{\Delta y}{\Delta x}$$

As Δx approaches zero, the ratio $\Delta y/\Delta x$ becomes $\partial y / \partial x$ (The partial derivative indicates that we are interested in the variation of x with position at a fixed time.) Therefore,

or $\Delta P = -B \dfrac{\partial y}{\partial x}$

If $y = A \sin(\omega t - kx)$ .... **(i)**

$= BAK \cos(\omega t - kx)$ .... **(ii)**

equation **(i)** and **(ii)** represents that same sound wave where, p is excess pressure (at position x), i.e. pressure over and above the average atmospheric pressure and the pressure amplitude p_0 is given by

$$p_0 = \frac{BA\omega}{v} = BAK \quad \text{......... (iii)}$$

(B = Bulk modulus of the medium, K = angular wave number)

Note from equation **(i)** and **(ii)** we can above that the displacement of a particle and excess pressure at any position are out of phase by $\dfrac{\pi}{2}$. Hence a displacement maxima corresponds to a pressure minima and vice-versa.

Ex. The equation of a sound wave in air is given by

$p = 0.2 \sin[3000\, t - 9x]$, where all variables are in S.I. units.

(a) Find the frequency, wavelength and the speed of sound wave in air.

(b) If the equilibrium pressure of air is 1.0×10^5 N/m^2, what are the maximum and minimum pressures at a point as the wave passes through that point ?

Sol. **(a)** Comparing with the standard form of a travelling wave

$p = p_0 \sin[\omega(t - x/v)]$

we see that ω 3000 s^{-1}. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{3000}{2\pi} Hz$$

Also from the same comparison, $\omega/v = 9.0$ m^{-1}

or, $$v = \frac{\omega}{9.0m^{-1}} = \frac{3000}{9.0m^{-1}} \quad \frac{1000}{3} m/s^{-1}$$

The wavelength is $$\lambda = \frac{v}{f} = \frac{(1000/3)}{(3000/2\pi\pi)} = \frac{2\pi}{9} \text{ m}$$

(b) The pressure amplitude is $p_0 = 0.02$ N/m^2. Hence, the maxima and minima pressures at a point in the wave motion will be $(1.01 \times 10^5 \pm 0.02)$ N/m^2.

Ex. A sound wave of wavelength 40cm travels in air. If the difference between the maximum and minimum $P_{max} - P_{min} = (P + p_0) - (P - p_0) = 2p_0$. The bulk modulus of air is 1.4×10^5 N/m^2.

Sol. The pressure amplitude is

$$p_0 = \frac{4.0 \times 10^{-3} N/m^2}{2} = 2 \times 10^{-3} N/m^2$$

The displacement amplitude A is given by

$p_0 = BAK$

or $$A = \frac{p_0}{BK} = \frac{p_0 \lambda}{2\pi B} \Rightarrow = \frac{2 \times 10^{-3} \times (40 \times 10^{-2})}{2 \times \pi \times 14 \times 10^4} = \frac{200}{7\pi} \mathring{A} \Rightarrow A = 13.2 \mathring{A}$$

2. Frequency and Pitch of Sound Waves

Frequency :

Each cycle of a sound wave includes one compression and one rarefaction, and frequency is the number of cycles per second that passes by a given location. This is normally equal to the frequency of vibration of the (tuning fork) source producing sound. If the source, vibrates in SHM of a single frequency, sound produced has a single frequency and it is called a pure tone..

However a sound source may not always vibrate in SHM (this is the case with most of the common sound sources e.g. guitar string, human vocal card, surface of drum etc.) and hence the pulse generated by it may not have the shape of a sine wave. But even such a pulse may be considered to be obtained by superposition of a large number of sine waves of different frequency and amplitudes. We say that the pulse contain all these frequencies.

Audible Frequency Range for Human :

A normal person hears all frequencies between 20 & 20 KHz. This is a subjective range (obtained experimentally) which may vary slightly from person to person. The ability to hear the high frequencies decreases with age and a middle-age person can hear only upto 12 to 14 KHz.

Infrasonic Sound :

Sound can be generated with frequency below 20 Hz called infrasonic sound.

Ultrasonic Sound :

Sound can be generated with frequency above 20 kHz called ultrasonic sound.

Even through humans cannot hear these frequencies, other animals may. For instance Rhinos communicate through infrasonic frequencies as low as 5Hz, and bats use ultrasonic frequencies as high as 100 KHz for navigating.

Pitch :

Frequency as we have discussed till now is an objective property measured its units is Hz and which can be assigned a unique value. However a person's perception of frequency is subjective. The brain interprets frequency primarily in terms of a subjective quality called Pitch. A pure note of high frequency is interpreted as high-pitched sound and a pure note of low frequency as low-pitched sound

Ex. A wave of wavelength 4 mm is produced in air and it travels at a speed of 300 m/s. Will it be audible ?

Sol. From the relation $v = \nu\lambda$, the frequency of the wave is

$$\nu = \frac{v}{\lambda} = \frac{300\ \text{m/s}}{4 \times 10^{-3}\ \text{m}} = 75000\ \text{Hz}.$$

This is much above the audible range. It is an ultrasonic wave and will not be audible to humans, but it will be audible to bats.

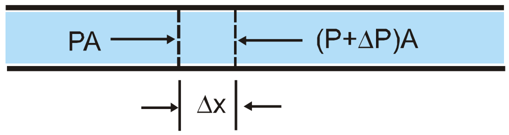
Speed of Sound Waves

We now derive equation by direct application of Newton's laws. Let a single pulse in which air is compressed travel (from right to left) with speed v through the air in a long tube, like that in figure. Let us run along with the pulse at that speed, so that the pulse appears to stand still in our reference frame. Figure shows the situation as it is viewed from that frame. The pulse is standing still, and air is moving at speed v through it from left to right.

Let the pressure of the undisturbed air be P and the pressure inside the pulse be $P + \Delta P$, where ΔP is positive due to the compression. Consider an element of air thickness Δx and face area A, moving toward the pulse at speed v. The time interval that is taken by the pulse to cross the element.

$$\Delta t = \frac{\Delta x}{v}$$

The force acting on the leading and trailing faces (due to air pressure) as shown.



$F = PA - (P + \Delta P) A$
$= - \Delta P A$ (net force)

The minus sign indicates that the net force on the air element is directed to the left in figure.
The volume of the element is A(Δx) so with the aid of equation, we can write its mass as

$$\Delta m = \rho \Delta v = \rho A \Delta x = \rho A v (\Delta t)$$

The average acceleration of the element during Δt is

$$a = \frac{\Delta v}{\Delta t}$$

Thus, from Newton's second law (F = ma), we have,

$$-\Delta P A = (\rho A v \Delta t)\frac{\Delta v}{\Delta t}$$

which we can write as

$$\rho v^2 = \frac{\Delta P}{\Delta v / v}$$

The air that occupies a volume V (=AvΔt) outside the pulse is compressed by an amount ΔV(=AvΔt) as it enters the pulse. Thus,

$$\frac{\Delta V}{V} = \frac{A \Delta v \Delta t}{A v \Delta t} = \frac{\Delta v}{v}$$

Substituting equation and then equation into equation leads to

$$\rho v^2 = \frac{\Delta P}{\Delta v / v} = \frac{\Delta P}{\Delta V / V} = B$$

$$\therefore \quad v = \sqrt{\frac{B}{\rho}}$$

Speed of longitudinal (Sound) waves

Newton Formula $v_{medium} = \sqrt{\frac{E}{\rho}}$ (Use for every medium)

Where E = Elasticity coefficient of medium & ρ = Density of medium

(i) For solid medium $v_{solid} = \sqrt{\frac{Y}{\rho}}$ Where E = Y = Young's modulas

(ii) For liquid Medium $v_{liquid} = \sqrt{\frac{B}{\rho}}$ Where E = B, where B = volume elasticity coefficient of liquid

(iii) For gas medium

The formula for velocity of sound in air was first obtained by Newton. He assumed that sound propagates through air and temperature remains constant. (i.e. the process is isothermal) so Isothermal Elasticity = P $\therefore\ v_{air} = \sqrt{(P/\rho)}$

At NTP for air $P = 1.01 \times 10^5\ N/m^2$ and $\rho = 1.3\ kg/m^3$ so $v_{air} = \sqrt{\dfrac{1.01\times10^5}{1.3}} = 279$ m/s

However, the experimental value of sound in air is 332 m/s which is much higher than given by Newton's formula.

(iv) Laplace Correction

In order to remove the discrepancy between theoretical and experimental values of velocity of sound, Laplace modified Newton's formula assuming that propagation of sound in air is adiabatic process, i.e.

Adiabatic Elasticity = γp so that $v = \sqrt{\dfrac{\gamma P}{\rho}}$ i.e. $v = \sqrt{1.41} \times 279 = 331.3$ m/s [as $\gamma_{air} = 1.41$]

Which is in good agreement with the experimental value (332 m/s). This in turn establishes that sound propagates adiabatically through gases.

The velocity of sound in air at NTP is 332 m/s which is much lesser than that of light and radio–waves ($= 3 \times 10^8$ m/s). This implies that –

(a) If we set our watch by the sound of a distant siren it will be slow.

(b) If we record the time in a race by hearing sound from starting point it will be lesser than actual.

(c) In a cloud–lightening, though light and sound are produced simultaneously but as c > v, light proceeds thunder. An in case of gases –

$$v_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{\text{mass}}}\left[\text{as } \rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{V}\right] \text{ or } v_s = \sqrt{\frac{\gamma \mu RT}{M}} \text{ [as } PV = \mu RT] \text{ or } v_s = \sqrt{\frac{\gamma RT}{M_w}}$$

$$\left[\text{as } \mu = \frac{\text{mass}}{M_w} = \frac{M}{M_w} \text{ where } M_w = \text{Molecular weight}\right]$$

And from kinetic-theory of gases $v_{rms} = \sqrt{(3RT/M_w)}$ So $\dfrac{v_s}{v_{rms}} = \sqrt{\dfrac{\gamma}{3}}$

Effect of Various Quantities

(1) Effect of temperature

For a gas γ & M_w is constant $v \propto \sqrt{T} \Rightarrow \dfrac{v_2}{v_1} = \sqrt{\dfrac{T_2}{T_1}} = \sqrt{\dfrac{t+273}{273}} \Rightarrow v_t = v_0\left[1 + \dfrac{t}{273}\right]^{\frac{1}{2}}$

By applying Binomial theorem.

(i) For any gas medium $v_t = v_0\left[1 + \dfrac{t}{546}\right]$ (ii) For air : $v_t = v_0 + 0.61\,t$ m/sec. ($v_0 = 332$ m/sec.)

(2) Effect of Relative Humidity

With increase in humidity, density decreases so in the light of $v = \sqrt{\gamma P/\rho)}$ We conclude that with rise in humidity velocity of sound increases. This is why sound travels faster in humid air (rainy season) than in dry air (summer) at same temperature. Due to this in rainy season the sound of factories siren and whistle of train can be heard more than summer.

(3) Effect of Pressure

As velocity of sound $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

So pressure has no effect on velocity of sound in a gas as long as temperature remain constant. This is why in going up in the atmosphere, though both pressure and density decreases, velocity of sound remains constant as long as temperature remains constant. Further more it has also been established that all other factors such as amplitude, frequency, phase, loudness pitch, quality etc. has partially no effect on velocity of sound.

Velocity of sound in air is measured by resonance tube or Hebb's method while in gases by Quinke's tube. Kundt's tube is used to determine velocity of sound in any medium solid, liquid or gas.

(4) Effect of Motion of Air

If air is blowing then the speed of sound changes. If the actual speed of sound is v and the speed of air is w, then the speed of sound in the direction in which air is blowing will be (v+ w), and in the opposite direction it will be (v – w).

(5) Effect of Frequency

There is no effect of frequency on the speed of sound. Sound waves of different frequencies travel with the same speed in air although their wavelength in air are different. If the speed of sound were dependent on the frequency, then we could not have enjoyed orchestra.

Ex. A piezo electric quartz plate of thickness 0.005 m is vibrating in resonant conditions. Calculate its fundamental frequency if for quartz $Y = 8 \times 10^{10}$ N/m^2 and $\rho = 2.65 \times 10^3$ kg/m^3

Sol. We known that for longitudinal waves in solids $v = \sqrt{\frac{Y}{\rho}}$, So $v = \sqrt{\frac{8 \times 10^{10}}{2.65 \times 10^3}} = 5.5 \times 10^3$ m/s

Further more for fundamental mode of plate – $(\lambda/2) = L$ So $\lambda = 2 \times 5 \times 10^{-3} = 10^{-2}$ m

But as $v = f\lambda$, i.e., $f = (v/\lambda)$ so $f = [5.5 \times 10^3/10^{-2}] = 5.5 \times 10^5$ Hz = 550 kHz

Ex. Determine the change in volume of 6 liters of alcohol if the pressure is decreased from 200 cm of Hg to 75 cm. [velocity of sound in alcohol is 1280 m/s, density of alcohol = 0.81 gm/cc, density of Hg = 13.6 gm/cc and g = 9.81 m/s^2]

Sol. For propagation of sound in liquid $v = \sqrt{(B/\rho)}$ i.e., $B = v^2\rho$

But by definition $B = -V\frac{\Delta P}{\Delta V}$ So $-V\frac{\Delta P}{\Delta V} = v^2\rho$, i.e. $\Delta V = \frac{V(-\Delta P)}{\rho v^2}$

Here $\Delta P = H_2\rho g - H_1\rho g = (75 - 200) \times 13.6 \times 981 = -1.667 \times 10^6$ dynes/cm^2

So $\Delta V = \frac{(6 \times 10^3)(1.667 \times 10^6)}{0.81 \times (1.280 \times 10^5)^2} = 0.75$ cc

Ex. **(a)** Speed of sound in air is 332 m/s at NTP. What will the speed of sound in hydrogen at NTP if the density of hydrogen at NTP is (1/16) that of air.

(b) Calculate the ratio of the speed of sound in neon to that in water vapour at any temperature. [Molecular weight of neon = 2.02×10^{-2} kg/mol and for water vapours = 1.8×10^{-2} kg/mol]

Sol. The velocity of sound in air is given by $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

(a) In terms of density and pressure $\frac{v_H}{v_{air}} = \sqrt{\frac{P_H}{\rho_H} \times \frac{\rho_{air}}{P_{air}}} = \sqrt{\frac{\rho_{air}}{\rho_H}}$ [as $P_{air} = P_H$]

$$\Rightarrow \quad v_H = v_{air} \times \sqrt{\frac{\rho_{air}}{\rho_H}} = 332 \times \sqrt{\frac{16}{1}} = 1328 \text{ m/s}$$

(b) In terms of temperature and molecular weight $\frac{v_{Ne}}{v_W} = \sqrt{\frac{\gamma_{Ne}}{M_{Ne}} \times \frac{M_W}{\gamma_W}}$ [as $T_N = T_W$]

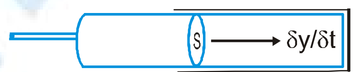
Now as neon is mono atomic ($\gamma = 5/3$) while water vapours poly atomic ($\gamma = 4/3$) so

$$\frac{v_{Ne}}{v_W} = \sqrt{\frac{(5/3)(1.8 \times 10^{-2})}{(4/3)(2.02 \times 10^{-2})}} = \sqrt{\frac{5}{4} \times \frac{1.8}{2.02}} = 1.055$$

Intensity of Sound Wave

Consider a harmonic sound wave propagating along a tube of cross-sectional area S, as shown in figure. The quantity p is the excess pressure caused by the wave, and $\delta y/\delta t$ is the velocity of an element of the fluid. The instantaneous power (P) supplied by the wave to the element is

$$P = Fv = pA\frac{\delta y}{\delta t}$$



The motion of a piston or Longitudinal pulse, produces an excess pressure p on one side of an element of air. The speed of the element is $\delta s/\delta t$

Using equation

$$P = [-p_0 \cos(kx - \omega t)]\, S[-\omega A \cos(kx - \omega t)]$$

or $\quad P = p_0 A \omega S \cos^2(kx - \omega t)$

At any position say x = 0, the average of $\cos^2 \omega t$ over one period is ; hence the average power transmitted by the wave is

$$P_{av} = \frac{1}{2}\rho S(\omega A)^2 v$$

Note that this has the same from as equation for the power transported by a wave on a string.

The intensity I of a wave is defined as the energy incident per second per unit area normal to the direction of propagation :

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{S}$$

The SI unit of intensity is W/m^2

from equation we know $p_0 = BAK$

Since $K = \frac{\omega}{v}$ and $B = \rho v^2$, therefore $p_0 = \rho\omega v A$

Thus $\quad I_{av} = \frac{p_0^2}{4\pi r^2}$

That is, $I \propto I/r^2$, the intensity decreases as the inverse square of the distance from a point source. For cylindrical waves $I \propto \frac{1}{r}$

Ex. Measurement of sound waves show that the maximum pressure variations in the loudest sound that the ear can tolerate without pain are of the order of 30 Pa. Find the corresponding maximum displacement, if the frequency is 1000 Hz and v = 350 m/s.

Sol. $k = \frac{\omega}{v} = \frac{2(3.14)(1000)}{350} = 18m^{-1}$

B = γp = (1.)(10^5)

$$A = \frac{P}{Bk} = \frac{30}{(1.4\times10^5)(18)} = 1.18\times10^{-5} = 0.0118mm$$

Ex. In the previous example, if the density of air r = 1.22 kg/m^2, then find the intensity of a sound wave of the largest amplitude tolerable to the human ear.

Sol. Using $I = \frac{p_m^2}{2\rho v} = \frac{(30)^2}{2(1.22)(350)} = 1.05W/m^2$

Appearance of Sound to Human Ear

A normal person hears all frequencies between 20 & 20 KHz. This is a subjective range)obtained experimentally) which may very slightly from person to person. The ability to near the high frequencies decreases with age and a middle-age person can hear only upto 12 to 14 K Hz.

Sound can be generated with frequency below 20 Hz called infrasonic sound and above 20 Hz called ultrasonic sound. Even through humans cannot hear these frequencies, other animals may. For instance Rhinos communicate through infrasonic frequencies as low as 5 Hz, and base use ultrasonic frequencies as high as 100 KHz for navigating.

The appearance of sound to a human ear is characterised by three parameters (a) pitch (b) loudness and (c) quality.

Pitch and Frequency

Pitch of a sound is that sensation by which we differentiate a buffalo voice with cat's voice. A male voice is of low pitch, and a female voice has (generally) higher pitch. This sensation primarily depends on the dominant frequency present in the sound (Higher the frequency, higher will be the pitch) and vice versa. The dominant frequency of a buffalo voice is smaller than that of a cat's voice.

Loudness and Intensity

Intensity Level : The decibel scale

The sound intensities that the human ear can hear range from 10^{-12} W/m^2. The intensity of a sound is perceived by the ear as the subjective sensation of loudness. However, if the intensity doubles, the loudness does not increase by a factor of 2. Experiments first carried out by A.G. Bell showed that to produce an apparent doubling in loudness, the intensity of sound must be increased by a factor of about 10. Therefore, it is convenient to specify the intensity level β in terms of the decibel (dB) which is defined as

$$\beta = 10\log\frac{I}{I_0} \quad \text{....... (i)}$$

Where I is the measured intensity and I_0 is some reference value. If one takes I_0 to be 10^{-12} W/m^2, then the threshold of hearing corresponds to β = 10 I = 0 dB. At the threshold of pain, 1 W/m^2 the intensity level is

$$b = 10\log\left(\frac{I}{10^{-12}}\right) = 120dB$$

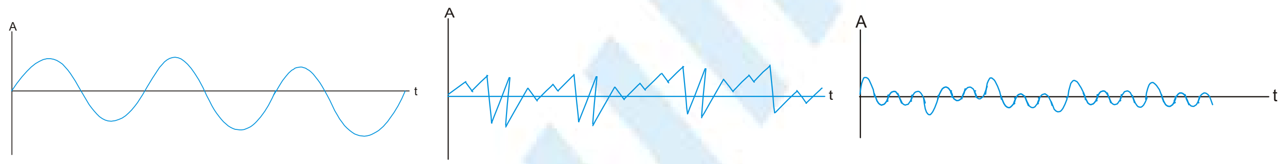
A list of the intensity level of various sources is given in Table

Intensity levels (dB)

Threshold of hearing	0
Leaves rustling	10
Quiet hall	25
Office	60
Conversation	60
Heavy traffic (3 m)	80
Loud classical music	95
Loud rock music	120
Jet engine (20 m)	130

Quality and Waveform

A sound generated by a source may contain a number of frequency components in it. Different frequency components have different amplitudes and superposition of them results in the actual waveform. The appearance of sound depends on this waveform apart from the dominant frequency and intensity. Figure shows waveforms for a tuning fork, a clarinet and a cornet playing the same note (fundamental frequency = 440 Hz) with equal loudness.



We differentiate between the sound from a table and that from a mridang by saying that they have different quality. A musical sound has certain well-defined frequencies which have considerable amplitude. These frequencies are generally harmonics of a fundamental frequency. Such a sound is particularly pleasant to the ear. On the other hand, a noise has frequencies that do not bear well-defined relationship among themselves.

Ex. The sound emitted by a source reaches a particular position with an intensity I_1. What is the change in intensity level when another identical source is placed next to the first (there is no fixed phase relation between the sources.)

Sol. If the initial and final intensities are I_1 and I_2, then the two intensity levels are

$$\beta_1 = 10\log\frac{I_1}{I_0}\ ; \qquad b_2 = 10\log\frac{I_2}{I_0}$$

The change in level is is

$$\beta_2 - \beta_1 = 10\log\frac{I_2}{I_1}$$

$= 10 \log 2 = 3$ dB

Thus when the intensity doubles the intensity level changes by 3 dB. The response of the ear roughly corresponds to this logarithmic scale. The smallest change in level that can be

Ex. A speaker emits 0.8W of acoustic power. Assume that it behaves as a point source which emits uniformly in all direction. At distance will that intensity level be 85 dB ?

Sol. From equation we know that the intensity of waves from a point source decreases as the inverse square of the distance r ; that is

$$I = \frac{P}{4\pi r^2} \qquad \text{........ (i)}$$

We must find the intensity corresponding to an 85 - dB sound level :

$$85 = 10 \log \frac{I}{I_0}$$

Thus, $\log (I/I_0) = 8.5$

or $I = 10^{-12} \times 10^{8.5} = 3.16 \times 10^{-4}\ W/m^2$ .... **(ii)**

Using **(ii)** and **(i)** we find

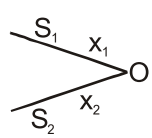
$$r^2 = \frac{P}{4\pi I}$$

$$= \frac{(0.8W)}{4(3.14)(3.16 \times 10^{-4} W/m^2)} = 201\ m^2$$

Thus, r = 14.1 m

where I is the intensity of the sound and I_0 is a constant reference intensity $10^{-12}\ W/m^2$. The reference intensity represents roughly the minimum intensity that is just audible at intermediate frequencies. For $I = I_0$, the sound level β=0. Table shows the approximate sound levels of some of the sounds commonly encountered.

Coherent Sources



If $p_1 = p_{01} \sin (\omega_1 t - k_1 x_1 + \phi_1)$

and $p_2 = p_{02} \sin (\omega_2 t - k_2 x_2 + \phi_2)$

Where p_1 and p_2 are pressure at point O due to S_1 and S_2 respectively.

The phase difference of the or arriving wave is

$$\Delta\theta = \theta_2 - \theta_1 = (k_1 x_1 - k_2 x_2) + (\omega_2 - \omega_1)t + (\phi_2 - \phi_1)$$

The sources are called coherent if the phase difference between them does not change with time Δθ is independent of time if

(a) $\omega_2 - \omega_1 = 0 \quad \Rightarrow \quad \omega_2 = \omega_1$

Thus for sources to be coherent, their frequencies must be equal.

(b) $k_1 = k_2$

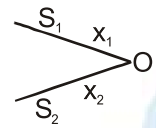
So $\Delta\theta = k(x_1 - x_2) + (\phi_2 - \phi_1) = k(\Delta x) + \Delta\phi$

Where Δx is called path difference of the waves.

If source are coherent the resultant intensity at any point is constant.

Source are called incoherent, if their frequencies are not equal. For incoherent sources, phase difference of the waves continuously changes with time and interference effects can not be observed at a point.

Interference of Sound Waves



If $\quad p_1 = p_{01}\sin(\omega t - k_1x_1 + \phi_1)$

and $\quad p_2 = p_{02}\sin(\omega t - k_2x_2 + \phi_2)$

Phase difference

$a = k(\Delta x) + (\phi)$

If initial phases for the two waves are same i.e. the sources vibrate in same source

i.e $\quad \Delta\phi = 0$

then phase difference is due to the path difference only i.e. $\alpha = k(\Delta x) = \frac{2\pi}{\lambda}(\Delta x)$

Resultant amplitude is

$$p_0\sqrt{p_{0_1}^2 + p_{0_2}^2 + 2(p_{0_1})(p_{0_2})\cos\alpha}$$

Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\alpha$

Condition for Constructive Interference

$\cos\alpha = +1$

i.e. $\alpha = 0, 2\pi, 4\pi, ...$ i.e. $2n\pi$

i.e. $\frac{2\pi}{\lambda} = (\Delta x) = 2n\pi \Rightarrow \Delta x = n\lambda \qquad (n = 0,1,2,3,..........) \qquad [n \in I]$

$$p_0 = p_{0_1} + p_{0_2} \Rightarrow I = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

Condition for destructive Interference

$$p_0 = |p_{0_1} - p_{0_2}| \Rightarrow I = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

When $\cos\alpha = -1$

i.e. $\quad \alpha = \pi, 3\pi, 5\pi, ... \Rightarrow \quad \alpha = (2n-1)\pi$

and $\quad \Delta x = (2n-1)\frac{\lambda}{2} = (n - \frac{1}{2})\lambda \qquad$ (where $n = 1,2,3, ...$) $[n \in I]$

If $\quad I_1 = I_2 = I_0$, then

$$I = I_0 + I_0 + 2\left(\sqrt{I^2{}_0}\right)\cos\alpha = 2I_0(1 + \cos\alpha)$$

$$! = 4I_0\cos^2\left(\frac{\alpha}{2}\right) \Rightarrow I_{max} = 4I_0 \qquad [\text{when } \alpha = 2n\pi]$$

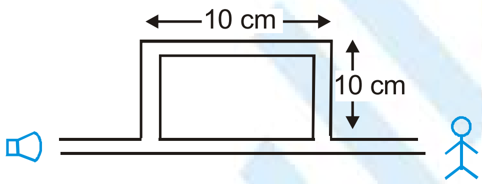
and $\quad I_{min} = 0 \qquad$ [when $\alpha = (2n-1)\pi$]

Reflection of Sound Waves

Reflection of sound waves from a rigid boundary (e.g. closed end of an organ pipe) is analogous to reflection of a string wave from rigid boundary, reflection accompained by an inversion i.e. and abrupt phase change of π. This is consistent with the requirement of displacement amplitude to remain zero at the rigid end, since a medium particle at the rigid end, since a medium particle at the rigid end can not vibrate. As the excess pressure and displacement corresponding to the same sound wave vary by $\pi/2$ in term of phase, a displacement minima at the rigid end will be a point of pressure maxima. This implies that the reflected pressure wave from the rigid boundary will have same phase as the incident wave, i.e., a compression pulse is reflected as a compression pulse and a rarefaction pulse is reflected as a rarefaction pulse.

On the other hand, reflection of sound wave from a low pressure region (like open end of an organ pipe) is analogies to reflection of string wave from a free end. This point corresponds to a displacement maxima, so that the incident & reflected displacement wave at this point must be in phase. This would imply that this point would be a minima for pressure wave (i.e. pressure at this point remains at its average value), and hence the reflected pressure wave would be out of phase by π with respect to the incident wave i.e. a compression pulse is reflected as a rarefaction pulse and vice-versa.

Ex. Figure shows a tube having sound source at one end and observer at other end. Source produces frequencies upto 10000 Hz. Speed of sound is 400 m/s. Find the frequencies at which person hears maximum intensity.



Sol. : The sound wave bifurcates at the junction of the straight and the rectangular parts. The wave through the straight part travels a distance p_1 = 10 cm and the wave through the rectangular part travels a distance $p_2 = 3 \times 10$ cm = 30 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore.

$$\Delta p = p_2 - p_1 = 30\text{ cm} - 10\text{ cm} = 20\text{ cm}$$

The wavelength of either wave is $\frac{v}{\upsilon} = \frac{400\text{ m/s}}{\upsilon}$. For constructive interference, $\Delta p = n\lambda$, where n is an integer.

or, $\Delta p = n.\frac{v}{\nu}$ $\Rightarrow$ $\nu = \frac{n.v}{\Delta p}$

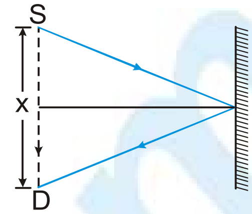
$\Rightarrow$ $\nu = \frac{400}{0.1} = 4000\,n$

Thus, the frequencies within the specified range which cause maximum of intensity are

4000×1 Hz, 4000×2 Hz

Ex. A source emitting sound of frequency 165 Hz is placed in front of a wall at a distance of 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the distance between the source and the detector for which the detector detects phase difference of 2π between the direct and reflected wave. Speed of sound in air = 330 m/s.

Sol. : The situation is shown in figure. Suppose the detector is placed at a distance of x meter from the source. The direct wave received from the source travels a distance of x meter. The wave reaching the detector after reflection from the wall has travelled a distance of



$2[(2)^2 + x^2/4]^{1/2}$ meter. The path difference between the two waves is

$$\Delta = \left\{2\left[(2)^2 + \frac{x^2}{4}\right]^{1/2} - x\right\} \text{ meter.}$$

$$\Delta = \lambda \quad \text{for } \Delta\phi = 2\pi \qquad \text{..........(i)}$$

The wavelength is $\lambda = \frac{\upsilon}{\nu} = \frac{330\,m/s}{165\,s^{-1}} = 2$ m.

Thus, by (i) $2\left[(2)^2 + \frac{x^2}{4}\right]^{1/2} - x = 2$

or, $\left[4 + \frac{x^2}{4}\right]^{1/2} = 1 + \frac{x}{2}$ or, $4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$

or, $x = 3$

Thus, the detector should be placed at a distance of 3 m from the source. Note that there is no abrupt phase change.

Vibration of Air Columns

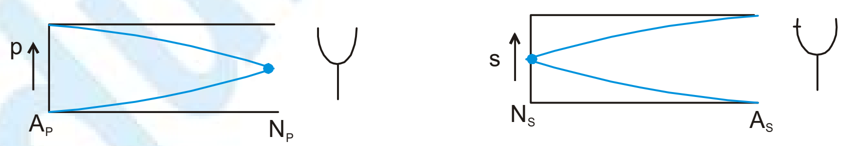
Standing Sound Wave

Standing waves can be set up in air-columns trapped inside cylindrical tubes if frequency of the tuning fork sounding the air column matches one of the natural frequency of air columns. In such a case the sound of the tuning fork becomes markedly louder, and we say there is resonance between the tuning fork and air column. To determine the natural frequency of the air column, notice that there is a displacement node (pressure antinode) at each closed end of the tube as air molecules there are not free to move, and a displacement antinode (pressure-node) at each open end of the air-column.

In reality antinodes do not occurs exactly at the open end but a little distance outside. However if diameter of tube is small compared to its length, this end correction can be neglected.

(i) Closed Organ Pipe

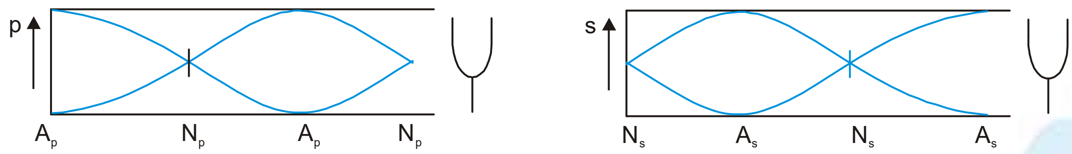
(In the diagram, A_P = Pressure antinode, A_S = displacement antinode, N_P = pressure node, N_S = displacement node)



Fundamental Mode :

The smallest frequency (largest wavelength) that satisfies the boundary condition for resonance (i.e. displacement node at left end and antinode at right end is $\lambda_0 = 4l$, where l = length of closed pipe the corresponding frequency.

$$v_0 = \frac{v}{\lambda} = \frac{v}{4L} \text{ is called the fundamental frequency.}$$



First Overnote :

Here there is one node and one antinode apart from the nodes and antinodes at the ends.

$$\lambda_1 = \frac{4l}{3} = \frac{\lambda_0}{3}$$

and corresponding frequency,

$$v_1 = \frac{v}{\lambda_1} = 3v_0$$

This frequency is 3 times the fundamental frequency and hence is called the 3rd harmonic.

nth Overnote :

In general, the nth overnote will have n nodes and n antinodes between the two nodes at the ends. The corresponding wavelength is

$$\lambda_n = \frac{4l}{2n+1} = \frac{\lambda}{2n+1} \qquad \text{and} \qquad v_n = (2n+1)\, v_0$$

This corresponds to the (2n + 1)th harmonic. Clearly only odd harmonic are allowed in a closed pipe.

(ii) Open Organ Pipe

Fundamental mode :

The smallest frequency (largest wave length) that satisfies the boundary condition for resonance (i.e. displacement antinodes at both ends) is,

$$\lambda_0 = 21$$

corresponding frequency, is called the fundamental frequency

$$v_0 = \frac{v}{21}$$

1st Overnote :

Here there is one displacement antinode between the two antinodes at the ends.

$$\lambda_1 = 1 = \frac{21}{2} \Rightarrow \lambda = \frac{\lambda_0}{2}$$

and corresponding frequency

$$v_1 = \frac{v}{\lambda_1} = 2v_0$$

The frequency is 2 times the fundamental frequency and is called the 2nd harmonic.

n^{th} Overnote :

The n^{th} overnote has n displacement antinodes between the two antinode at the ends.

$$\lambda_n = \frac{2l}{n+1} = \frac{\lambda_0}{n+1}$$

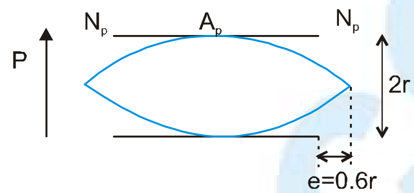
and $v_n = (n+1)\, v_0$

This correspond to $(n+1)^{th}$ harmonic, clearly both even and odd harmonics are allowed in an open pipe.

End Correction

As mentioned earlier the displacement antinode at an open end of an organ pipe lies slightly outside the open end. The distance of the antinode from the open end is called end correction and its value is given by

$e = 0.6\, r$



where r = radius of the organ pipe.

with end correction, the fundamental frequency of a closed pipe (f_c) and an open argon pipe (f_0) will be given by

$$f_c = \frac{v}{4(l + 0.6r)} \quad \text{and} \quad f_0 = \frac{v}{2(l + 1.2r)}$$

(i) A rod clamped at one end or a string fixed at one end is similar to vibration of closed end organ pipe.

(ii) A rod clamped in the middle is similar to the vibration of open end organ pipe.

(iii) If an open pipe is half submerged in water, it becomes a closed organ pipe of length half that of open pipe i.e. frequency remains same.

(iv) Due to finite motion of air molecular in organ pipes reflection takes place not exactly at open end but some what above it so in an organ pipe antinode is not formed exactly at free–end but above it at a distance e = 0.6r (called end correction or Rayleigh's correction) with r being the radius of pipe. So for closed organ pipe $L \to L + 0.6r$ while for open $L \to L + 2 \times 0.6r$ (as both ends are open)

so that – $f_C = \dfrac{v}{4(L+0.6r)}$ while $f_0 = \dfrac{v}{2(L+1.2r)}$

This is why for a given v and L narrower the pipe higher will the frequency or pitch and shriller will be the sound.

(v) For an organ pipe (closed **or** open) if v = constant. $f \propto (1/L)$

So with decrease in length of vibrating air column, i.e., wavelength ($\lambda \propto L$), frequency or pitch will increase and vice–versa.

This is why the pitch increases gradually as an empty vessel fills slowly.

(vi) For an organ pipe if f = constant. $v \propto \lambda$ or $v \propto L$, $f = \dfrac{v}{\lambda}$ = constant i.e. the frequency of an organ pipe will remain unchanged if the ratio of speed of sound in to its wave length remains constant.

(v) As for a given length of organ pipe ● = constant. $f \propto v$ So

(a) With rise in temperature as velocity will increase ($v \propto \sqrt{T}$), the pitch will increase.

(Change in length with temperature is not considered unless stated)

(b) With change in gas in the pipe as v will change and so f will change ($v \propto \sqrt{\gamma / M}$)

Ex. For a certain organ pipe, three successive resonant frequencies are observed at 425, 595 and 765 Hz respectively. Taking the speed for sound in air to be 340 m/s **(a)** Explain whether the pipe is closed at one end **or** open at both ends **(b)** determine the fundamental frequency and length of the pipe.

Sol. **(a)** The given frequencies are in the ratio

$425 : 595 : 765,$ i.e., $5 : 7 : 9$

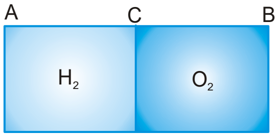
And as clearly these are odd integers so the given pipe is closed pipe.

(b) From part **(b)** it is clear that the frequency of 5th harmonic (which is third overtone) is 425 Hz

So $425 = 5f_C \Rightarrow f_C = 85$ Hz Further as $f_C = \frac{v}{4L}, L = \frac{v}{4f_C} = \frac{340}{4 \times 85} = 1$ m

Ex. AB is a cylinder of length 1 m fitted with a thin flexible diaphragm C at middle and two other thin flexible diaphragm A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of the same frequency. What is the minimum frequency of these vibrations for which diaphragm C is a node ? Under the condition of the experiment the velocity of sound in hydrogen is 1100 m/s and oxygen 300 m/s.

Sol. As diaphragm C is a node, A and B will be antinode (as in a organ pipe either both ends are antinode or one end node and the other antinode), i.e., each part will behave as closed end organ pipe so that



$f_H = \frac{v_H}{4L_H} = \frac{1100}{4 \times 0.5} = 550$ Hz And $f_0 = \frac{v_0}{4L_0} = \frac{300}{4 \times 0.5} = 150$Hz

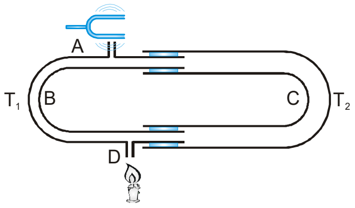
As the two fundamental frequencies are different, the system will vibrate with a common frequency f such that $f = n_H f_H = n_0 f_0$ i.e. $\frac{n_H}{n_0} = \frac{f_0}{f_H} = \frac{150}{550} = \frac{3}{11}$

i.e., the third harmonic of hydrogen and 11th harmonic of oxygen or 6th harmonic of hydrogen and 22nd harmonic of oxygen will have same frequency. So the minimum common frequency

$f = 3 \times 550 \text{ or } 11 \times 150 = 1650$ Hz

Apparatus for determining speed of sound

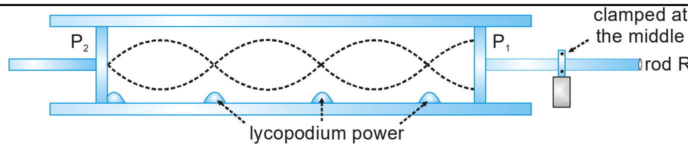
1. Quinck's Tube :



It consists of two U shaped metal tubes. Sound waves with the help of tuning fork are produced at A which travel through B & C and comes out at D where a sensitive flame is present. Now the two waves coming through different path interfere and flame flares up. But if they are not in phase, destructive interference occurs and flame remains undisturbed.

Suppose destructive interference occurs at D for some position of C. If now the tube C is moved so that interference condition is disturbed and again by moving a distance x, destructive interference occurs so that $2x = \lambda$. Similarly if the distance moved between successive constructive and destructive interference is x then $2x = \frac{\lambda}{2}$ Now by having value of x, speed of sound is given by $v = n\lambda$.

2. Kundt's tube : It is the used to determine speed of sound in different gases. It consists of a glass tube in which a small quantity of lycopodium powder is spread. The tube is rotated so that powder starts slipping. Now rod CD is rubbed at end D so that stationary waves form. The disc C vibrates so that air column also vibrates with the frequency of the rod. The piston P is adjusted so that frequency of air column become same as that of rod. So resonance occurs and column is thrown into stationary waves. The powder sets into oscillations at antinodes while heaps of powder are formed at nodes.



Let n is the frequency of vibration of the rod then, this is also the frequency of sound wave in the air column in the tube.

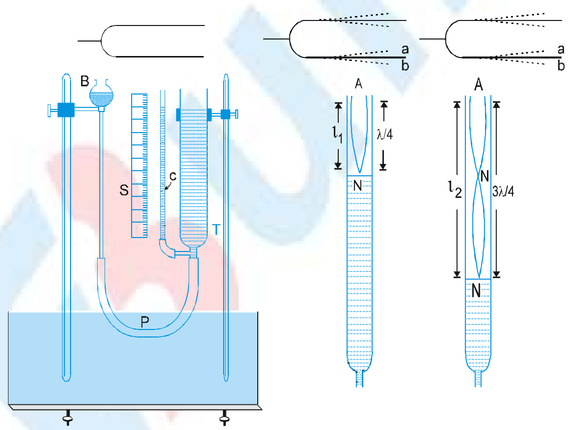
For rod : $\frac{\lambda_{rod}}{2} = l_{rod}$ **For air :** $\frac{\lambda_{air}}{2} = l_{air}$

Where $\bullet_{air}$ is the distance between two heaps of powder in the tube (i.e. distance between two nodes). If v_{air} and v_{rod} are velocity of sound waves in the air and rod respectively, then $n = \frac{v_{air}}{\lambda_{air}} = \frac{v_{rod}}{\lambda_{rod}}$

Therefore, $\frac{v_{air}}{v_{rod}} = \frac{\lambda_{air}}{\lambda_{rod}} = \frac{l_{air}}{l_{rod}}$ Thus knowledge of v_{rod} determines v_{air}.

3. Resonance Tube

Construction : The resonance tube is a tube T (figure) made of brass or glass, about 1 meter long and 5 cm in diameter and fixed on a vertical stand. Its lower end is connected to a water reservoir B by means of a flexible rubber tube. The rubber tube carries a pinch-cock P. The level of water in T can be raised or lowered by water adjusting the height of the reservoir B and controlling the flow of water from B to T or from T to B by means of the pinch-cock P. Thus the length of the air–column in T can be changed. The position of the water level in T can be read by means of a side tube C and a scale S.



Determination of the speed of sound in air by resonance tube

First of all the water reservoir B is raised until the water level in the tube T rises almost to the top of the tube. Then the pinch–cock P is tightened and the reservoir B is lowered. The water level in T stays at the top. Now a tuning fork is sounded and held over the mouth of tube .The pinch–cock P is opened slowly so that the water level in T falls and the length of the air–column increases. At a particular length of air–column in T, a loud sound is heard. This is the first state of resonance. In this position the following phenomenon takes place inside the tube.

(i) For first resonance $\bullet_1 = \lambda/4$

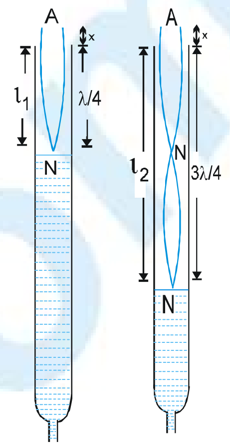
(ii) For second resonance $\bullet_2 = 3\lambda/4 \Rightarrow \bullet_2 - \bullet_1 = \lambda/2 \Rightarrow \lambda = 2(\bullet_2 - \bullet_1)$

If the frequency of the fork be n and the temperature of the air-column be t°C, then the speed of sound at t°C is given by $v_t = n\lambda = 2n(\bullet_2 - \bullet_1)$

The speed of sound wave at 0°C $v_0 = (v_t - 0.61\ t)$ m/s.

End Correction :

In the resonance tube, the antinode is not formed exactly at the open but slightly outside at a distance x. Hence the length of the air -column in the first and second states of resonance are $(l_1 + x)$ and $(\bullet_2 + x)$ then



(i) For first resonance $\bullet_1 + x = \lambda/4$ ....(i)

(ii) For second resonance $\bullet_2 + x = 3\lambda/4$ ....(ii)

Subtract Equation (ii) from Equation (i)

$$\bullet_2 - \bullet_1 = \lambda/2$$

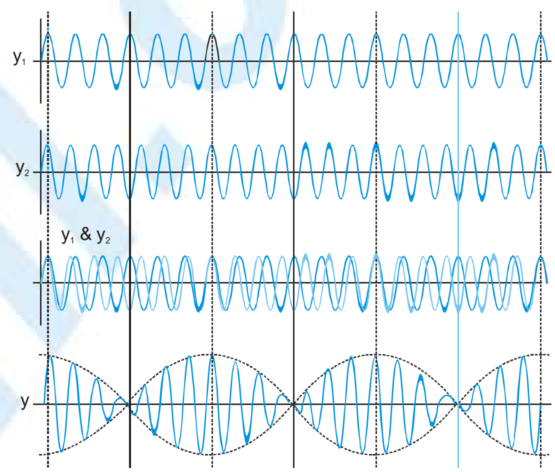
$$\lambda = 2(\bullet_2 - \bullet_1)$$

Put the value of λ in Equation (i) $\bullet_1 + x = \dfrac{2(l_2 - l_1)}{4}$

$$\Rightarrow \bullet_1 + x = \frac{l_2 - l_1}{2} \Rightarrow x = \frac{l_2 - 3l_1}{2}$$

BEATS

When two sound waves of same amplitude travelling in same direction with slightly different frequency superimpose, then intensity varies periodically with time. This effect is called Beats. Suppose two waves of frequencies f_1 and f_2 ($<f_1$) are meeting at some point in space. The corresponding periods are T_1 and T_2 ($>T_1$). If the two waves are in phase at t=0, they will again be in phase when the first wave has gone through exactly one more cycle than the second. This will happen at a time t=T, the period of the beat. Let *n* be the number of cycles of the first wave in time T, then the number of cycles of the second wave in the same time is (n–1).



Hence, $T = nT_1 = (n-1)\,T_2$

Eliminating n we have $T = \dfrac{T_1 T_2}{T_2 - T_1} = \dfrac{1}{\frac{1}{T_1} - \frac{1}{T_2}} = \dfrac{1}{f_1 - f_2}$

The reciprocal of the beat period is the beat frequency $f = \dfrac{1}{T} = f_1 - f_2$

Waves Interference On The Bases Of Beats :

Conditions ⁙ Two equal frequency waves travel in same direction.

Mathematical analysis

If displacement of first wave $y_1 = a \sin \omega_1 t \longrightarrow (N_1, a)$ $I \propto N^2 a^2$

Displacement of second wave $y_2 = a \sin \omega_2 t \longrightarrow (N_2, a)$

By superposition $y = y_1 + y_2$

Equation of resulting wave $y = a\,\{\sin 2\pi N_1 t + \sin 2\pi N_2 t\}$

$$y = a\left\{2 \sin 2\pi\, t\frac{(N_1 + N_2)}{2} \cos 2\pi t \frac{(N_1 - N_2)}{2}\right\} = \left\{2a \cos 2\pi t \;\frac{(N_1 - N_2)}{2}\right\} \sin 2\pi t \;\frac{(N_1 + N_2)}{2} = A \sin 2\pi N't$$

Amplitude $A = 2a \cos 2\pi t \left(\dfrac{N_1 - N_2}{2}\right) = 2a \cos \pi t\;(N_1 - N_2)$ **Frequency** $N' = \dfrac{N_1 + N_2}{2}$

For max Intensity ($A = \pm 2a$): –

If $\cos \pi (N_1 - N_2) t = \pm 1 \Rightarrow \cos \pi (N_1 - N_2) t = \cos n\pi$, n = 0, 1, 2, ...

$$\Rightarrow \pi (N_1 - N_2)t = n\pi \Rightarrow t = \frac{n}{N_1 - N_2} = 0, \frac{1}{\Delta N}, \frac{2}{\Delta N}, \frac{3}{\Delta N} \ldots\ldots\ldots\ldots$$

For Minimum Intensity (A = 0) :

$$\Rightarrow \cos \pi (N_1 - N_2) t = 0 \Rightarrow \cos \pi (N_1 - N_2) t = \cos (2n+1)\frac{\pi}{2} \quad n = 0, 1, 2 \ldots\ldots\ldots\ldots$$

$$\Rightarrow \pi (N_1 - N_2) t = (2n+1)\frac{\pi}{2} \Rightarrow t = \frac{2n+1}{2(N_1 - N_2)} = \frac{1}{2\Delta N}, \frac{3}{2\Delta N}, \frac{5}{2\Delta N} \ldots\ldots\ldots\ldots$$

(i) When we added wax on tuning fork then the frequency of fork decreases.

(ii) When we file the tuning fork then the frequency of fork increases.

Ex. A tuning fork having n = 300 Hz produces 5 beats/s with another tuning fork. If impurity (wax) is added on the arm of known tuning fork, the number of beats decreases then calculate the frequency of unknown tuning fork.

Sol. The frequency of unknown tuning fork should be $300 \pm 5 = 295$ Hz or 305 Hz.

When wax is added, if it would be 305 Hz, beats would have increases but with 295 Hz beats is decreases so frequency of unknown tuning fork is 295 Hz.

Ex. A tuning fork having n = 158 Hz, produce 3 beats/s with another. As we file the arm of unknown, beats become 7 then calculate the frequency of unknown.

Sol. The frequency of unknown tuning fork should be $158 \pm 3 = 155$ Hz or 161 Hz.

After filling the number of beats = 7 so frequency of unknown tuning fork should be

$158 \pm 7 = 165$ Hz or 151 Hz.

As both above frequency can be obtain by filing so frequency of unknown = 155/161 Hz.

Ex. Wavelength of two notes in air are $\left(\frac{90}{175}\right)$ m and $\left(\frac{90}{173}\right)$ m. Each note produces four beats per second with a third note of a fixed frequency. Calculate the velocity of sound in air.

Sol. Given $\lambda_1 = \frac{90}{175} m$ and $\lambda_2 = \frac{90}{173} m$

Let f_1 and f_2 be the corresponding, frequencies and v be the velocity of sound in air.

$v = \lambda_1 f_1$ and $v = \lambda_2 f_2$

$\therefore \quad f_1 = \frac{v}{\lambda_1}$ and $f_2 = \frac{v}{\lambda_2}$

Since $\lambda_2 > \lambda_1 \quad \therefore \quad f_1 > f_2$

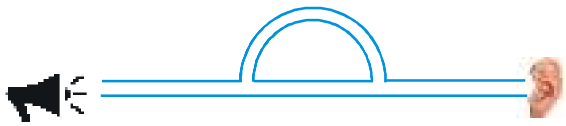
Let be the frequency of the third note.

$\therefore \quad f_1 - f = 4$ and $f - f_2 = 4$

$\therefore \quad f_1 - f_2 = 8 \qquad \therefore \quad \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 8$

$v\left[\frac{175}{90} - \frac{173}{90}\right] = 8 \qquad \therefore \quad v\left[\frac{2}{90}\right] = 8$

Ex. Figure shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of 10.0 cm. The frequency of the sound source can be varied from 1 to 10 kHz. Find the frequencies at which the ear perceives maximum intensity. The speed of sound in air = 342 m/s.



Sol. The sound wave bifurcates at the junction of the straight and the semiconductor parts. The wave through the straight part travels a distance $s_1 = 2 \times 10$ cm and the wave through the curved part travels a distance $s_2 = \pi\ 10$ cm = 31.4 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore,

$$\Delta s = s_2 - s_1 = 31.4 \text{ cm} - 20 \text{ cm} = 11.4 \text{ cm}$$

The wavelength of either wave is $\frac{v}{f} = \frac{330\ m/s}{f}$. For constructive interference, $\Delta p = n\lambda$, where n is an integer.

or, $\Delta p = n.\frac{v}{f} \quad \Rightarrow \quad f = \frac{n.v}{\Delta p} \quad \Rightarrow \quad \frac{n.342}{(0.114)} = 3000n$

Thus, the frequencies within the specified range which cause maximum of intensity are

$3000 \times 1 \qquad 3000 \times 2 \qquad$ and $\qquad 3000 \times 3$ Hz

Ultrasonic, Infrasonic and Audible (sonic) Sound :

Sound waves can be classified in three groups according to their range of frequencies.

(i) Infrasonic Waves

Longitudinal waves having frequencies below 20 Hz are called infrasonic waves. They cannot be heard by human beings. They are produced during earthquakes. Infrasonic waves can be heard by snakes.

(ii) Audible Waves

Longitudinal waves having frequencies lying between 20-20,000 Hz are called audible waves.

(iii) Ultrasonic Waves

Longitudinal waves having frequencies above 20,000 Hz are called ultrasonic waves. They are produced and heard by bats. They have a large energy content.

Applications of Ultrasonic Waves

Ultrasonic waves have a large range of application. Some of them are as follows:

(i) The fine internal cracks in metal can be detected by ultrasonic waves.

(ii) Ultrasonic waves can be used for determining the depth of the sea, lakes etc.

(iii) Ultrasonic waves can be used to detect submarines, icebergs etc.

(iv) Ultrasonic waves can be used to clean clothes, fine machinery parts etc.

(v) Ultrasonic waves can be used to kill smaller animals like rates, fish and frogs etc.

Shock Waves

If the speed of the body in air is greater than the speed of the sound, then it is called supersonic speed. Such a body leaves behind a conical region of disturbance which spreads continuously. Such a disturbance is called a **'Shock Wave'**. This wave carries huge energy. If it strikes a building, then the building may be damaged.

ECHO

Multiple reflection of sound is called an echo. If the distance of reflector from the source is d then,

$$2d = vt$$

Hence, v = speed of sound and t, the time of echo. $\therefore d = \frac{vt}{2}$

Since, the effect of ordinary sound remains on our ear for 1/10 second, therefore, if the sound returns to the starting point before 1/10 second, then it will not be distinguished from the original sound and no echo will be heard. Therefore, the minimum distance of the reflector is, $d_{min} = \frac{v \times t}{2} = \left(\frac{330}{2}\right)\left(\frac{1}{10}\right) = 16.5\,m$

Acoustic Doppler effect (Doppler effect for Sound waves)

The apparent change in the frequency of sound when the source of sound, the observer and the medium are in relative motion is called Doppler effect. While deriving these expressions, we make the following assumptions

(i) The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.

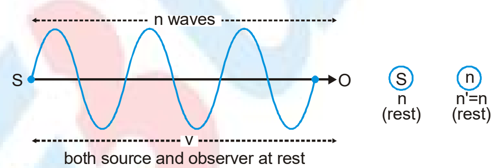
(ii) The velocity of the source and the observer is less than velocity of sound.

Doppler effect takes place both in sound and light. In sound it depends on whether the source **or** observer **or** both are in motion while in light it depends on whether the distance between source and observer is increasing or decreasing.

Notations

$n \rightarrow$ actual frequency	$n' \rightarrow$ observed frequency (apparent frequency)
$\lambda \rightarrow$ actual wave length	$\lambda' \rightarrow$ observed (apparent) wave length
$v \rightarrow$ velocity of sound	$v_s \rightarrow$ velocity of source
$v_0 \rightarrow$ velocity of observer	$v_w \rightarrow$ wind velocity

Case I : Source in motion, observer at rest, medium at rest :

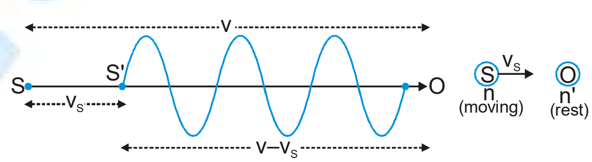


Suppose the source S and observer O are separated by distance v. Where v is the velocity of sound. Let n be the frequency of sound emitted by the source. Then n waves will be emitted by the source in one second. These n waves will be accommodated in distance v.

So, wave length $\lambda = \frac{\text{total distance}}{\text{total number of waves}} = \frac{v}{n}$

(i) Source moving towards stationary observer :

Let the sources start moving towards the observer with velocity v_s. After one second, the n waves will be crowded in distance $(v - v_s)$. Now the observer shall feel that he is listening to sound of wavelength λ' and frequency n'



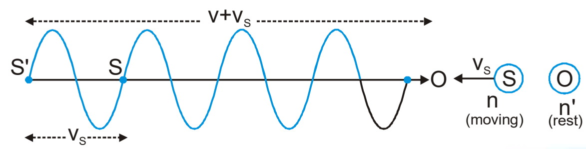
Now apparent wavelength $\lambda' = \frac{\text{total distance}}{\text{total number of waves}} = \frac{v - v_s}{n}$

and changed frequency, $$n' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v - v_s}{n}\right)} = n\left(\frac{v}{v - v_s}\right)$$

So, as the source of sound approaches the observer the apparent frequency n' becomes greater than the true frequency n.

(ii) When source move away from stationary observer :-

For this situation n waves will be crowded in distance $v + v_s$.



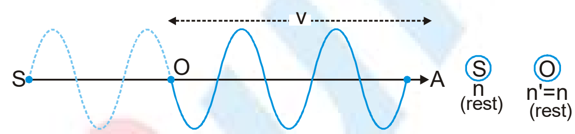
So, apparent wavelength $$\lambda' = \frac{v + v_s}{n}$$

and

Apparent frequency $$n' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v + v_s}{n}\right)} = n\left(\frac{v_s}{v + v_s}\right)$$ So $n' < n$

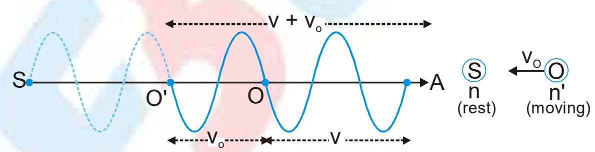
Case II : Observer in motion, source at rest, medium at rest

Let the source (S) and observer (O) are in rest at their respective places. Then n waves given by source 'S' would be crossing observer 'O' in one second and fill the space OA (=v)



both source S and O at rest

(i) Observer moves towards stationary source



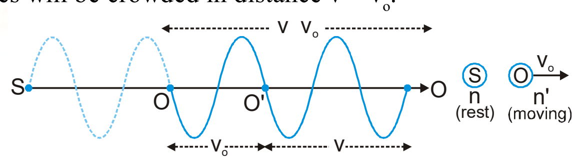
When observer 'O' moves towards 'S' with velocity v_o, it will cover v_o distance in one second. So the observer has received not only the n waves occupying OA but also received additional number of Δn waves occupying the distance OO' (= v_o).

So, total waves received by observer in one second i.e., apparent frequency (n') = Actual waves (n) + Additional waves (Δn)

$$n' = \frac{v}{\lambda} + \frac{v_o}{\lambda} = \frac{v + v_o}{(v/n)} = n\left(\frac{v + v_o}{v}\right)\left(Q\ \lambda = \frac{v}{n}\right) \quad (\text{so, } n' > n)$$

(ii) Observer moves away from stationary source :-

For this situation n waves will be crowded in distance $v - v_o$.



When observer move away from source with v_o velocity then he will get Δn waves less than real number of waves. So, total number of waves received by observer i.e.

Apparent frequency (n') = Actual waves (n) – reduction in number of waves (Δn)

$$n' = \frac{v}{\lambda} - \frac{v_o}{\lambda} = \frac{v - v_o}{\lambda} = \frac{v - v_o}{(v/n)} = \left(\frac{v - v_o}{v}\right) n \qquad \left(\because \lambda = \frac{v}{n}\right) \text{ (so } n' < n)$$

Table : Doppler Frequencies (f') for different situations

	Source Stationary	Source Toward Observer	Source Away from Observer
Observer Stationary	f	$f\left(\frac{v}{v - v_s}\right)$	$f\left(\frac{v}{v + v_s}\right)$
Observer toward source	$f\left(\frac{v + v_o}{v}\right)$	$f\left(\frac{v + v_o}{v - v_s}\right)$	$f\left(\frac{v + v_o}{v + v_s}\right)$
Observer away from source	$f\left(\frac{v - v_o}{v}\right)$	$f\left(\frac{v - v_o}{v - v_s}\right)$	$f\left(\frac{v - v_o}{v + v_s}\right)$

Wind Effect

The above formula can be modified by taking the wind effects into account. The velocity of sound should be taken as $v + v_w$ or $v - v_w$ if the wind is blowing in the same or opposite direction as SO (source to observer)

ETOOS KEY POINTS

(i) If medium (air) is also moving with v_m velocity in direction of source to observer. Then velocity of sound relative to observer will be $v \pm v_m$ (–ve sign, if v_m is opposite to sound velocity). So, $n' = n\left(\frac{v \pm v_m + v_o}{v \pm v_m - v_s}\right)$

(ii) If medium moves in a direction opposite to the direction of propagation of sound, then $n' = \left(\frac{v - v_m - v_o}{v - v_m - v_s}\right) n$

(iii) Source in motion towards the observer. Both medium and observer are at rest. $n' = \left(\frac{v}{v - v_s}\right) n$

So, when a source of sound approaches a stationary observer, the apparent frequency is more than the actual frequency.

(iv) Source in motion away from the observer. Both medium and observer are at rest. $n' = \left(\frac{v}{v + v_s}\right) n$. So, when a source of sound moves away from a stationary observer, the apparent frequency is less than actual frequency.

(v) Observer in motion towards the source. Both medium and source are at rest. $n' = \left(\frac{v + v_o}{v}\right) n$. So, when observer is in motion towards the source, the apparent frequency is more than the actual frequency.

(vi) Observer in motion away from the source. Both medium and source are at rest. $n' = \left(\frac{v - v_o}{v}\right) n$. So, when observer is in motion away from the source, the apparent frequency is less than the actual frequency.

(vii) Both source and observer are moving away from each other. Medium at rest. $n' = \left(\frac{v - v_o}{v + v_s}\right) n$

Doppler's Effect in Reflection of Sound (ECHO)

When the sound is reflected from the reflector the observer receives two notes one directly from the source and other from the reflector. If the two frequencies are different then superposition of these waves result in beats and by the beat frequency we can calculate speed of the source.

If the source is at rest and reflector is moving towards the source with speed u,

then apparent frequency heard by reflector $n_1 = \left(\frac{v+u}{v}\right)n$

Now this frequency n_1 acts as a source so that apparent frequency received by observer is

$$n_2 = \left(\frac{v}{v-u}\right)n_1 = \left(\frac{v}{v-u}\right)\times\left(\frac{v+u}{v}\right)n = \left(\frac{v+u}{v-u}\right)n$$

If $u << v$ then $\quad n_2 = n\left(1+\frac{u}{v}\right)\left(1-\frac{u}{v}\right)^{-1} \approx n\left(1+\frac{u}{v}\right)^2 \approx n\left(1+\frac{2u}{v}\right)$

Beat frequency $\Delta n = n_2 - n = \left(\frac{2u}{v}\right)n$ $\qquad$ So speed of the source $u = \frac{v}{2}\left(\frac{\Delta n}{n}\right)$

Conditions When Doppler's effect is not observed for sound waves

(i) When the source of sound and observer both are at rest then Doppler effect is not observed.

(ii) When the source and observer both are moving with same velocity in same direction.

(iii) When the source and observer are moving mutually in perpendicular directions.

(iv) When the medium only is moving.

(v) When the distance between the source and observer is constant.

Ex. When both source and observer approach each other with a speed equal to the half the speed of sound, then determine the percentage change in frequency of sound as detected by the listener.

Sol. Source $\bullet\xrightarrow{v/2}$ $\xleftarrow{v/2}\bullet$ Observer $n' = \left(\frac{v+\frac{v}{2}}{v-\frac{v}{2}}\right)n = \left(\frac{\frac{3}{2}v}{\frac{1}{2}v}\right)n = 3n$

$$\%\text{ change} = \frac{n'-n}{n}\times 100 = \frac{3n-n}{n}\times 100 = \frac{2n}{n}\times 100 = 200\,\%$$

Ex. Two trains travelling in opposite directions at 126 km/hr each, cross each other while one of them is whistling. If the frequency of the node is 2.22 kHz find the apparent frequency as heard by an observer in the other train:

(a) Before the trains cross each other

(b) After the trains have crossed each other. (v_{sound} = 335 m/sec)

Sol. Here $\quad v_1 = 126 \times \frac{5}{18} = 35$ m/s

(i) In this situation $\circ\!\!\longrightarrow v_1 \qquad v_1 \longleftarrow\!\!\circ$

Observed frequency $\qquad n' = \left(\frac{v+v_1}{v-v_1}\right)n = \left(\frac{335+35}{335-35}\right)\times 2220 = 2738$ Hz

(ii) In this situation $v_1 \longleftarrow\!\!\circ \qquad \circ\!\!\longrightarrow v_1$

Observed frequency $\qquad n' = \left(\frac{v-v_1}{v+v_1}\right)n = \left(\frac{335-35}{335+35}\right)\times 2220 = 1800$ Hz

Ex. A stationary source emits sound of frequency 1200 Hz. If wind blows at the speed of 0.1v, deduce

(a) The change in the frequency for a stationary observer on the wind side of the source.

(b) Report the calculations for the case when there is no wind but the observer moves at 0.1v speed towards the source. (Given : velocity of sound = v)

Sol.

(a) Medium moves in the direction of sound propagation i.e. from source to observer

so effective velocity of sound $v_{eff} = v + v_m$

since both source and observer are at rest $n' = \left(\frac{v + v_m + 0}{v + v_m + 0}\right)n = \left(\frac{v + 0.1v}{v + 0.1v}\right)n = n$

so there is no change in frequency

(b) When observer move towards source $n' = \left(\frac{v + v_0}{v}\right)n = \left(\frac{v + 0.1\,v}{v}\right)n$

$= 1.1\ n = 1.1 \times 1200\ \text{Hz} = 1320\ \text{Hz}$

Ex. A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall ?

Sol.

The apparent frequency heard by the bat of reflected sound

$$n' = \left(\frac{v + v_0}{v - v_s}\right)n = \left(\frac{v + 0.03\,v}{v - 0.03\,v}\right) \times 40 = \frac{1.03\,v}{0.97\,v} \times 40 = 42.47\ \text{kHz}$$

A wave is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

1. Classification of Waves

- Medium Necessity
  - Mechanical (Elastic) waves
  - Non - mechanical waves (EM waves)
- Propagation of energy
  - Progressive waves
  - Stationary (standing) waves
- Vibration of medium particle
  - Transverse waves
  - Longitudinal waves
- Dimension
  - 1-D (Waves on strings)
  - 2-D (Surface waves or ripples on water)
  - 3-D (Sound or light waves)

(a) A mechanical wave will be transverse or longitudinal depending on the nature of medium and mode of excitation.

(b) In strings, mechanical waves are always transverse.

(c) In gases and liquids, mechanical waves are always longitudinal because fluids cannot sustain shear.

(d) Partially transverse waves are possible on a liquid surface because surface tension provide some rigidity on a liquid surface. The waves are called as ripples as they are combination of transverse & longitudinal.

(e) In solids mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation.

(f) In longitudinal wave motion, oscillatory motion of the medium particles produce regions of compression (high pressure) and rarefaction (low pressure).

2. Plane Progressive Waves

(a) Wave equation : $y = A\sin(\omega t - kx)$ where $k = \dfrac{2\pi}{\lambda}$ = wave propagation constant.

(b) Differential equation : $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v_P^2}\dfrac{\partial^2 y}{\partial t^2}$

Wave velocity (phase velocity) $v = \dfrac{dx}{dt} = \dfrac{\omega}{k}$ ✈ $\omega t - kx$ = constant $\Rightarrow \dfrac{dx}{dt} = \dfrac{\omega}{k}$

(c) Particle Velocity $v_p = \dfrac{dy}{dt} = A\omega\cos(\omega t - kx)$ ✈ $v_p = -v \times \text{slope} = -v\left(\dfrac{dy}{dx}\right)$

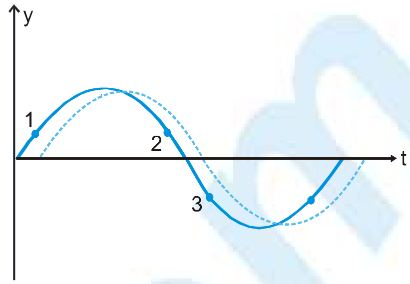
(d) Particle acceleration : $a_p = \dfrac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(\omega t - kx) = -\omega^2 y$

For particle 1 : $v_p \downarrow$ and $a_p \downarrow$

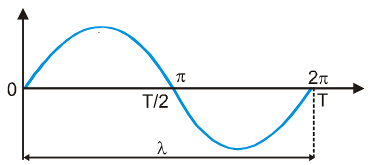
For particle 2 : $v_p \uparrow$ and $a_p \downarrow$

For particle 3 : $v_p \uparrow$ and $a_p \uparrow$

For particle 4 : $v_p \downarrow$ and $a_p \uparrow$



(e) Relation between phase difference, path difference & time difference



$$\frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T}$$

3. Energy in Wave Motion

(a) $$\frac{\text{KE}}{\text{volume}} = \frac{1}{2}\left(\frac{\Delta m}{\text{volume}}\right)v_p^2 = \frac{1}{2}\rho v_p^2 = \frac{1}{2}\rho\omega^2 A^2 \cos^2(\omega t - kx)$$

(b) $$\frac{\text{PE}}{\text{volume}} = \frac{1}{2}\rho v^2\left(\frac{dy}{dx}\right)^2 = \frac{1}{2}\rho\omega^2 A^2 \cos^2(\omega t - kx)$$

(c) $$\frac{\text{TE}}{\text{volume}} = \rho\omega^2 A^2 \cos^2(\omega t - kx)$$

(d) Energy density [i.e. Average total energy / volume] $u = \frac{1}{2}\rho\omega^2 A^2$

(e) **Power :** P = (energy density) (volume / time) $P = \left(\frac{1}{2}\rho\omega^2 A^2\right)(Sv)$

[where S = Area of cross - section]

(f) **Intensity :** $I = \frac{\text{Power}}{\text{area of cross section}} = \frac{1}{2}\rho\omega^2 A^2 v$

4. Speed of transverse wave on string :

$v = \sqrt{\frac{T}{\mu}}$ where μ = mass / length and T = tension in the string

5. A wave can be represented by function $y = f(kx \pm \omega t)$ because it satisfy the differential equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\left(\frac{\partial^2 y}{\partial t^2}\right)$

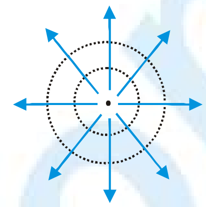
where $v = \frac{w}{k}$.

6. A pulse whose wave function is given by $y = 4 / [(2x + 5t)^2 + 2]$ propagates in –x direction as this wave function is of the form $y = f(kx + \omega t)$ which represent a wave travelling in –x direction.

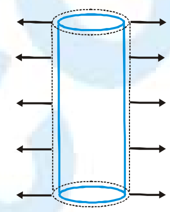
7. Longitudinal waves can be produced in solids, liquids and gases because bulk modulus of elasticity is present in all three.

8. Wave Front

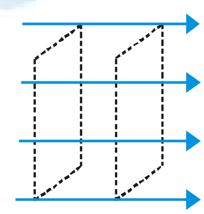
(a) Spherical wave from (source → point source)



(b) Cylindrical wave front (source → linear source)



(c) Plane wave front (source → point / linear source at very large distance)



9. Intensity of Wave

(a) Due to point source $I \propto \frac{1}{r^2}$ $\qquad y(r,t) = \frac{A}{r}\sin\left(\omega t - \vec{k}.\vec{r}\right)$

(b) Due to cylindrical source $I \propto \frac{1}{r}$ $\qquad y(r,t) = \frac{A}{\sqrt{r}}\sin(\omega t - \vec{k}.\vec{r})$

(c) Due to plane source I = constant $\qquad y(r,t) = A\sin(\omega t - \vec{k}.\vec{r})$

10. Interference of Waves

$y_1 = A_1\sin(\omega t - kx)$, $\qquad y_2 = A_2\sin(\omega t - kx + \phi_0)$

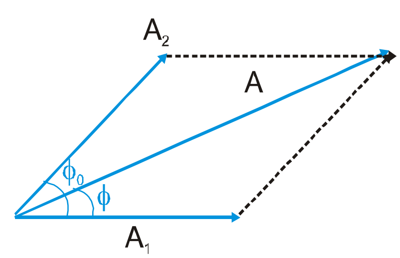
$y = y_1 + y_2 = A\sin(\omega t - kx + \phi)$

where $\quad A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi_0}$

and $\quad \tan\phi = \frac{A_2\sin\phi_0}{A_1 + A_2\cos\phi_0}$

As $\quad I \propto A^2$

So $\quad I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi_0$



(a) For constructive interference [Maximum intensity]

$\phi_0 = 2n\pi$ or path difference $= n\lambda$ where $n = 0, 1, 2, 3, \ldots\ldots\ldots\ldots$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

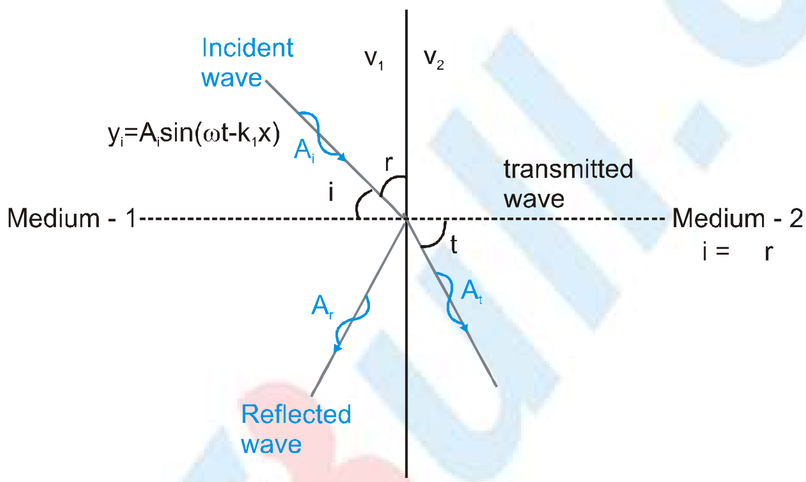
(b) For destructive interference (Minimum Intensity]

$\phi_0 = (2n + 1)$ p or path difference $= (2n+1)\dfrac{\lambda}{2}$

where $n = 0, 1, 2, 3, \ldots\ldots\ldots$ $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

Degree of hearing $= \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100$

11. Reflection and Refraction (transmission) of waves



(a) The frequency of the wave remain unchanged.

(b) Amplitude of reflected wave $\rightarrow A_r = \left(\dfrac{v_2 - v_1}{v_1 + v_2}\right) A_i$

(c) Amplitude of transmitted wave $\rightarrow A_t = \left(\dfrac{2v_2}{v_1 + v_2}\right) A_i$

(d) $v_2 > v_1$ i.e. medium - 2 is rarer.

$A_r > 0 \Rightarrow$ no phase change in reflected wave

(e) If $v_2 < v_1$ i.e. medium - 1 is rarer

$A_r < 0 \Rightarrow$ There is a phase change of π in reflected wave

(f) As A_t is always positive whatever be v_1 & v_2 the phase of transmitted wave always remains unchanged.

(g) In case of reflection from a denser medium or support or fixed end, there is invention of reflected wave i.e. phase difference of π between reflected and incident wave.

(h) The transmitted wave is never inverted.

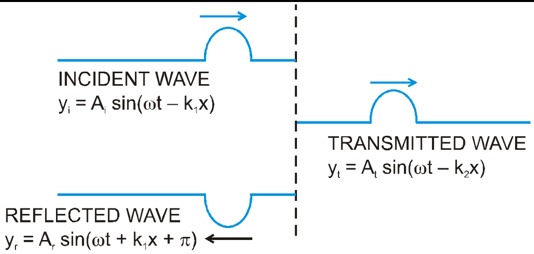


Fig. : Reflection at denser boundary

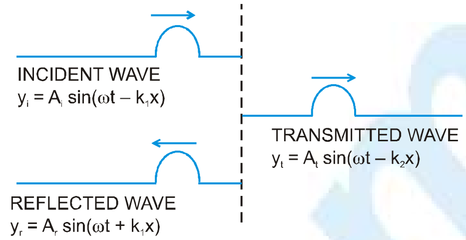


Fig. : Reflection at rarer boundary

Beats :

When two sound waves of nearly equal (but not exactly equal) frequencies travel in same direction, at a given point due to their super position, intensity alternatively increases and decreases periodically. This periodic waxing and waning of sound at a given position is called beats.

Beat frequency = difference of frequencies of two interfering waves

12. Stationary waves or standing waves

When two waves of same frequency and amplitude travel in opposite direction at same speed, their superstition gives rise to a new type of wave, called stationary waves or standing waves.

Formation of standing wave is possible only in bounded medium.

(i) Let two waves are $y_1 = A\sin(\omega t - kx)$; $y_2 = A\sin(\omega t + kx)$ by principle of superposition $y = y_1 + y_2 = 2A\cos kx \sin \omega t$ ← Equation of stationary wave

(ii) As this equation satisfies the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$, it represent a wave.

(iii) **Nodes** → amplitude is minimum : $\cos kx = 0 \Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$

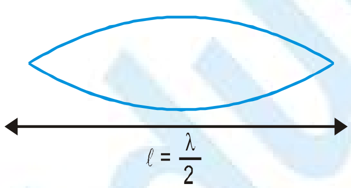
(iv) **Antinodes** → amplitude is maximum : $\cos kx = 1 \Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$

(v) The nodes divide the medium into segments (loops). all the particles in a segment vibrate in same phase but in opposite phase with the particles in the adjacent segment.

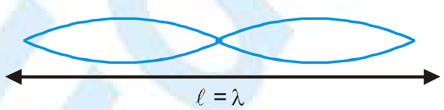
(vi) As nodes are permanently at rest, so no energy can be transmitted across them, i.e. energy of one region (segment) is confined in that region.

13. Transverse stationary waves in stretched string

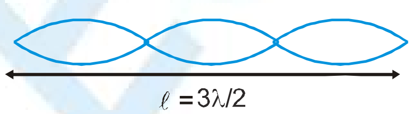
(i) **Fixed at both ends** [fixed end → Node & free end → Antinode]



Fundamental or first harmonic $f = \frac{v}{2l}$

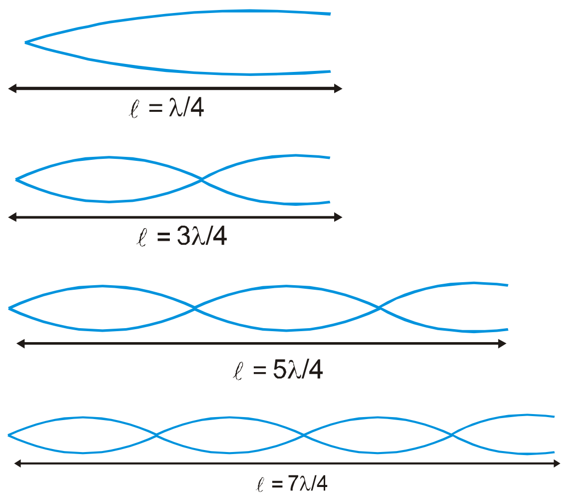


Second harmonic first overtone $f = \frac{2v}{2l}$



Third harmonic first overtone $f = \frac{3v}{2l}$

(ii) Fixed at one end



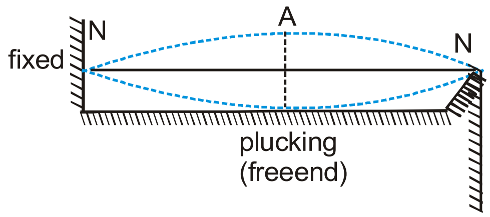
Fundamental $f = \frac{v}{4l}$

Third harmonic first overnote $f = \frac{3v}{4l}$

Fifth harmonic first overnote $f = \frac{5v}{4l}$

seventh harmonic first overnote $f = \frac{7v}{4l}$

14. Sonometer



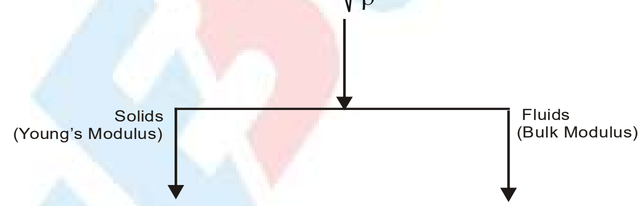
$$f_n = \frac{p}{2l}\sqrt{\frac{T}{\mu}}$$

[p : number of loops]

15. Sound Waves

Velocity of sound in a medium of elasticity E and density ρ is

$$v = \sqrt{\frac{E}{\rho}}$$



$$v = \sqrt{\frac{Y}{\rho}} \qquad v = \sqrt{\frac{B}{\rho}}$$

(i) **Newton's Formula :** Sound propagation is isothermal $B = P \Rightarrow v = \sqrt{\frac{P}{\rho}}$

(ii) **Laplace correction :** Sound propagation is adiabatic $B = \gamma P \Rightarrow v = \sqrt{\frac{\gamma P}{\rho}}$

16. With rise in temperature, velocity of sound in a gas increases as $v = \sqrt{\frac{\gamma RT}{M_w}}$

17. With rise in humidity velocity of sound increases due to presence of water in air.

18. Pressure has no effect on velocity of sound in a gas as long as temperature remains constant.

19. Displacement and pressure wave

A sound wave can be described either in terms of the longitudinal displacement suffered by the particles of the medium (called displacement wave) or in terms of the excess pressure generated due to compression and rarefaction (called pressure wave).

Displacement wave $y = A\sin(\omega t - kx)$

Pressure wave $p = p_0 \cos(\omega t - kx)$

where $p_0 = ABk = \rho A v \omega$

Note : As sound-sensors (e.g. ear or mike) detect pressure changes, description of sound as pressure wave is preferred over displacement wave.

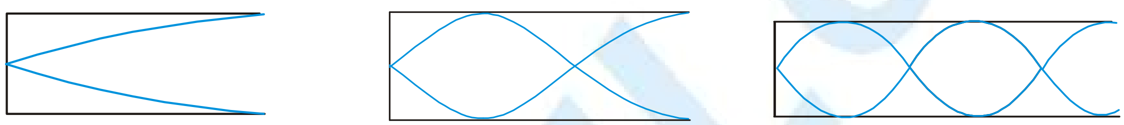
20. The pressure wave is 90° out of phase w.r.t. displacement wave, i.e. displacement will be maximum when pressure is minimum and vice-versa.

21. Intensity in terms of pressure amplitude $I = \frac{p_0^2}{2\rho v}$

22. Vibrations of organ pipes

Stationary longitudinal waves closed end → displacement node, open end → displacement antinode

(i) Closed end organ pipe



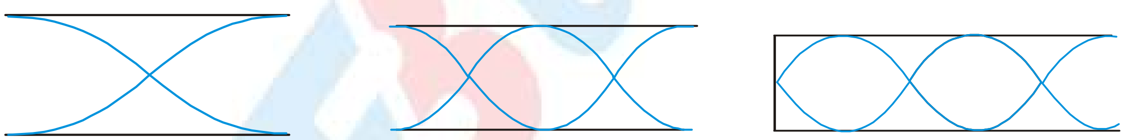
$l = \frac{\lambda}{4} \Rightarrow f = \frac{v}{4l}$ $\quad l = \frac{3\lambda}{4} \Rightarrow f = \frac{3v}{4l}$ $\quad l = \frac{5\lambda}{4} \Rightarrow f = \frac{5v}{4l}$

(a) Only odd harmonics are present

(b) Maximum possible wavelength = 4 ●

(c) Frequency of m[th] overnote = $(2m+1)\frac{v}{4l}$

(ii) Open end organ pipe



$l = \frac{\lambda}{2} \Rightarrow f = \frac{v}{2l}$ $\quad l = \lambda \Rightarrow f = \frac{2v}{2l}$ $\quad l = \frac{3\lambda}{2} \Rightarrow f = \frac{3v}{2l}$

(a) All harmonics are present

(b) Maximum possible wavelength is 2 ●

(c) Frequency of m[th] overnote = $(m+1)\frac{v}{2l}$

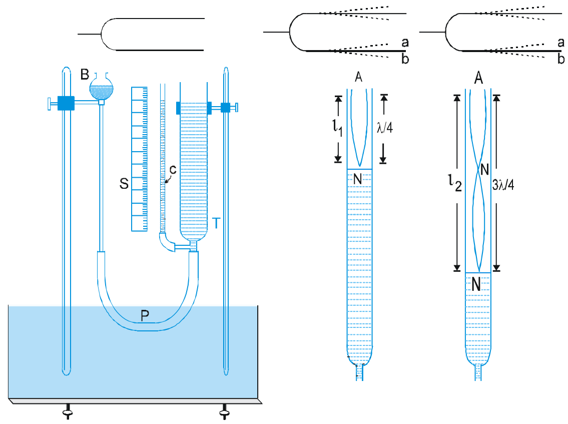
(iii) End correction :

Due to finite motion of air momentum of air molecules in organ pipes reflection takes place not exactly at open end but some what above it so antinode is not formed exactly at free end but slightly above it.

In closed organ pipe $f_1 = \frac{v}{4(l+e)}$ where $e = 0.6R$ (R = radius of pipe)

In open organ pipe $f_1 = \frac{v}{2(l+2e)}$

23. Resonance Tube



Wavelength $\lambda = 2(\bullet_2 - \bullet_1)$ End correction $e = \dfrac{l_2 - 3l_1}{2}$

24. Intensity of sound in decibels

Sound level, $SL = 10\log_{10}\left(\dfrac{1}{10}\right)$

where I_0 = threshold of human ear = 10^{-12} W/m^2

25. Characteristics of sound

(a) Loudness → Sensation received by the ear due to intensity of sound.

(b) Pitch → Sensation received by the ear due to frequency of sound.

(c) Quality (or Timber) → Sensation received by the ear due to waveform of sound.

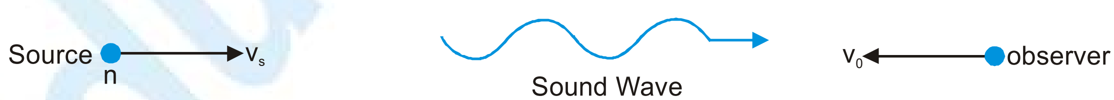
26. Doppler's effect in sound

A stationary source emits wave fronts that propagate with constant velocity with constant separation between them and a stationary observer encounters them at regular constant intervals at which they were emitted by the source.

A moving observer will encounter more or lesser number of waveforms depending on whether he is approaching or receding the source.

A source in motion will emit different wave front at different places and therefore alter wavelength i.e. separation between the wavefronts.

The apparent change in frequency or pitch due to relative motion of source and observer along the line of sight is called Doppler effect.



Observed frequency $n' = \dfrac{\text{Speed of sound wave w.r.t. observer}}{\text{observed wavelength}}$

$$n' = \frac{v + v_0}{\left(\dfrac{v - v_s}{n}\right)} = \left(\frac{v + v_0}{v - v_s}\right)n$$

If v_0, v_s <<< v then $n' \approx \left(1+\frac{v+v_s}{v}\right)n$

$$\text{Mach Number} = \frac{\text{speed of source}}{\text{speed of sound}}$$

27. Doppler's effect in light :

Case I : **Observer** **Light Source**

$$\left.\begin{array}{l}\text{Frequency } \nu' = \left(\sqrt{\dfrac{1+\frac{v}{c}}{1-\frac{v}{c}}}\right)\nu \approx \left(1+\dfrac{v}{c}\right)\nu \\ \text{Wavelength } \lambda' = \left(\sqrt{\dfrac{1-\frac{v}{c}}{1+\frac{v}{c}}}\right)\lambda \approx \left(1-\dfrac{v}{c}\right)\lambda\end{array}\right\}\text{Violet Shift}$$

Case I : **Observer** **Light Source**

$$\left.\begin{array}{l}\text{Frequency } \nu' = \left(\sqrt{\dfrac{1-\frac{v}{c}}{1+\frac{v}{c}}}\right)\nu \approx \left(1-\dfrac{v}{c}\right)\nu \\ \text{Wavelength } \lambda' = \left(\sqrt{\dfrac{1+\frac{v}{c}}{1-\frac{v}{c}}}\right)\lambda \approx \left(1+\dfrac{v}{c}\right)\lambda\end{array}\right\}\text{Red Shift}$$