PERMUTATIONS AND COMBINATIONS

Before going deep into Permutation & Combination, let us figure out a term known as 'factorial'. The product of the numbers starting from 1 up to a number 'n' is known as the factorial of number 'n'.

It means **n! = 1x2x3x4x5x6……..x(n-2)x(n-1)xn**

1! = 1

2! = 1x2 = 2

3! = 1x2x3 = 6

4! = 1x2x3x4 = 24

5! = 1x2x3x4x5 = 120

Key points related to Factorial:

0! & 1! are equal to 1.

We can't find factorial of a negative number.

Application of factorial:

Factorial has the most common application in an **arrangement**. Let us understand how factorial helps in arranging things.

Suppose we have 5 persons and we want to arrange them in 5 vacant places. Then we will start with the first place. We can choose 1 person out of 5 for the first place. We can do that in 5 ways.

Now only 4 seats are vacant and 4 persons are left. We will choose 1 person out of 4 for second place now. We can do that in 4 ways.

In the same manner, for the third place, 3 ways, for the fourth place, 2 ways, and for the last vacant place only 1 way of selection is possible. As we know that we have to do all of these activities, so we will **multiply all these ways** to get the final answer for getting the different ways of arrangement.

So total ways = 5x4x3x2x1 **which is 5!**

= 120

Or we can say that whenever we have to arrange 'n' things at 'n' places then total arrangements that can be made will always be equal to n!

Q.1) In how many ways can the letters of the word PATNA be rearranged?

Solution: PATNA has total 5 words. So we will arrange 5 letters at 5 places in 5! = 120 ways.

But in this question, A is coming twice. Whenever any letter is more than once in the word, then we have to divide by the number of repetitions of the word. So we have to divide the total 120 ways by 2!= 2.

So total different words that can be made will be 120/2 = 60.

Direct answer: 5!/2! = 60

Q.2) How many different words can be made using letters of PATNA starting with P?

Solution: PATNA has total 5 words. According to the question, P is fixed at first place, so we will arrange remaining 4 letters at 4 places in 4! = 24 ways. But in this question, A is coming twice, so we have to divide the total 24 ways by 2!= 2. So total different words starting from P will be 24/2 = 12.

Direct answer: 4!/2! = 4*3 = 12

- **Whenever we have to choose certain things from a group and no arrangement is done. In that case combination comes into picture. So let us see concept of combination.**

Combination

In combination, we select the things at random & check out the different possible ways of selection. So this is a one step process. Combination is also known as collection. The formula used for combination is ${}^{n}C_{r}$

${}^{n}C_{r} = n! / [r! \times (n-r)!]$

${}^{n}C_{r} = [n \times (n-1) \times (n-2) \times (n-3) \times \ldots\ldots x(n-r+1) \times (n-r) X \ldots\ldots x\ 1] / [1x\ 2 \times 3 \ldots\ldots \times r] X [(n-r) x \ldots\ldots 3 \times 2 \times 1]$

${}^{n}C_{r} = [n \times (n-1) \times (n-2) \times (n-3) \ldots\ldots \times (n-r+1)] / [1x\ 2 \times 3 \ldots\ldots \times r]$

For example: ${}^{12}C_{2} = 12!/ [2! X (12-2)!] = 12!/ (2! X 10!) = [12 \times 11] / [1 \times 2] = 66$

${}^{5}C_{2} = [5 \times 4] / [1 \times 2] = 10$

${}^{n}C_{r} = {}^{n}C_{(n-r)}$

For example: ${}^{5}C_{3} = [5 \times 4 \times 3] / [1 \times 2 \times 3] = [5 \times 4]/[1 \times 2] = {}^{5}C_{2} = 10$

${}^{10}C_{7} = {}^{10}C_{3} = [10 \times 9 \times 8]/[1 \times 2 \times 3] = 120$

Q.3) In a class there are 4 boys and 5 girls. In how many different ways a class monitor can be chosen?

Solution: As we can clearly see that we have to choose a student from total 9. So we will use combination concept here which will give us the answer as ${}^{9}C_1 = 9/1 = 9$

Q.4) In a class there are 4 boys and 5 girls. In how many different ways a boy and a girl can be selected for group leaders of two groups?

Solutions: We have to choose a boy from 4 boys and a girl from 5 girls for two groups.

So total ways of selection = ${}^{4}C_1 x {}^{5}C_1 = 4x5 = 20$

Q.5) In how many different ways a cricket team can be selected from total 16 players?

Solution: We need to select 11 players from total 16 players.

So the answer will be ${}^{16}C_{11} = 16!/5!*(16-5)! = 16!/5!*11! = (16*15*14*13*12)/(1*2*3*4*5) = 4368$

Q.6) An urn contains 5 red and 3 blue balls. In how many different ways, 2 red and 1 blue balls can be drawn?

Solution: The urn contains 5 red and we want 2 red balls. So ways of selecting red balls $={}^{5}C_2 = 10$

Similarly ways of selecting 1 blue ball from 3 blue balls = ${}^{3}C_1 = 3$

So total ways to select 2 red and 1 blue ball will be = 10*3 = 30

Q.7) In how many different ways a team of 11 can be selected from 15 players if 2 particular players are never selected?

Solution: It is given that 2 particular players are never selected. So we will do selection from rest of the players which means we will select 11 players out of 13 players.

So total ways of selection = ${}^{(15-2)}C_{11} = {}^{13}C_{11} = {}^{13}C_2 = (13*12)/(1*2) = 78$

Q.8) In how many different ways a team of 11 can be selected from 15 players if 2 particular players are always selected?

Solution: It is given that we have to select two particular players always which means that we have choice of selection only for remaining 9 players and the possible options are only 13.

So total ways of selection = ${}^{(15-2)}C_{(11-2)} = {}^{13}C_9 = {}^{13}C_4 = (13*12*11*10)/(1*2*3*4) = 715$

- **Whenever we have to choose certain things from a group and arrangement of those chosen things is to be done. In that case permutation comes into picture. So let us see concept of permutation.**

Permutation

In permutation, we select the things and then arrange them to check out different possible ways of arrangement. So basically permutation is a two-step process.

The formula used for permutation is ${}^{n}P_{r} = n!/(n-r)!$

Suppose we have 5 persons and we have to arrange them on 3 vacant places. Then first of all, we will choose 3 persons from 5. We can do that in ${}^{5}C_{3}$ different ways. After choosing 3 persons, we will have to arrange them on the 3 vacant places, for that we will use factorial concept. The total ways to arrange 3 persons on 3 places are 3!

So total ways to arrange 3 persons from total 5 on 3 vacant places will be:

${}^{5}C_{3}*3! = {}^{5}C_{2}*3! = 5!/(2!*3!) * 3! = 5!/2! = 60$ ways.

Q.9) A wicket-keeper and a bowler are to be chosen out of a team having 11 players. In how many different ways we can do this?

Solution: First of all, we will select 2 players from total 11 players. The ways of selection are ${}^{11}C_{2} = (11*10)/(1*2) = 55$. After doing selection, we can arrange 2 players on 2 different positions in 2! = 2 ways. So total ways of selecting a wicket-keeper and a bowler = ${}^{11}C_{2}*2! = 55*2 = 110$

Direct answer: ${}^{11}P_{2} = 110$

Q.10) In how many ways can the letters of the word EQUATION be arranged so that all the vowels come together?

Solution: In word 'EQUATION', we have 5 vowels (E,U,A,I,O) and 3 consonants (Q,T,N). According to question, all five vowels should come together so we will assume these 5 vowels to take one place and other 3 consonants will be arranged on 3 places, so total 4 places.

So ways to arrange these on 4 places will be 4! = 24

One important thing is that we can arrange the vowels order as well and we can do that in 5!= 120 ways.

So total ways = 24*120 = 2880

Direct answer: 4!*5! = 24*120 = 2880

Q.11) There are 7 candidates for 4 different posts. In how many ways we can fill the posts?

Solution: First of all, we will select 4 candidates out of total 7 candidates. The ways of selection are,

${}^{7}C_{4} = {}^{7}C_{3} = (7*6*5)/(1*2*3) = 35$

After doing selection, we can arrange 4 candidates for 4 different posts in 4! ways = 24 ways.

So total possible number of ways to fill the posts = 35*24 = 840

Direct answer: ${}^{7}P_{4}$ = (7*6*5*4) = 840

Q.12) Twenty students are participating in a race. In how many ways can the first three prizes be won?

Solution: First of all, we will select 3 candidates from total 20 candidates. The ways of selection are,

${}^{20}C_{3}$ = (20*19*18)/(1*2*3) = 1140

After doing selection, we can arrange 3 candidates on 3 positions in 3! = 6 ways.

So total possible number of ways in which the first three prizes can be won = 1140*6 = 6840.

Direct answer: ${}^{20}P_{3}$ = (20*19*18) = 6840 ways

Key points related to Permutation & Combination:

- Whenever we want to **arrange n things at n places**, we have total n! ways of arrangement.
- Whenever we have to **select r things out of n,** we have total ${}^{n}C_{r}$ ways of selection.
- Whenever we have to **select r things from n and then arrange those r things at r places,** we have total ${}^{n}P_{r}$ ways.
- **${}^{n}C_{r}$ = n! / [r! x (n-r)!]**
- **${}^{n}C_{r}$ = $nC_{(n-r)}$**
- **${}^{n}P_{r}$ = n!/(n-r)!**