Nuclei

13

INTRODUCTION

* Rutherford's α-scattering experiment established that the mass of atom is concentrated within small positively charged region at the centre which is called 'nucleus'.
* Radius of nucleus (R)

  $R = R_0A^{1/3}$

  Where A = mass number $R \propto A^{1/3}$

  volume of nucleus = $\frac{4}{3}\pi R^3$

  $\therefore$ Volume $\propto$ A

Solved Examples

Ex.1 The ratio of the radii of the nuclei $^{27}_{31}Al$ and $_{52}Te^{125}$ is approximately -

Sol. $R_{A\ell}/R_{Te} = \frac{(27)^{1/3}}{(125)^{1/3}} = \frac{3}{5} = \frac{6}{10}$

Ex.2 The radius of the $_{30}Zn^{64}$ nucleus is nearly (in fm)

Sol. $R = R_0\ A^{1/3} = 1.2 \times 10^{-15} \times (64)^{1/3}$

$= 1.2 \times 10^{-15} \times 4 = 4.8$ fm

* $A \propto R^3$
* A = Nucleon number or mass number
* Any element X with mass number A and charge number Z can be represented by $_ZX^A$ or ^A_ZX.

  Number of neutron = A – Z

  mass number = A = P + N
* 1 amu = $\frac{1}{12}$ th mass of 12gm of $_6C^{12}$ atom.

Solved Examples

Ex.3 How many electrons, protons, and neutrons are there in a nucleus of atomic number 11 and mass number 24 ?

Sol. Number of protons in nucleus = atomic number = 11

Number of electrons = number of protons = 11

Number of neutrons = mass number A – atomic number Z

N = 24 – 11 = 13

Ex.4 How many electrons, protons and neutrons are there is a 6gm of $_6C^{12}$.

Sol. $\because$ 6 gm of $_6C^{12}$ contains atoms = $\frac{6\times10^{23}}{2}$ and each atom of $_6C^{12}$ contains electron, protons and neutrons = 6, 6, 6

$\therefore$ No. of electron, protons and neutron in 6 gm of ${}_6C^{12}$ = 18×10^{23}, 18×10^{23}, 18×10^{23}

Isotopes → Those set of nuclei containing same number of protons but different numbers of neutrons.

Ex. ${}_6C^{12}$ and ${}_6C^{14}$

${}_1H^1$, ${}_1H^2$, ${}_1H^3$

${}_{92}U^{234}$, ${}_{92}U^{235}$, ${}_{92}U^{238}$

${}_{10}Ne^{20}$, ${}_{10}Ne^{22}$

Isobars → Those set of nuclei's containing same mass number but different atomic number are known as isobars.

Example : ${}_6C^{14}$ and ${}_7N^{14}$

Isotones → Those set of nuclei's containing same numbers of neutrons are known as isotones.

Example : ${}_6C^{14}$ and ${}_8O^{16}$

Mirror nuclei → Nuclei with same mass number A but with proton and neutron numbers interchanged i.e. the number of protons in one is equal to the number of neutrons in the other are called mirror nuclei.

Example : ${}_4Be^7$ (Z = 4, N = 3) and ${}_3Li^7$ (Z = 3, N = 4)

* $\dfrac{\text{Radius of atom}}{\text{Radius of nucleus}} \cong 10^5$
* $\dfrac{\text{Volume of atom}}{\text{Volume of nucleus}} \cong 10^{15}$
* Density of nucleus (ρ)

(a) ρ is of the order of $\simeq 10^{17}$ kg/m^3 for almost all nuclei's.

* $\rho = \dfrac{\text{mass of nucleus}}{\text{volume of nucleus}}$
* Nuclear density is independent of its mass number.

Solved Examples

Ex.5 Nuclear radius of ${}_8O^{16}$ is 3×10^{-15} m. Find the density of nuclear matter.

Sol. Use ρ = mass/volume

$$= \frac{1.66 \times 10^{-27} \times 16}{(4/3)\pi(3 \times 10^{-15})^2}$$

$= 2.35 \times 10^{17}$ kg m^{-3}

* Neutron is electrically neutral particle and not deflected by electric or magnetic field.
* Neutron do not ionize gases
* Angular momentum of electron = $\pm \dfrac{1}{2}\left(\dfrac{h}{2\pi}\right)$
* When a neutron strikes with the atoms it penetrates in to the atoms. This important property of neutron is used for nuclear fission.
* Neutron was discovered by Chadwick.
* Proton was discovered by Goldstein.

Nuclear stability →

For stable nuclides the following points many be noted.

The light nuclides have almost equal number of proton and neutrons. In general light nuclei A < 20 contains approximately equal number of neutrons and protons.

Example :

${}^4_2He(Z = 2,\ N = 2)$, ${}_6C^{12}(Z = 6,\ N = 6)$,

${}_4Be^9(Z = 4,\ N = 5)$

In heavier stable nuclides, there are more neutrons than protons. The heaviest having about 50 percent more.

Example :

${}_{47}Ag^{107}(Z = 47,\ N = 60)$, ${}_{79}Au^{197}(Z = 79,\ N = 118)$,

${}_{82}Pb^{208}(Z = 82,\ N = 126)$

* Most of the stable nuclides have both an even number of protons and even number of neutrons. Examples of the most stable nuclides are

${}^4_2He(Z = N = 2)$, ${}^{16}_8O(Z = N = 8)$, ${}^{28}_{14}Si(Z = N = 14)$,

${}^{56}_{26}Fe(Z = 26, N = 30)$

For unstable nuclides the following points may be noted.

Nuclei with Z > 83 spontaneously disintegrate with emission of α or β particles.

A heavy nuclide disintegrated by α-decay

Nuclear forces →

Forces acting between n – n, n – p and p – p inside the nucleus is called nuclear forces.

They are short ranging forces (10^{-15} m – 10^{-14}m)

They are most strongest in nuclear ranges and their strength becomes zero when distance between nucleus becomes of the order of 10^{-12} m.

Order of strength of various nuclear forces.

$F_{nuclear} : F_{FM} : F_{weak} : F_{grav.} :: 10^{39} : 10^{36} : 10^{33} : 1$

Nuclear force are charge independent or they act alike between various nucleons.

$F_{n-n} = F_{n-p} = F_{pp}$

Net force between the nucleon

$F_{n-n} = F_{n-p} > F_{pp}$

Nuclear forces are non-central. The nuclear force depends on the relative spin orientation of nucleons.

MASS DEFECT

* When proton and neutron combines together in order to from nucleus, then in this process some mass gets lost and this lost mass is known as mass defect (Δm). More is the mass defect, more is presumed to be stability of product nuclei.
* Mathematically :

Δm = Mass of nucleons – mass of nucleus

= A – M

= $[Zm_p + Nm_n] - M$

= $[Zm_p + (\Delta - Z) m_n]$ – [mass of atom - mass of electron orbiting in circular orbits]

* Packing fraction (f) = mass defect per nucleons

$$= \frac{\Delta m}{A} = \frac{M - A}{A} = \frac{[Zm_p + (A - Z)m_n] - M}{A}$$

More the negative value of packing fraction, more in the stability of product nuclei.

Solved Examples

Ex.6 Consider the decay of radium -226 atom into an alpha particle and radon -222. Then, what is the mass defect of the reaction -

Mass of radium – 226 atom = 226.0256 u

Mass of radon - 222 atom = 222.0715 u

Mass of helium - 4 atom = 4.0026 u

Sol. Mass defect = Δm = M (Ra 226) – M (Rn 222) – M (α) 226.0256 – 222.0175 – 4.00026 = 0.0053 u

NUCLEAR BINDING ENERGY

* In nucleus, proton and neutron are contained in very small sphere of radius of the order 10^{-15}m. At such small distance, two proton exerts a very large force of repulsion on each other. Therefore certain amount of energy needed to bind the nucleus. This energy is known as binding energy.
* When nucleus is formed, the mass of nucleons forming it decreases. The mass defect supplies the required binding energy.
* The energy equivalent to mass defect is called binding energy.

By Einstein mass energy relation Binding energy = (Mass defect) × (velocity of light)^2 = $\Delta E = (\Delta m)c^2$

Here Δm : mass defect

Binding energy of a nucleus may also be defined as the amount of work required to separate the nucleons at infinite distance.

Binding energy/nucleon : Binding energy per nucleon

$$= \frac{\Delta E}{A} = \frac{\Delta mc^2}{A}$$

It is the measure of stability of nucleus.

Greater the binding energy per nucleons greater will be nucleus stability

Solved Examples

Ex.7 If mass equivalent to one mass of proton is completely converted into energy then determine the energy produced

Sol. $E = mc^2$

$= (1.66 \times 10^{-27})(3 \times 10^8)^2$ J

$= 1.49 \times 10^{-10}$ J

$= \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-13}}$ MeV = 931.49 MeV

1 amu = 931.49 MeV

Ex.8 If mass equivalent to one mass of electron is completely converted into energy then determine the energy liberated.

Sol. $E = mc^2$

$= (9.1 \times 10^{-31})(3 \times 10^8)^2$ J

= 0.51 meV

Ex.9 If the mass defect in the formation of helium from hydrogen is 0.5% then the energy obtained, in KWH, in forming helium from 1kg of hydrogen will be-

Sol. $\Delta E = \Delta mC^2$

$\Delta m = \frac{0.5}{100} \times /kg = 0.005$ kg

$C = 3 \times 10^8$ m/s

$\Delta E = 0.005 \times (3 \times 10^8)^2$

$\Delta E = 4.5 \times 10^{14}$ J or watt-sec

$\Delta E = \frac{4.5 \times 10^{14}}{60 \times 60} = 1.25 \times 10^{11}$ watt hour

$\Delta E = 1.25 \times 10^8$ KWH

Ex.10 Binding energy per nucleon of an α-particle from the following data :

Mass of the helium nucleus = 4.001265 amu

Mass of proton = 1.007277 amu

Mass of neutron = 1.00866 amu

(1amu = 931.4812 MeV)

Sol. Mass of two protons = 2 × 1.007277 = 2.014554 amu

Mass of two neutron = 2 × 1.008666 = 2.017332 amu

Total initial mass of two proton and neutrons = 2.014554 + 2.017332 = 4.031886 amu

Mass defect Δm = 4.031816 – 4.001265, Δm = 0.030621 amu

$\therefore$ Binding energy of α particle = 0.030621 × 931.4812 = 28.5221 MeV

Binding energy of nuclueon = 28.5221/4 = 7.10525 MeV

BINDING ENERGY CURVE

Nature of the curve :

The curve plotted between binding energy per nucleon and mass number of atom is known as binding energy curve.

The curve shows that BE per nucleon first increases, attains a maximum value and then decreases

The average binding energy per nucleon in a heavy nucleus is approximately 8 MeV.

Positive value of BE per nucleon for most nuclei indicates that all the nucleus are stable.

For $30 \leq A \leq 170$, the nuclei are more highly bound than those for A > 170 or A < 30.

Light nuclei can fuse and form a heavier nuclei, and release energy (fusion)

Stable nuclei have N/Z ratio = 1.3 to 1.4

Nuclei with even A, even Z are usually stable and most abundant. Nuclei with odd A and odd Z are unstable, in general. Nuclei with odd Z and even A are also unstable with exceptions like ^2H, ^6Li, ^{10}B, ^{14}N, which are stable.

A nucleus may be unstable in its ground state (may beta decay for example). But an atom in its ground state is always stable.

The heaviest stable nuclides is $^{209}_{83}Bi$ (Half life > 2 × 10^{16} year greater than the age of the universe). All nuclide with Z > 83 are unstable.

NUCLEAR FISSION

It was discovered by scientist Otto Hann and F. Strassman

Process of splitting up of heavier nuclei by the bombardment of neutron into two almost equal weight nucleus is known as nuclear fission and in this process huge energy is liberated on account of mass defect.

Natural uranium is obtained in the following proportions :

${}_{92}U^{234} : {}_{92}U^{275} : {}_{92}U^{236} :: 0.006 : 0.7 : 99.3$ (approx)

U^{238} is not used in fission as its fission is possible only by neutron of energy approximately 1MeV.

Fission of U^{235} is possible by slow neutrons of energy (~ 1eV) or even by thermal neutrons of energy (0.025eV)

Fission reaction →

${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236}$ (unstable nucleus)

(a) Because of unstability, ${}_{92}U^{235}$ again fission into two stable nucleus Ba and Kr with the three other neutrons.

$${}_{92}U^{236} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + (200\,\text{MeV energy})$$

(b) During $_{92}U^{235}$ fission we don't get only isotopes of Ba and Kr but we can get other type of pairs also like

$$_{92}U^{235} + _0n^1 \longrightarrow _{92}U^{236} \longrightarrow _{38}Sr^{95} + _{54}$$

(Unstable)

$$Xe^{139} + 2_0n^1 + 200 \text{ (Mev energy)}$$

(c) By fission of one $_{92}U^{235}$ we get approximately 200MeV energy

By fission of $_{92}U^{235}$, on an average 2.5 neutrons are liberated.

Most part of liberated energy is in the form the kinetic energy of fragmented nuclei.

Neutrons emitted from nuclear fission have energy equal to 2.0 MeV. These fast neutrons can escape from the reaction so as to proceed the chain reaction they are need to slow down.

CHAIN REACTION

When Uranium is bombard by neutrons, each Uranium nucleus is broken into two nearly equal fragmented and along with it huge energy and two or more (avg. 2.5 neutron) fresh neutrons are liberated. Under formable condition, three neutrons fission other uranium nuclei in the same way. Thus a chain of nuclear fission, is established which continues until the whole of Uranium is consumed.

There are following main difficulties in establishing a chain reaction -

(a) Leaking or escaping of neutron from the system

(b) Absorption of neutron by unbroken nuclei present in the system

(c) Absorption of Neutrons by U^{238}

Critical mass : Minimum mass of Uranium system or any other system for which chain reaction is possible is called critical mass. It is 10 kg for U^{235}.

The necessary condition for nuclear fission is that there must be at least one neutron which takes part in nuclear fission. This condition is defined by multiplication factor or reproduction factor (K)

$$K = \frac{\text{No. of neturons present in a step of neuclear fission reaction}}{\text{No. of neutrons present in one step before the above reaction}}$$

If K < 1 then chain reaction will finally stop because in further step no. of neutrons decreases.

If $K \geq 1$ chain reaction will continue

No. of neutrons emitted during fission reaction depends upon volume of system (or $\propto r^3$)

Rate of leaking of neutrons from the system is directly proportional to area of system ($\propto r^2$)

TYPES OF CHAIN REACTIONS

(a) controlled (b) uncontrolled

1. Controlled chain reaction :

* Rate of reproduction in controlled reaction is equal to one.
* K = 1
* Energy liberated in this types of reaction is always less than explosive energy and it proceed with a certain velocity.
* This type of energy is used in constructive work.
* Nuclear reactor uses this principle exactly.

Nuclear Reactor :

Nuclear reactor is device in which a self sustaining controlled chain reaction is established to produce energy.

In this reactor at least one neutron liberated in a step can fission next nucleus it is control by artificial manner.

A modern reactor has following important parts :-

Fission substance (Fuel) : This substance is used for fission. Natural uranium or U^{235} rich uranium are used for this purpose. Other fuels are Pu^{239} and U^{233}. Quantity of fissionable materials should be equal to the critical mass.

Moderator : It is used to slow down the fast neutrons and changes to thermionic neutron. Heavy water, graphite or beryllium oxide is used for this purpose. Heavy water is best moderator.

Neutron's source : It is used to start the chain reaction

Controller : Cadmium rods are used as controller to control the rate of fission in the reaction. Cd is very good absorber of neutrons.

Coolant : The energy released inside the reactor in the form of heat is removed by coolant. For this purpose air, ice, cold water or CO_2 is flown in the reactor.

Shield : Various types of harmful intense rays are emitted from these reactor so it is surrounded by 7 to 8 ft. thick concrete walls.

Uses :

(i) In energy production

(ii) In manufacturing of radioactive isotopes which are used in agriculture, medical, industry, biological & scientific discoveries etc.

(iii) In manufacturing of Pu^{239} which is used in atom bomb

Uncontrolled chain reaction :

There is no control over this type of nuclear reaction

In this type of reaction more than one neutron takes part into reaction. Due to this speed nuclear fission increases.

For this type of reaction. Reproduction factor (K) > 1

A huge amount of energy is liberated in this type of reaction.

In atom bomb, uncontrolled chain reaction takes place.

Atom bomb :

This is based on uncontrolled chain reaction.

In atom bomb manufacturing two or more parts of U^{235} or Pu^{239} is used whose mass slightly more than half of critical mass. These parts are arranged in a sphere of metal.

T.N.T is used to separate these parts.

These parts are totally safe till these parts are separate.

When these individual sizes are joined together, their combined size becomes larger than the critical size.

Thus speed of fission enhances & more energy is liberated.

Due to explosion in fraction of minute the temperature and atmospheric pressure multiplies and reaches to 10^7 K.

First nuclear test for peace work was at Pokhran in Rajathan in 18th may 1974.

NUCLEAR FUSION

Process of combining two light weight nucleus into one heavy weight nucleus at high temperature and pressure is called nuclear fusion.

This reaction is possible only at high pr ($\cong 10^6$ atm) and high temperature $(\cong 10^7 - 10^8 \,°C)$

The vast amount of energy emitted in this reaction is also called thermonuclear energy and this reaction is called thermonuclear reaction.

Four hydrogen atom combined into one $_2He^4$ nucleus and 24.7 MeV energy emits in the whole reaction.

$$4\ _1H^1 \longrightarrow\ _2He^4 + 2\ _{-1}e^0 + 24.7 \text{ MeV}$$

Hydrogen bomb is also based on above principle and it is much more disastrous than the atom bomb.

Hydrogen bomb explosion needs atom bomb explosion so that $10^7 - 10^8$ K temperature could be reached for fusion reaction.

Fusion reactor is still not available to get energy from nuclear fusion.

Solved Examples

Ex.11 In a nuclear fusion process 4 hydrogen atom combine to form a helium atom and two positrons. Calculate the energy released in the process.

Sol. Initial mass = 4 × 1.007825

= 4.091300 amu

Total final mass = 4.003702 amu

Decreases in mass = 4.031300 – 4.003702

= 0.026598 amu

1 amu = 931 MeV

Energy released = 0.027598 × 931 = 25.69 Mev

POINTS TO REMEMBER

Rutherford discovered nucleus.

Neutron was discovered by chadwick.

Packing fraction = $\frac{\Delta m}{A} = \left(\frac{M-A}{A}\right)$

nuclei with negative packing fraction are more stable.

Greater than binding energy per nucleon, greater is the stability.

Nuclear fission was discovered by Otto Hahn & Strassmann.

Nuclear forces are the strongest known force in nature.

Relative strength of the gravitational, coulomb's and nuclear force is

$F_g : F_e : F_n :: 1 : 10^{35} : 10_{36}$

The high density of the nucleus suggests that the nucleus is compact.

The fist nuclear reactor was made by fermi.

Neutrons having energy ~ 0.03 are called thermal neutrons.

Critical mass of uranium is 10kg

Nuclear reactor is based on controlled chain reaction.

The nuclear forces between two protons, two neutrons and a neutron and a proton have the same magnitude.

Fusion takes place at high temperature of 10^7 ºC.

The fusion of nuclei is an uncontrolled process.

Density of the nuclei of all the atoms is same.

The actual disconvery of pi-measons was made from the study of cosmic radiation.

Neutron is neutral and its spin angular momentum is $\left(\frac{1}{2}.\frac{h}{2\pi}\right)$

Escape of neutron is proportional to r^2.

$_1H^2 +_1 H^2 \rightarrow_2 He^2 + n + 3.27 \text{ MeV}$

$_1H^2 +_1 H^3 \rightarrow_2 He^4 + n + 17.59 \text{ MeV}$

To achieve fusion in laboratory a Tokomak is used. It is a device to confine plasma (collection of deuterium, tritium nuclei, in this case)

Research atomic reactions -

Apsara - This is 1MW reactor and was made in Bombay. An alloy of Uranium and aluminum is used as fuel in it. This is also known as swimming pool type reactor.

Cirus - It is a 40 MW reactor, made in collaboration with canada and is used to produce radioisotopes.

Zerlina - OMW reactor

Purnima - OMW reactor

Power reactors -

In Tarapur (Maharashta) 400 MW

In Rana pratap sagar (Rajesthan) 400 MW

In Kalpakkam (Madras) 220 MW

In Narora (U.P) 200 MW

In Kaiga (Karnataka) 200 MW

In Kakapar (Gujarat) 200 MW

Fusion reactors are better than fission reactors because no unwanted radioactive. substances are produced in them and these are available in abundance.

Difference between nuclear fission and nuclear fusion.

Nuclear fission

Nuclear fusion

Neutrons are required for it.

Protons are required for it.

It is possible at normal temperature and

It is possible at high temperature and

pressure.

pressure.

For this the energy released per nucleon

For this $\frac{\Delta E}{A} = \frac{27}{4} = 6.75$

$\frac{\Delta E}{A} = \frac{200}{236} \sim 0.8 \text{MeV}$

The fissionable materials are expensive.

The materials used in it are cheap (e.g heavy water)

Difference between atom bomb and hydrogen bomb-

Atom bomb

Hydrogen bomb

It is based on the fission process

It is based on fusion process

In this critical size is important

There is no limit to critical size

In this the explosion is possible at

In this high temperature and pressure are required

normal temperature and pressure are required.

In this the harmful radiations

In this the harmful radiations are not

are produced

produced

In all nuclear reactions following quantities remains conserved

Mass - energy

Linear momentum

Angular momentum

Nucleon number

Charge number

Enriched uranium - The uranium, in which the fraction of $_{92}U^{233}$ is increased rom 0.7% to 2.3% is known as enriched uranium.

Solved Examples

Ex.12 In the nuclear reaction, $_{92}U^{238} \rightarrow {}_ZTh^A + {}_2He^4$, the value is of A and Z are -

Sol. A = 238 – 4 = 234, Z = 92 – 2 = 90

Ex.13 The mass of helium nucleus is less than that of its constituent particles by 0.03 amu. The binding energy per nucleon of $_2He^4$ nucleus will be

Sol. $\Delta m = 0.03$ amu, $A = 4 \Rightarrow \Delta E = \frac{\Delta m \times 931}{A} \Rightarrow \Delta E = \frac{0.03 \times 931}{4} = 7 MeV$

Ex.14 If the binding energy of deuterium is 2.23 MeV, then the mass defect will be - (in amu)

Sol. $\because \Delta E = \Delta m \times 931$ MeV $\Rightarrow \Delta m \frac{\Delta E}{931} = \frac{2.23}{931}$ = 0.0024 amu

Ex.15 Energy of each photon obtained in the pair production process will be, if the mass of electron or positron is 1/2000 amu

Sol. $\because$ equivalent mass of each photon = 1/2000 amu

$\because$ 1amu = 931 MeV

$\therefore$ Energy of each photon = $\frac{931}{2000} = 0.465$ MeV

Ex.16 The binding energies of deutron ($_1H^2$) and α- particle ($_2He^4$) are 1.125 and 7.2 MeV/nucleon respectively. In the process $_1H^2 + {}_1H^2 \rightarrow {}_2He^4$, amount of energy transferred is & which nucleus is more stable ?

Sol. Binding energy of deutron = 2 × 1.125 = 2.25 MeV

$\therefore$ Total Binding energy ($_1H^2 + {}_1H^2$) = 2.25 + 2.25 = 4.5 MeV

Binding energy of $_2He^4$ is = 4 × 7.2 = 28.8 MeV

Thus $_2He^4$, in comparison to ($_1H^2 + {}_1H^2$), has (28.8 – 4.5) = 24.3 MeV more energy . So 24.3 MeV energy will be liberated.

Also higher the binding energy per nucleon for a nucleus, more stable is the nucleus so $_2He^4$ is more stable

Ex.17 Deuterium is an isotope of hydrogen having a mass of 2.01470amu. Find binding energy in MeV of this isotope

Sol. Deuterium, the isotope of hydrogen consists of one proton & neutron. Therefore mass of nuclear constituents of deuterium = mass of proton + mass of neutron

= 1.00759 + 1.00898 = 2.01657 amu

mass of nucleus of deuterium = 2.01470 amu

Mass defect = 2.01657 – 2.01470 = 0.00187 amu

Binding energy = ΔE = 0.00187 × 931 MeV = 1.741 MeV.

Ex.18 The binding energy per nucleon for $_3Li^7$ will be, if the mass of $_3Li^7$ is 7.0163 amu.

Sol. $E = \frac{\Delta E}{A} = \frac{\Delta m \times 931}{A} MeV$

$\Delta m = (3m_p + 4m_n)$ – mass of Li^7

$\Delta m = (3 \times 1.00759 + 4 \times 1.00898) - 7.01653$

$\Delta m = 0.04216$ amu

$\Delta E = \frac{0.04216 \times 931}{7} = \frac{39.25}{7} = 5.6$ MeV

Ex.19 Sun radiates energy in all direction. The average energy received at earth is 1.4 KW/m^2. The average distance between the earth and the sun is 1.5×10^{11} m. If this energy is released by conservation of mass into energy, then the mass lost per day by sun is approximately (use 1 day = 86400 sec)

Sol. The sun radiates energy in all directions in a sphere. At a distance R, the energy received per unit area per second is 1.4KJ (given). Therefore the energy released in area $4\pi R^2$ per sec is $1400 \times 4\pi R^2$ joule the energy released per day

$= 1400 \times 4\pi R^2 \times 86400$ J

where $R = 1.5 \times 10^{11}$ m, thus

$\Delta E = 1400 \times 4 \times 3.14 \times (1.5 \times 10^{11})^2 \times 86400$

The equivalent mass is $\Delta m = \Delta E/C^2$

$$\Delta m = \frac{1400 \times 4 \times 3.14 \times (1.5 \times 10^{11})^2 \times 86400}{9 \times 10^{16}}$$

$\Delta m = 3.8 \times 10^{14}$ kg

Ex.20 The energy released per fission of uranium 235 is about 200 MeV . A reactor using U-235 as fuel is producing 1000 kilowatts power. The number of U-235 nuclei undergoing fission per sec is, approximately -

Sol. The energy produced per second is $= 1000 \times 10^3$

$J = \frac{10^6}{1.6 \times 10^{-19}}$ eV $= 6.25 \times 10^{24}$ eV

The number of fissions should be, thus number

$= \frac{6.25 \times 10^{24}}{200 \times 10^6} = 3.125 \times 10^{16}$

Ex.21 Calculate power output of $_{92}U^{235}$ reactor if it takes 30 days to use up 2kg of fuel, and if each fission given 185 MeV of usable energy.

Sol. No . of atoms in 2kg $_{92}U^{235} = \frac{2}{235} \times N_A = \frac{2}{235} \times (6.02 \times 10^{23}) = 5.12 \times 10^{24}$

Fission rate $= \frac{5.12 \times 10^{24}}{30 \times 24 \times 60 \times 60} = 1.975 \times 10^{18}$ per sec

Usable energy per fission = 185 MeV

$\therefore$ Power output $= (185 \times 10^6)(1.975 \times 10^{18})(1.6 \times 10^{-19})$ watt $= 58.4 \times 10^6$ watt = 58.46 MW

Ex.22 On disintegration of one atom of U^{235} the amount of energy obtained is 200 MeV. The power obtained in a reactor is 1000 KW. Calculate atoms disintegrated per second in the reactor & decay in mass per hour.

Sol. Power received from the reactor, m

$P = 1000$ KW $= 1000 \times 1000$ W $= 10^6$ J/s

$P = \frac{10^6}{1.6 \times 10^{-19}}$ eV/sec or

$P = 6.25 \times 10^{18}$ MeV/sec

$\therefore$ Number of atoms disintegrated per sec =

$\frac{6.25 \times 10^{18}}{200} = 3.125 \times 10^{16}$

Energy released per hour $= 10^6 \times 60 \times 60$ joule

mass decay per hour $= \Delta m = \frac{\Delta E}{C^2}$

$\Rightarrow \Delta m = \frac{10^6 \times 60 \times 60}{(3 \times 10^8)^2} \Rightarrow \Delta m = 4 \times 10^{-8}$ kg

Ex.23 A star initially has 10^{40} deuterons. It produces energy via the processes $_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + P$ and $_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$. If the average power radiated by the star is 10^{16} W, in how much time the deuteron supply of the star get exhausted.

Sol. Adding the two processes, we get

$3\ {}_1H^2 \rightarrow {}_2He^4 + P + n$

Mass defect $= 3 \times 2.014 - 4.001 - 1.007 - 1.008$

$= 0.026$ amu $= 0.026 \times 931$ MeV

Power of the star $= 10^{16}$W $= 10^{16}$ J/s

Number of deutrons used in one second

$$= \frac{10^{16}}{0.026 \times 931 \times 10^6 \times 1.6 \times 10^{-19}} \times 3 = 7.75 \times 10^{27}$$

Now the time in which the deutrons supply exhausted

$$= \frac{\text{Number of deutrons}}{\text{number of deutrons used in one sec}}$$

$$= \frac{10^{40}}{7.75 \times 10^{27}} = 1.3 \times 10^{12} \text{ sec} \simeq 10^{12} \text{ sec}$$

RADIOACTIVITY

INTRODUCTION

* The phenomenon of radioactivity was discovered by Henry Bequerel in uranium salts (1896)
* After the discovery of radioactivity in uranium, Piere curie & Madame curie discovered a new radioactive element called 'radium'. It is 10^6 times more radioactive than uranium. Radium was extracted from pitch blend (a kind of coal tar) for which curies were honoured by nobel prize in 1903.
* Definition : The self, sapontaneous, disintegration (i.e. decay) of unstable radioactive nuclei's is known as radioactivity & the nuclelis exhibiting this phenomenon are known as radioactive nuclelis.
* Some example of radio active substance (or elements) are;

  Uranium, Radium, Thorium, Polonium, Neptunium etc.
* All elements having atomic number greater than 83 are radioactive element.
* Lead (Z = 82) is the most stable element in high atomic number elements that's why all radio active elements emits radio active radiation till they are converted into Lead.
* Radioactivity is a nuclear event & not atomic. Hence electronic configuration of atom don't have any relationship with radio activity.
* Decay processes are random. Here one simply knows that in a radioactive element, radioactivity is taking place or it is definite that a certain no. of atoms will decay in a given time interval but one never knows that which particular radioactive nuclei will decay when. It is just a matter of chance i.e. probability which is explained by quantum mechanics. It is statistical in nature.
* This process is spontaneous i.e, it can neither be started, stopped accelerated nor retarded by any physical (i.e., by changing temperature pressure force) or chemical change.
* In this phenomenon parent-daughter chain continues. Original radio active element are called parent element or nucleus or nucleus but new element is called Daughter element or disintegrated nucleus.

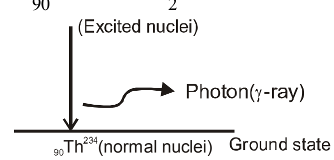
  $_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_2\alpha^4$

  (Parent) (Daughter)

  $_{90}Th^{234} \rightarrow {}_{91}Pa^{234} + {}_{-1}\beta^0 + \bar{\nu}$

  (Parent) (Daughter) (Antineutrino)
* In this phenomenon, α, β particles and γ-rays are emitted. For a given nuclei at a particular time emission of either α or β takes place, never both at time.
* γ-rays are emitted in when an excited nuclei makes transition to any lower or ground energy state the form of photon.

  $_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_2\alpha^4$



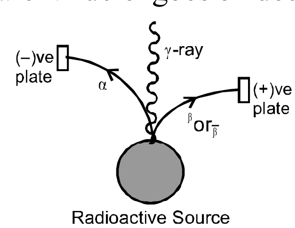
$_{90}Th^{234} \rightarrow {}_{90}Th^{234} + \gamma$ -ray

(higher energy nuclei) (lower energy nuclei)

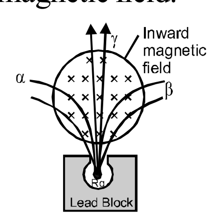
(lower stability) (higher stability)

excited nuclei and normal nuclei are known as radioactive isomers.

* Due to emission of α, β, γ during radioactive decay, mass of parent nuclei goes on decreasing



* Deflection of radioactive radiations was measured by rutherford in electric field & it is as below.
* Deflection in magnetic field.



* ***Remember*** : Proton is not emitted during radioactive decay.

COMPARISON OF PROPERTIES OF α, β AND γ RADIATIONS

Property	α-rays	β-rays	γ-rays
1. Nature	These are doubly ionized helium atom $_2He^4$ charge $q = +2e = 3.2 \times 10^{-19}C$ mass $m = 2p + 2n = 4amu$ $= 4 \times 1.6 \times 10^{-27}$ kg	These are beam of fast moving electrons(β^-) and positions (β^+) and charge $\beta^- = -e = -1.6 \times 10^{-19}$ $\beta^+ = +e = 1.6 \times 10^{-19}C$ $m(\beta^-) = m(b) = 9.1 \times 10^{-31}kg$	These are electromagnetic radiation of high frequency & travel in form of photons Ccharge q = 0 (chargeless) rest mass = 0 effective mass $= \frac{hv}{c^2} = \frac{h}{\lambda c}$
2. Velocity	Speed ranges between 1.4×10^7 to 2.20×10^7 m/s $v_\alpha \sim 0.05$ c	speed ranges from 1% to 90% of velocity of light $v_\beta - 0.9$ c	speed equals velocity of light $v_\gamma = c$
3. Ionising power	These have maximum ionizing power (1000)	There ionizing power is less than α particles and more than γ rays (100)	There ionizing power is less (1)
4. Penetration power	The penetration power is smallest. Can only penetrate through 0.01 mm thick Al sheet (1)	Penetration power is about 100 times that of α rays can penetrate through 1 mm thick Al sheet (100)	Penetration power is very large. Can penetrable about 30 cm thick Al sheet (10000)
5. Range	Range is very small (few cms in air)	Range is more than α rays. (few meters in air)	Range is very large (many hundreds of meter is air)
6. Nature of spectrum	Line spectrum	continuous spectrum	line spectrum
7. Interaction with matter	produces heat	produces heat	produces photoelectric effect Compton effect, pair production
8. Effect of electric and magnetic field	Suffers small deflection	suffers large deflection	pass undeflected
9. Effect of photo graphic plate and ZnS	Affects photographic plate and produces fluorescense	Affects photographic plate and produces fluorescence	Affects photographic plate and produces fluorescence

LAWS OF RADIOACTIVE DISINTEGRATION

* If N = No. of active nuclei at time t

N – dN = No. of active nuclei after time interval of dt.

$\therefore$ Rate of decay = $-\frac{dN}{dt}$

* According to the Rutherford and Soddy law for radioactivity, "At any instant the rate of decay of radioactive atom is proportional to the number of atoms present at that instant" Rate of decay $\propto$ N

$-\frac{dN}{dt} \propto N$ or $-\frac{dN}{dt} = \lambda N$

λ is called decay constant.

* If N_i = No of active nuclei at time t_i

N_f = No of active nuclei left at time t_f

$\int_{N_i}^{N_f} \frac{dN}{N} = -\int_{t_i}^{t_f} \lambda dt$ $\qquad [\ell n \; n]_{N_i}^{N_f} = -\lambda[t]_{t_i}^{t_f}$

$\ell n \frac{N_f}{N_i} = -\lambda[t_f - t_i]$ $\qquad \frac{N_f}{N_i} = e^{-\lambda[t_f - t_i]}$

If $t_f = t$ & $N_f = N$

& $t_i = 0$ $\quad N_i = N_0$

then $\frac{N}{N_0} = e^{-\lambda t}$ $\quad N = N_0 \, e^{-\lambda t}$ ....(1)

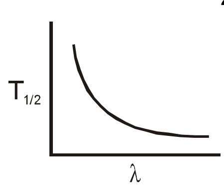
* Eq. (1) is known as Ruther ford & soddy's exponential equation of radioactivity.

* If $t = \infty$ then N = 0

Therefore, for a complete decay of a radioactive nuclei, it takes infinite time.

* **Half life($T_{1/2}$)** $\rightarrow$ Half life of a radioactive element is the time interval is which 50% of radioactive nuclei will disintegrates.

or when $t = T_{1/2}$ then $N = \frac{N_0}{2}$



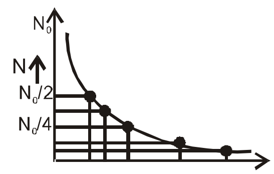
From $N = N_0 \, e^{-\lambda t}$

$\frac{N_0}{2} = N_0 \, e^{-\lambda T_{1/2}}$ $\qquad T_{1/2} = \frac{\ell n 2}{\lambda} = \frac{0.693}{\lambda}$

* **Physical significance of Half life**

If half life of a radio active nuclei is more than in given time interval probability of its availability is also more. Half life of a element does not depend on physical & chemical circumstances. $\frac{N}{N_0} = \frac{1}{(2)^{t/T}}$

* Graph between no. of active nuclei left & time of decay



* **Average life or mean life** (τ) $\rightarrow$

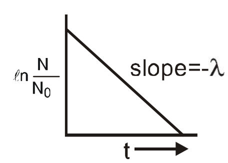
(i) The time, for which a radioactive material remains active, is defined as mean life of that material

(ii) $\tau = \frac{\text{Sum of lives of all nuclei's}}{\text{Total no. of nuclei present}} = \frac{\int_0^\infty t \, dN}{N_0}$

(iii) The average time taken in decaying by the nuclei of an element is defined as its mean life τ.

(iv) $\tau = \frac{1}{\lambda}$

(v) $\ell n \frac{N}{N_0} = -\lambda t$



The magnitude of inverse of slope of $\ell n \frac{N}{N_0}$ & t curve is known as mean life.

(vi) In $N = N_0 \, e^{-\lambda t}$

if $t = \frac{1}{\lambda}$

then $N = N_0 e^{-1} = \frac{N_0}{e} = \frac{N}{2.718} = 0.3676 \, N_0 \approx 0.37 \, N_0$

Therefore if $\tau = t = \frac{1}{\lambda}$ then $N = \frac{N_0}{e} \approx 0.37 N_0$

Mean life of a radio active nuclei is equal to that time in which no of nuclei left becomes either $\frac{1}{e}$ times the original nuclei or approximately 37% of the original nuclei .

or, Number of decayed nuclei in mean life = N_0

$- N = N_0 - \frac{N_0}{e} = N_0\left[1 - \frac{1}{e}\right] \approx 0.63 \, N_0$

Mean life is equal to that time in which no. of decayed becomes either $\left(\frac{e-1}{e}\right)$ times original nuclei or approximately 63% of original nuclei .

(vii) Fraction of active nuclei left = $\frac{N}{N_0}$ (probability of survival)

(viii) Fraction of decay nuclei = 1 – $\frac{N}{N_0}$ = $\frac{N_0 - N}{N_0}$ (probability of decay)

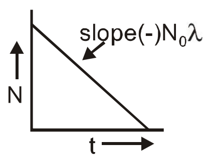
(ix) When decay process is too slow

then $N = N_0\ [e^{-\lambda t}]$

$N = N_0\ [1 - \lambda t]$

$N = [-N_0\lambda]\ t + N_0$

On comparing y = mx + c



(x) Half life, mean life & decay constant for a particular nuclei always remains constant.

(xi) **Decay constant (λ) :**

(i) $\lambda = \frac{(-)\left(\frac{dN}{dt}\right)}{N}$

$= \frac{\text{Rate of decay at a given instant}}{\text{No. of active nuclei at that instant}}$

(ii) $\lambda = \frac{(-)\left(\frac{dN}{dt}\right)}{dt}$ = Probability of decay in unit.

(iii) If $\lambda = t^{-1}$ then $N = \frac{N_0}{e} = 0.3676\,N_0 \approx 0.37N_0$

Decay constant is equal to inverge of that time in which no. of active nuclei left becomes either $\frac{1}{e}$ times or approx 37% original nuclei.

(iv) $N = N_0^{-\lambda t}$

can be written in the form of mass as below

$m = m_0e^{-\lambda t}$

when m_0 = mass of radio active nuclei at time t

& m = mass of radioactive nuclei at time

t = 0 $\frac{m}{m_0} = \frac{1}{(2)}$ t/α

(v) Decay constant is equal to inverse of the time in which 63% of initial atoms (N_0) is being decayed.

(vi) Unit : $(\text{second})^{-1}$ or $(\text{minute})^{-1}$ or $(\text{year})^{-1}$

ACTIVITY (A or R)

* The number of decays per unit time or decay rate is called activity.

Activity $A = \frac{dN}{dt} = N_0\,\lambda\,e^{-\lambda t} = A_0e^{-\lambda t} = N\lambda$

where $N_0\lambda = A_0$ is initial activity

* $A = A_0e^{-\lambda t}$ is the activity law which shows activity decreases exponentially with time.

* Activity is proportional to number of active atoms ($A \propto N$) which depends on mass of radioactive sample.

* The activity of one gram of radioactive substance called **specific activity**.

* **Half life** is the time in which activity of radioactive substance is reduced to half.

* **Mean life** is the time in which the activity reduces to 37% of the original value.

* The variation of Activity with time is $\frac{A}{A_0} = \left(\frac{1}{2}\right)^{t/T}$ where T is half life.

* **Units of activity**

Curie : The specific activity of 1 gm of Radium 226 is called one curie.

1 curie = 3.7×10^{10} disintegrations per second.

Rutherford 1 rutherford = 10^6 disintegrations ser second

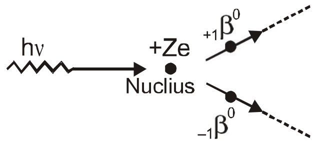
Becqueral 1 Becquerel = 1 disintergration per second

* **Sirvert (Sv)** : It is a unit of radiation dose

1 Sv is the amount of any radiation which will produce same biological effect as produced by absorption of 1 joule of X-rays or gamma rays by 1 kg of body tissue.

PAIR PRODUCTION & PAIR ANNIHILATION

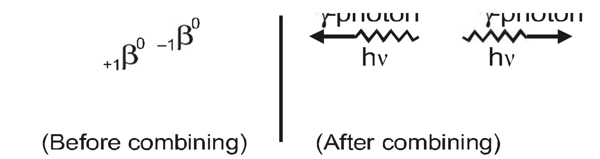
Collision of γ-ray photon by a nucleus & production of electron positron pair is known as pair production.



The rest mass of each of the electron & the positron is 9.1×10^{-31} kg. so, the rest mass energy of each of then is

$$E_0 = m_0 c^2 = (9.1 \times 10^{-31})(3 \times 10^8)^2$$
$$= 8.2 \times 10^{-14} \text{ joule}$$
$$= 0.51 \text{ MeV}$$

Hence for pair-production, it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02$ MeV.



$_{+1}\beta^0$ (positron) + $_{-1}\beta^0$ (electron) = hν (γ-photon) + hν (γ-p

FUNDAMENTAL PARTICLES & THEIR ANTIPARTICLES

The particles which are not constituted by any other particles i.e., which have no structure, are called 'fundamental particles'.

* **Electron :** It is the first fundamental particle which was discovered by Thomoson in 1897. It revolves around the nucleus of an atom indifferent orbits. Electron plays an important role in explaining the physical and chemical properties of substances. Its charge is -1.6×10^{-19} coulomb and mass is 9.1×10^{-31} kg. Its symbol is e^- (or $_{-1}\beta^0$).
* **Proton :** It was discovered by Rutherford in 1919 in artificial nuclear disintegration . It has a positive charge ($+ 1.6 \times 10^{-19}$ coulomb) equal to the electronic charge and its mass is (1.673×10^{-27} kg) 1836 times the electronic mass. In free state, the proton is a stable particles. Its symbol is p^+. It is also written as $_1H^1$.
* **Neutron :** It was discovered by Chadwick in 1932. It carries no charge. Its mass is 1839 times the electronic mass (1.675×10^{-27} kg). In free state the neutron is unable (its mean life is about 17 minutes), but it constitutes a stable nucleus along with proton. Its symbol is n or $_0n^1$.
* **Positron :** It was also discovered in 1932 by Anderson. Its charge and mass are same as those of electron, the only difference being that is positively-charged whereas the electron is negatively -charged. Its symbol is e^+ (or $_{+1}\beta^0$)
* **Antiproton :** It was discovered in 1955. Its charged and mass are same as those of proton, the only difference being that it is negatively charged. Its symbol is p^-.
* **Antineutron :** It was discovered in 1956. It has no charge and its mass is equal to the mass of neutron. The only difference between neutron and antineutron is that if they spin in the same direction, their magnetic moment will be in opposite directions. The symbol for antineutron is n^-.
* **Neutrino and Anti-neutrino :** The existence of these particles was predicated in 1930 by Pauli while explaining the emission of β-particles from radio-active nuclei, but they were observed experimentally in 1956. Their rest-mass and charge are both zero but they have energy and momentum. Both neutrino and anti-neutrino are stable particles. The only difference between them is that their spins are in opposite directions. Their symbols are v and $\bar{v}$ respectively.
* **Pi-mesons :** The existence of these particles was predicated by Yukawa in 1935 as originator of exchange force between the nucleons, but they were actually discovered in 1947 in cosmic rays. Pi-mesons are of three types.

(i) Positive pi-meason : It is a positively charged particle whose charge is equal to the electronic charge and whose mass is 274 times the electronic mass. It is an unstable particles. Its mean life is of the order of 10^{-8} second. Its symbol is π^+.

(ii) Negative pi-meason : It is a negatively charged particle whose charge is equal to the electronic charge and whose mass is 274 times the electronic mass. It mean life is also of the order of 10^{-8} second. Its symbol is π^-.

(iii) Neutral pi-meason : This particle has no charge. Its mass is nearly 264 times the electronic mass. Its mean life is of the order 10^{-15} second. Its symbol is π^0. On disintegration, it forms two γ-photons :

$\pi^0 \rightarrow \gamma + \gamma$

* **Photon :** These are the bundles of electromagnetic energy and travel with the speed of light. If the frequency of waves be v, then the energy of a photon is $h\nu$ and momentum is $h\nu/c$. Its symbol is γ.

Name of particle	Symbol	Antipartilce	Mass (in comparison to mass of electron	Average life (in second)
Photon	γ	(γ)	0	Stable
Electron	e^-	e^+	1	Stable
Proton	p^+	e^-	1836	Stable
Neutron	n	n^-	1839	1010
Neutrino	v	v^-	0	Stable
Charged pi-meson	π^+	π^-	274	2.6×10^{-8}
Uncharged pi-meson	π^0	(π^0)	264	0.9×10^{-16}

CHARACTERISTICS OF RADIOACTIVE RADIATIONS

1. Characteristics of α-decay

(1) α-particle are two times ionised Helium atoms.

(2) α-particle carry 2-proton and 2-neutrons.

(3) $_2He^4$ (Nucleus) = α-particle

(4) In general α-decay is given by

$$_ZX^A \xrightarrow{\alpha-\text{decay}} {}_{Z-2}Y^{A-4} + \underset{(\alpha-\text{particle})}{{}_2He^4} + \gamma \text{ (energy)},$$

atomic number decreases by 2 & mass no. decreases by 4.

(5) Mass of α-particle = (2p + 2n) = 6.68×10^{-27} kg

Charge of α-particle = 2e = $2 \times 1.6 \times 10^{-19}$ coulomb. (+) ve.

(6) In α-particle emission an element goes to two column backward in periodic table.

(7) Energy in α-particle emission

$= (M_x - M_y - M_{_2He^4})\, C^2$

* Energy in α-particle emission varies from 4.5 MeV to 11 MeV. Through calculation α-particle has to crop potential barrier of 21 MeV for their emission. Hence α-particle emission can not be explained on the basis of classical theory.
* Emission of α-particle can be explained on the basis of quantum mechanics (Tunnel effect)
* Energy spectrum of α-particle is line spectrum.
* Energy spectrum of α-particle also has micro details
* Energy spectrum of α-particle shows that a nucleus also has energy levels like atoms have.
* Energy of α-particle emitted from a single nucleus are not same. These emit in various energy groups.
* Range of α-particle $\propto$ (velocity)^3
* When α-particle emission takes place, γ–rays are also emitted.
* $R \propto E^{3/2}$

  or $R = 0.318\ E^{3/2}$ E : energy of α-particle
* (Geiger's and Nuttal law) : →

  Relation between decay constant of a element and range of α-particle as follows :

  $\log \lambda = A + B \log R$

  A and B are constant. B has equal value while A have different values for radioactive series.

2. Characteristics of β-decay

* β-particle are high energy electron or positron.

  (a) β^- or $_{-1}e^0$ (electron)

  (b) β^+ or $_{+1}e^0$ (position)
* Resultant charge on β-particle = $\pm 1.6 \times 10^{-19}$ coulomb.
* Rest mass of β-particle are equal to mass of electron.
* β-particle emission can be represented by following reactions :

$$_ZX^A \xrightarrow{\beta^--\text{particle}} {}_{Z+1}Y^A + {}_{-1}e^0 + Q$$

$$_ZX^A \xrightarrow{\beta^+-\text{particle}} {}_{Z-1}Y^A + {}_{+1}e^0 + Q$$

where Q = energy

* In β-particle, atomic no. increases by one and in β^+ emission, atomic no. decreases by one

* Mass number does not change in β-emission.
* Emission of β particle can be explained by conversion of neutron in proton & vice versa in the nucleus.

$${}_1p^1 \rightarrow {}_0n^1 + {}_1e^0(\beta^+)$$

or $${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0(\beta^-)$$

* To explain energy conservation, linear momentum conservation and angular momentum conservation, a hypothetical neutrino was considered
* Neutrino was first given by Pauli.
* According to neutrino hypothesis, some particle also emits with β-emission, which is called neutrino.
* Rest mass and charge of neutron are both zero and angular or spin momentum of neutrino is ±1/2 (h/2π). It travel with speed of light and it's spin value is ± 1/2.
* So by neutrino hypothesis, emission of β particle reaction

$${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{v} \text{ (anit-neutrino)}$$

$${}_1p^1 \rightarrow {}_0n^1 + {}_1e^0 + \bar{v} \text{ (neutrino)}$$

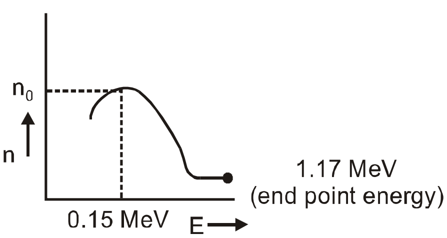
Hence reaction

$${}_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + v + Q$$

$${}_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{v} + Q$$

* Existance of neutino is practically explained by Rein's Collin
* Energy spectrum of β-emission is continuous.

(No. of β particle)



3. Characteristic of β spectrum

* Energy range of emitted β-particle has all possible energy β-particle.
* Maximum value of energy of β-particle is called end point energy.
* During β-emission the decreases in energy of parent nucleus is equal to end point energy of β-particle, latter it is shared by β-particle and neutrino.

4 Characteristics of γ-rays :

* γ-rays are electromagnetic waves of short wavelength ($\cong 10^{-12}$ m)
* They emit from nucleus.
* They travel with speed of light (3×10^8 m/sec)
* These are high energy rays (of photons)
* When α or β particle emission takes place, nucleus come in excited state and during coming back to normal state γ radiation emission takes place

$${}_{27}Co^{60} \rightarrow {}_{28}Ni^{60}\text{(exicited state)} + {}_{-1}e^0$$

$${}_{28}Ni^{60}\text{(exicited state)} \rightarrow {}_{28}N^{60}\text{(gound state)} + Q\ (\gamma\text{ rays})$$

* In γ-decay atomic no. and mass no. does not changes.
* Energy spectrum of γ-rays is line spectrum.
* This spectrum verifies that same energy levels are found in nucleus as that of in atom outside the nucelius.
* If affects photographic plate.
* It has ionising power.
* It also has penetration power.
* It is not affected by electric & magnetic field
* Intensity of γ-rays after travelling x-distance is $I = I_0 e^{-\mu x}$ (same as x-rays)

I_0 – initial intensity, μ – Absorption coefficient, I – intensity after travelling x-distance

* γ-rays also shows diffraction by crystal grating like x-rays.
* γ-rays radiations after entering into substance are absorbed in three process depending upon energy

(A) Photoelectric effect (B) Compton effect (C) Pair production

RADIOACTIVE SERIES

* There are mainly four radioactive series. Three are natural and one is artificial.

Thorium series (4n series)

$${}_{90}Th^{232} \xrightarrow[\alpha,\beta\text{ decay}]{\text{Irregular}} {}_{82}Pb^{208} \text{ (stable element)}$$

* Neptunium series (4n + 1 series)

  This is artificial series

  $_{93}Np^{237} \xrightarrow[\alpha,\beta \text{ decay}]{\text{Irregular}} {}_{83}Bi^{209}$ (stable)

* Uranium series (4n + 2 series)

  $_{92}U^{238} \xrightarrow[\alpha,\beta \text{ decay}]{\text{Irregular}} {}_{82}Pb^{206}$ (stable)

* Actinium series (4n + 3 series)

  $_{92}U^{235} \xrightarrow[\alpha,\beta \text{ decay}]{\text{Irregular}} {}_{82}Pb^{207}$

  Last element of radio active series is stable and decay constant of that element has value equal to zero.

USES OF RADIOACTIVE ISOTOPES

1. In Medicine

* Co^{60} for treatment of cancer
* Na^{24} for circulation of blood
* I^{131} for thyroid
* Sr^{90} for treatement of skin & eye
* Fe^{59} for location of brain tumor
* radiographs of castings and teeth

2. In Industries

for detecting leakage in water and oil pipe lines

for investigation of wear & tear, study of plastics & alloys, thickness measurement.

3. In Agriculture

C^{14} to study kinetics of plant photosynthesis.

P^{32} to find nature of phosphate which is best for given soil & crop

Co^{60} for protecting potato crop from earth worm.

sterilization of insects for pest control.

4. In Scientific research

* K^{40} to find age of meteorites
* S^{35} in factories

5. Carbon dating

* It is used to find age of earth and fossils
* The age of earth is found by Uranium disintegration and fossil age by disintegration of C^{14}.
* The estimated ege of earth is about 5×10^9 years.
* The half life of C^{14} is 7500 years.

6. As Tracers

* A very small quantity of radio isotope present in any specimen is called tracer.
* This technique is used to study complex biochemical reactions, in detection of cracks, blockage etc., tracing sewage or silt in sea

7. In Geology

* for dating geological specimens like ancient rocks, lunar rocks using Uranium
* for dating archaeological specimens, biological specimens using C^{14}.

Solved Examples

Ex.24 The mean lives of a radioactive material for α- and β-radiations are 1620 years and 520 years respectively. The material decays simultaneously for α- and β- radiation. The time after which one fourth of the material remains undecayed is -

Sol. $\tau = \frac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta} = \frac{1620 \times 520}{1620 + 520} = 394$ years

time of decay $t = \tau 2.303 \log_{10} \frac{N_0}{N}$

$t = 394 \times 2.303 \log_{10} 4$

$= 394 \times 2.303 \times 0.602$

t = 546 years

Ex.25 A freshly prepared radioactive sample, with half-life 2 hours, emits radiations whose intensity is 64 times higher than its safe level. The minimum time after which it will be safe to work with the sample will be -

Sol. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$ $\qquad \frac{N}{N_0} = \frac{1}{64}$

$\frac{1}{64} = \left(\frac{1}{2}\right)^{t/2}$ $\qquad$ or $\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{t/T}$

t = 6T = 12 hours

Ex.26 The counter used to determine the activity of a sample shows 4750 counts per minute at any instant of time. After five minutes it shows 2700 counts per minute. The value of decay constant will be -

Sol. $\lambda = \dfrac{2.303\log_{10}\dfrac{N_0}{N}}{t} = \dfrac{2.303\times\log_{10}\dfrac{4750}{2700}}{5}$

$= \dfrac{2.303\times 0.2455}{5} = 0.1130$ per minute

Ex.27 The half life of a specimen of thorium is 1.4×10^{10}. The time taken for 20% of thorium to decay will be -

Sol. $t = \dfrac{\left[2.303\log_{10}\dfrac{N_0}{N}\right]^{T}_{\frac{1}{2}}}{0.693}$

$t = \dfrac{\left[2.203\log_{10}\dfrac{100}{80}\right]\times 1.4\times 10^{10}}{0.693}$

$= 4.507 \times 10^{9}$ years

Ex.28 The half life of that radioactive substance, which reduces to 1/64 of its initial value in 15 hours, will be -

Sol. $N = \dfrac{N_0}{2^n} = \dfrac{N_0}{2^6}$

$\therefore n = 6$

$t = nT_{1/2}$

$T_{1/2} = ?$

$T_{1/2} = 15 = 2.50$ hours

Ex.29 When ${}_{90}Th^{228}$ gets converted into ${}_{83}Bi^{212}$, then the number of α– and β- particles emitted will respectively be -

Sol. ${}_{90}Th^{228} = {}_{83}Bi^{212} + x(2He^4) + y(-1e^0)$

According to law of conservation of charge

$90 = 83 + 2x - y$

or $2x - y = 7$ ....(1)

According to law of conservation of mass number

$228 = 212 + 4x$

or $4x = 16$

$x = 4$ ....(2)

$2 \times 4 - y = 7$

$y = 1$

Hence 4α & 1β will be emitted.

Ex.30 In the decay process ${}_{92}U^{238}$ $\xrightarrow{\alpha} X \xrightarrow{\beta^-} {}_Z X^A$, the values of Z and A will be -

Sol. ${}_{92}U^{238} = 2He^4 + {}_{90}X^{234}$

${}_{90}Th = {}_1e^{0+}\ {}_{91}Y^{234}$

$\therefore z = 91$ & $A = 234$

Ex.31 The intensity of γ-ray beam redcues to one fourth after traversing a distance of 5mm through lead. The half value thickness of lead in mm will be -

Sol. $\mu = \dfrac{2.303\log_{10}\dfrac{I_0}{I}}{x}$

$= \dfrac{\log_{10}4}{5} = \dfrac{\log_{10}2}{x_{\frac{1}{2}}}$ or $x_{1/2} = 2.5$ mm

Ex.32 A radioactive nucleus decays as follows -

$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$

If the mass number and charge number of A are 180 and 72 respectively, then for A_4 these values will respectively be

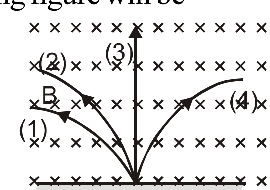
Sol. ${}_{72}A^{180} = {}_2He^4 + {}_{70}Al^{176}$

${}_{70}A_1^{176} = {}_{-1}e^0 + {}_{71}A_2^{176}$

${}_{71}A_2^{176} = {}_2He^4 + {}_{69}A_3^{172}$

${}_{69}A_3^{172} = \gamma + {}_{69}A_4^{172}$

Ex.33 A neutron, a proton, an electron and an α-particle enter a perpendicular uniform magnetic field, with the same uniform velocity. The path of electron in the following figure will be -



Sol. Electron is a negatively charged particle, hence it will be deflected towards right.

Ex.34 In the radioactive decay of an element it is found that the count rate reduces from 1024 to 128 in 3 minutes. Its half life will be -

Sol. $t = \dfrac{T\log\dfrac{N_0}{N}}{\log 2}$

or $3 = \dfrac{T\log\dfrac{1024}{128}}{\log 2} = \dfrac{T\log 8}{\log 2} = \dfrac{3T\log 2}{\log 2}$

$\therefore T = \dfrac{3}{3} = 1$ minute

Ex.35 The half life of a radioisotopes is 3 days, In how many days will its activity reduce to 1/16 of its initial value -

Sol. $T_{1/2} = 3$ days

$N = \frac{N_0}{(2^4)} = \frac{N_0}{2^n}$

$\therefore n = 4$

$t = nT_{1/2}$

$t = 4 \times 3 = 12$ days

Ex.36 The count rate of a radioactive source at t = 0 was 1600 counts/s and at t = 8 sec, it was 100 counts/s. The count rate (in counts) at t = 6 sec will be -

Sol. $t = \frac{T\log\frac{N_0}{N}}{\log 2}$

$8 = \frac{T\log\frac{1600}{100}}{\log 2} = \frac{T\log 16}{\log 2}$

$8 = \frac{T\log 2^4}{\log 2}$

$8 = 4T\,\frac{\log 2}{\log 2}$

$\therefore T = 2$ sec

$N = \frac{N_0}{2^{\frac{t}{T}}} = \frac{1600}{2^{\frac{6}{2}}}$

N = 200 counts/s

Ex.37 A radioactive element, with mass 8 gm and half life 100 second, after 5 minutes will reduce to -

Sol. $M = \frac{M_0}{2^n} = \frac{M_0}{2^3}$

$\therefore n = 3$

$\because M = \frac{8}{8} = 1$ gram

Ex.38 The half life of radium is 1600 years. In how much time will its $\frac{15}{16}$ fraction disintegrate -

Sol. time of decay $t = \frac{T\log\left(\frac{N_0}{N}\right)}{\log 2}$

or $t = \frac{1600\log\left(\frac{16}{1}\right)}{\log 2}\ 2 = \frac{4 \times 1600\ \log 2}{\log 2}$

or 6400 years

Ex.39 A radioactive material remains 25% after 16 days. Its half life will be-

Sol. Time of decay $t = \frac{T\log\left(\frac{N_0}{N}\right)}{\log 2}$

or $16 = \frac{T\log\left(\frac{4}{1}\right)}{\log 2} = \frac{2T\log 2}{\log 2}$

$\therefore T = 8$ days

Ex.40 The half life of radon is 3.8 days. 15 mg of radon after 38 days will reduce to -

Sol. $M = \frac{M_0}{(2)^{t/T}}$ $\qquad M = \frac{15}{(2)^{38/3.8}}$

$M = \frac{15}{(2)^{10}} = \frac{15}{1024}$ $\qquad = 0.01465$ mg

Ex.41 A specimen of radioactive material contains 10^6 radioactive nuclei. Its half life is 20 second. How many nuclei will remain undecayed after 10 second -

Sol. $N = 10^6\left(\frac{1}{2}\right)^{10/20} = \frac{10^6}{\sqrt{2}} = \frac{10^6}{1.41} = 7.09 \times 10^5$

Ex.42 The half life of radioisotope is 5 years. Its fraction, which will decay in 15 days, will be -

Sol. Fraction remaining after 15 days

$\frac{N}{N_0} = \frac{1}{\frac{t}{2T}} = \frac{1}{\frac{15}{25}} = \frac{1}{8}$

Hence the fraction decayed in 15 days will be

$1 - \frac{N}{N_0} = 1 - \frac{1}{8} = \frac{7}{8}$

Ex.43 A sample contains two substances P and Q, each of mass 10^{-2} kg. The ratio of their atomic weights is 1 : 2 and their half lives are 4s and 8s respectively. The masses of P and Q that remain after 16s will respectively be -

Sol. $\because N = \frac{N_0}{2^n}$ $\quad M = \frac{M_0}{2^n}$

for P $\quad n = \frac{16}{4} = 4$

$\therefore N_p = \frac{10^{-2}}{16} = 6.25 \times 10^{-4}$ Kg

for Q $n = \frac{16}{8} = 2$

$\therefore N_Q = \frac{10^{-2}}{2^2} = 2.5 \times 10^{-3}$