PROBABILITY

1. DEFINITIONS

1.1 Trial and Event : An experiment is called a **trial** if it results in anyone of the possible outcomes and all the possible outcomes are called **events**.

1.2 Exhaustive Events : Total possible outcomes of an experiment are called its **exhaustive events**.

1.3 Favourable Events : Those outcomes of a trial in which a given event may happen, are called **favourable cases** for that event.

1.4 Equally likely events : Two or more events are said to be **equally likely events** if they have same number of favourable cases.

1.5 Mutually exclusive or disjoint events : Two or more events are said to be **mutually exclusive**, if the occurrence of one prevents or precludes the occurrence of the others. In other words they cannot occur together.

1.6 Simple and Compound events : If in any experiment only one event can happen at a time then it is called a **simple event**. If two or more events happen together then they constitute a **compound event.**

1.7 Independent and Dependent events : Two or more events are said to be **independent** if happening of one does not affect other events. On the other hand if happening of one event affects (partially or totally) other event, then they are said to be **dependent events.**

1.8 Sample Space : The set of all possible outcomes of a trial is called its **sample space**. It is generally denoted by S and each outcome of the trial is said to be a point of sample of S.

2. MATHEMATICAL DEFINITION OF PROBABILITY

Let there are n exhaustive, mutually exclusive and equally likely cases for an event A and m of those are favourable to it, then probability of happening of the event A is defined by the ratio m/n which is denoted by P(A). Thus

$$P(A) = \frac{m}{n} = \frac{\text{No. of favourable cases to A}}{\text{No. of exhaustive cases to A}}$$

Note :

It is obvious that $0 \le m \le n$. If an event A is certain to happen, then m = n thus P (A) = 1. If A is impossible to happen then m = 0 and so P (A) = 0. Hence we conclude that

$$0 \le P(A) \le 1$$

Further, if $\bar{A}$ denotes negative of A i.e. event that A doesn't happen, then for above cases m, n ; we shall have

$$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$$\therefore \quad P(A) + P(\bar{A}) = 1$$

3. ADDITION THEOREM OF PROBABILITY

Case I : When events are mutually exclusive:

If A and B are mutually exclusive events then

$n(A \cap B) = 0 \Rightarrow P(A \cap B) = 0$

$\therefore \quad P(A \cup B) = P(A) + P(B)$

For any three events A, B, C which are mutually exclusive then $P(A \cap B) = 0$, $P(B \cap C) = 0$, $P(C \cap A) = 0$ and

$P(A \cap B \cap C) = 0$

$\therefore \quad P(A \cup B \cup C) =$
$P(A) + P(B) + P(C)$

The probability of happening of any one of several mutually exclusive events is equal to the sum of their probabilities, i.e. if $A_1, A_2 .... A_n$ are mutually exclusive events then

$P(A_1 + A_2 + ...+ A_n) = P(A_1) + P(A_2) + ... + P(A_n)$

i.e. $P(\sum A_i) = \sum P(A_i)$

Case II : When events are not mutually exclusive.

If A & B are two events which are not mutually exclusive then.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or $P(A + B) = P(A) + P(B) - P(AB)$

For any three events A, B, C

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

or $P(A + B + C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$

4. SOME IMPORTANT RESULTS

(a) Let A and B be two events, then

(i) $P(A) + P(\bar{A}) = 1$

(ii) $P(A + B) = 1 - P(\bar{A}\,\bar{B})$

(iii) $P(A + B) = P(AB) + P(\bar{A}B) + P(A\bar{B})$

(iv) $A \subset B \Rightarrow P(A) \le P(B)$

(v) $P(\bar{A}B) = P(B) - P(AB)$

(vi) $P(AB) \le P(A)P(B) \le P(A+B) \le P(A)+P(B)$

(vii) $P(AB) = P(A) + P(B) - P(A + B)$

(viii) P (Exactly one event)

$= P(A\bar{B}) + P(\bar{A}B) = P(A)+P(B)-2p(AB)$

$= P(A+B) - P(AB)$

(ix) P(neither A nor B) $= P(\bar{A}\,\bar{B}) = 1 - p(A+B)$

(x) $P(\bar{A}+\bar{B}) = 1 - P(AB)$

(b) Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times) = $\mathbf{2^n}$

(c) Number of exhaustive cases of throwing n dice simultaneously (or throwing one dice n times) = $\mathbf{6^n}$

(d) Playing Cards :

(i) Total : 52 (26 red, 26 black)

(ii) Four suits : Heart, Diamond, Spade, Club - 13 cards each

(iii) Court Cards : 12 (4 Kings, 4 queens, 4 jacks)

(iv) Honour Cards : 16 (4 aces, 4 kings, 4 queens , 4 jacks)

SOLVED PROBLEMS

Ex.1 One card is drawn from a pack of playing cards, then find the probability that it is a card of king

Sol. Probability of one card to be king

$p = \frac{4}{52} = \frac{1}{13}$

($\because$ favourable cases = 4, Total cases = 52)

Ex.2 If $P(A) = \frac{3}{8}$, then find the odds in against of A

Sol. $P(A) = \frac{3}{8} \Rightarrow P(\bar{A}) = 1 - \frac{3}{8} = \frac{5}{8}$

$\therefore$ odds in against of

$A = \frac{P(\bar{A})}{P(A)} = \frac{5}{3} = 5:3$

Ex.3 If the probability for A to fail in an examination is 0.2 and that of B to fail is 0.3, then find the probability that either A or B fails

Sol. Let A be event for A to fail and B be the event for B to fail, then

P (A) = 0.2 and P(B) = 0.3

Since A and B are independent events,

$\therefore$ P (AB) = P(A) P (B)

$\therefore$ Required probability

= P(A+ B)

= P(A) + P (B) – P (AB)

= P(A) + P (B) – P (A) P(B)

= 0.2 + 0.3 – 0.2 × 0.3

= 0.5 – 0.06 = 0.44

Ex.4 If two dice are thrown together then what is the probability that the sum of their numbers is greater than 9.

Sol. The sum of the numbers greater than 9 may be 10,11 and 12. If these events be A, B, C respectively, then

P (A) = 3/36

[$\because$ favourable cases are (6, 4), (5, 5), (4, 6)]

P (B) = 2/36

[$\because$ favourable cases are (6, 5), (5, 6)]

P (C) = 1/36

[$\because$ favourable case is (6, 6)]

Now since A, B, C are mutually exclusive, so P (A + B + C)

= P(A) + P(B) + P (C)

$= \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{1}{6}$

Ex.5 Two cards are drawn one by one from a pack of 52 cards. If the first card is not replaced in the pack , then what is the probability that first card is that of a king and second card is that of a queen?

Sol. Let A $\equiv$ first card is that of a king

B $\equiv$ second card is that of a queen

that $P(A) = \frac{4}{52} = \frac{1}{13}$, $P(B/A) = \frac{4}{51}$;

$\therefore$ P (AB) = P (A) P (B/A)

$= \frac{1}{13} \cdot \frac{4}{51} = \frac{4}{663}$

Ex.6 Three coins are tossed together. What is the probability of getting tail on first, head on second and tail on third coin?

Sol. Let the three events be denoted by A, B and C respectively, then

P (A) = P(B) = P (C) = 1/2 since the events A,B and C are independent

$\therefore$ P(ABC) = P(A)P(B)P(C) = 1/8

Ex.7 One person can kill a bird twice in 3 shots, second once in 3 shots and third thrice in 4 shots. If they shot together then what is the probability that the bird will be killed?

Sol. If A, B, C denote events of killing the bird by first second and third person respectively, then

$P(A) = 2/3$, $P(B) = 1/3$,

$P(C) = 3/4$

The bird will be killed if atleast one of these three independent events happens. So

Required probability

$$= 1 - P(\bar{A}_1)P(\bar{A}_2)P(\bar{A}_3)$$

$$= 1 - (1 - 2/3)(1 - 1/3)(1 - 3/4)$$

$$= 1 - 1/3 . 2/3 . 1/4 = 17/18$$

Ex.8 If from a factory a labourer is chosen, randomly. the probability that he is a male is 0.6 and is married is 0.7. Find the probability that the chosen labourer is a married woman

Sol. Let A and B respectively be two events that a chosen labourer is a man and is married, then required probability

$$= P(\bar{A}B) = \{1 - P(A)\}\ P(B)$$

$$= (1 - 0.6)\ (0.7) = 0.28$$

Ex.9 A speaks truth in 75% cases and B in 80% cases. What is the probability that they contradict each other in stating the same fact?

Sol. There are two mutually exclusive cases in which they contradict each other i.e. $A\bar{B}$ and $\bar{A}B$. Hence

Required probability

$$= P(A\bar{B} + \bar{A}B) = P(A\bar{B}) + P(\bar{A}B)$$

$$= P(A)\ P(\bar{B}) + P(\bar{A})\ P(B)$$

$$= \frac{3}{4} . \frac{1}{5} + \frac{1}{4} . \frac{4}{5} = \frac{7}{20}$$

Ex.10 The letters of the word HIRDESH are written in a row randomly. Find the probability of the words starting with H and ending with H

Sol. Total no. of case s $= \frac{7!}{2!}$ Since H is written at first and last places, therefore at the remaining 5 places, 5 letters can be written in 5! ways.

$$\therefore \text{ Required probability } = \frac{5!}{7!/2!} = \frac{1}{21}$$

Ex.11 If a dice is thrown twice, then find the probability of getting 1 in the first throw only

Sol. Probability of getting 1 in first throw $= \frac{1}{6}$

Probability of not getting 1 in second throw $= \frac{5}{6}$ Both are independent events, so the required probability $= \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

Ex.12 Two dice are thrown simultaneously. What is the probability of obtaining a multiple of 2 on one of them and a multiple of 3 on the other

Sol. Favourable cases for one are three i.e. 2,4 and 6 and for other are two i.e. 3,6.

Hence required probability

$$= \left[\left(\frac{3 \times 2}{36}\right)2 - \frac{1}{36}\right] = \frac{11}{36}$$

[As same way happen when dice changes numbers among themselves]

Ex.13 A target is hit by A, 4 times out of 5 attempts; by B, 3 times out of 4 attempts and by C, 2 times out of 3 attempts. Find the probability that the target is hit by two of them

Sol. The following mutually exclusive cases are possible.

(i) $AB\bar{C}$ (ii) $A\bar{B}C$ (iii) $\bar{A}BC$

Since A, B and C are independent event therefore $P(AB\bar{C}) = P(A)\ P(B)\ P(\bar{C})$

$$= \frac{4}{5} . \frac{3}{4}\left(1 - \frac{2}{3}\right) = \frac{12}{60}$$

Similarly

$$P(A\bar{B}C) = \frac{8}{60} \text{ and } P(\bar{A}BC) = \frac{6}{60}$$

Thus the required probability

$$= P(AB\bar{C}) + P(A\bar{B}C) + P(\bar{A}BC)$$

$$= \frac{12}{60} + \frac{8}{60} + \frac{6}{60} = \frac{26}{60}$$

Ex.14 Three numbers are selected one by one from whole numbers 1 to 20. Find the probability that they are consecutive integers

Sol. Total number of sequences of 3 numbers selected one by one from whole numbers 1 to 20

$$= {}^{20}P_3 = 20 \times 19 \times 18$$

Now sequences which will contain three consecutive integers are (1, 2, 3) (2, 3, 4), (3, 4, 5)......, (18,19,20).

These are 18 sequences. Hence

$\therefore$ required probability

$$= \frac{18}{20 \times 19 \times 18} = \frac{1}{380}$$

EXERCISE

Q.1 A coin is tossed once. Find the probability of getting a tail.

Q.2 A die is thrown. Find the probability of getting a 5

Q.3 In a single throw of two dice, find the probability of getting a sum less than 6

Q.4 In a single throw of two dice, find the probability an odd number on the first die and a 6 on the second.

Q.5 A bag contains 4 white and 5 black balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is white.

Q.6 An urn contains 9 red, 7 white and 4 black balls. A ball is drawn at random. Find the probability that the ball drawn is red

Q.7 In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize.

Q.8 If there are two children in a family, find the probability that there is at least one boy in the family.

Q.9 Three unbiased coins are tossed once. Find the probability of getting exactly 2 tails

Q.10 In a single throw of two dice, determine the probability of not getting the same number on the two dice.

Q.11 If a letter is chosen at random from the English alphabet, find the probability that the letter chosen is (i) a vowel, and (ii) a consonant.

Q.12 A card is drawn at random at from a well-shuffled pack of 52 cards. What is the probability that the card bears a number greater then 3 and less than 10 ?

Q.13 Tickets numbered from 1 to 12 are mixed up together and then a ticket is withdrawn at random. Find the probability that the ticket has a number which is a multiple of 2 or 3.

Q.14 What is probability that an ordinary year has 53 Tuesdays ?

Q.15 What is the probability that a leap year has 53 Sundays ?

Q.16 What is the probability that in a group of two people, both will have the same birthday, assuming that there are 365 days in a year and no one has his/her birthday on 29^{th} February ?

Q.17 Which of the following cannot be the probability of occurrence of an event ?

(i) 0 (ii) $\frac{-3}{4}$ (iii) $\frac{3}{4}$ (iv) $\frac{4}{3}$

Q.18 If $^{7}/_{10}$ is the probability of occurrence of an event, what is the probability that it does not occur ?

Q.19 The odds in favour of the occurrence of an event are 8 : 13. Find the probability that the event will occur.

Q.20 If the odds against the occurrence of an event be 4 : 7, find the probability of the occurrence of the event.

Q.21 If 5/14 is the probability of occurrence of an event, find

(i) the odds in favour of its occurrence

(ii) the odds against its occurrence

Q.22 Two dice are thrown. Find

(i) the odds in favour of getting the sum 6

(ii) the odds against getting the sum 7

Q.23 A combination lock on a suitcase has 3 wheels, each labelled with nine digits from 1 to 9. If an opening combination is a particular sequence of three digits with no repeats, what is the probability of a person guessing the right combination ?

Q.24 In a lottery, a person chooses six different numbers at random from 1 to 20. If these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability that he wins the prize in the game ?

Q.25 In a single throw of three dice, find the probability of getting (i) a total of 5 (ii) a total of at most 5.

Q.26 If A and B are two events associated with a random experiment for which
P(A) = 0.60, P(A or B) = 0.85 and
P(A and B) = 0.42, find P(B).

Q.27 Let A and B be two events associates with a random experiment for which
P(A) = 0.4, P(B) = 0.5 and P(A or B) = 0.6. Find P(A and B).

Q.28 In a random experiment, let A and B be events such that P(A or B) = 0.7,
P(A and B) = 0.3 and P($\overline{A}$) = 0.4.
Find P(B).

Q.29 If A and B are two events associated with a random experiment such that
P(A) = 0.25, P(B) = 0.4 and P(A or B) = 0.5, find the values of (i) P(A and B) (ii) P(A and $\overline{B}$)

Q.30 If A and B two events associated with a random experiment such that P(A) = 0.3, P(B) = 0.2 and P(A $\cap$ B) = 0.1, find
(i) P($\overline{A} \cap$ B) (ii) P(A $\cap \overline{B}$)

Q.31 If A and B are two mutually exclusive events such that P(A) = (1/2) and
P(B) = (1/3), find P(A or B).

Q.32 Let A and B be two mutually exclusive events of a random experiment such that P(not A) = 0.65 and P(A or B) = 0.65, find P(B).

Q.33 A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. If P(B) = 3/2 P(A) and P(C) = 1/2 P(B), find P(A).

Q.34 The probability that a company executive will travel by plane is (2/5) and that he will travel by train is (1/3). Find the probability of his traveling by plane or train.

Q.35 From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of its being a king or a queen.

Q.36 From a well-shuffled pack of cards, a card is drawn at random. Find the probability of its being either a queen or a heart.

Q.37 A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a spade or a king.

Q.38 A number is chosen from the numbers
1 to 100. find the probability of its being divisible by 4 or 6.

Q.39 A die is thrown twice. What is the probability that at least one of the two throws comes up with the number 4 ?

Q.40 Two dice are tossed once. Find the probability of getting an even number on the first die or a total of 8.

Q.41 Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4 ?

Q.42 In a class, 30% of the students offered mathematics, 20% offered chemistry and 10 % offered both. If a student is selected at random, find the probability that he has offered mathematics or chemistry.

Q.43 The probability that Hemant passes in English is (2/3) and the probability that he passes in Hindi is (5/9). If the probability of his passing both the subjects is (2/5), find the probability that he will pass in at least one of these subjects.

Q.44 The probability that a person will get an electrification contract is (2/5) and the probability that he will not get a plumbing contract is (4/7). If the probability of getting at least one contract is (2/3), what is the probability that he will get both ?

Q.45 The probability that a patient visiting a dentist will have a tooth extracted is 0.06, the probability that he will have a cavity filled is 0.2. and the probability he will a tooth extracted or a cavity filled is 0.23. What is the probability that he will have a tooth extracted as well as a cavity filled ?

Q.46 In a town of 6000 people, 1200 are over 50 years old and 2000 are females. It is known that 30 % of the females are over 50 years. What is the probability that a randomly chosen individual from the town is either female or over 50 years ?

EXERCISE

1. $\frac{1}{2}$ **2.** $\frac{1}{6}$ **3.** $\frac{5}{18}$ **4.** $\frac{1}{12}$ **5.** $\frac{4}{9}$ **6.** $\frac{9}{20}$

7. $\frac{2}{7}$ **8.** $\frac{3}{4}$ **9.** $\frac{3}{8}$ **10.** $\frac{5}{6}$ **11.** (i) $\frac{5}{26}$ (ii) $\frac{21}{26}$

12. $\frac{6}{13}$ **13.** $\frac{2}{3}$ **14.** $\frac{1}{7}$ **15.** $\frac{2}{7}$ **16.** $\frac{1}{365}$

17. (ii) and (iv) **18.** $\frac{3}{10}$ **19.** $\frac{8}{21}$ **20.** $\frac{7}{11}$ **21.** (i) 5 : 9 (ii) 9 : 5

22. (i) 5 : 31 (ii) 5 : 1 **23.** $\frac{1}{504}$ **24.** $\frac{1}{38760}$ **25.** (i) $\frac{1}{36}$ (ii) $\frac{5}{108}$

26. 0.67 **27.** 0.3 **28.** 0.4 **29.** (i) 0.15 (ii) 0.1

30. (i) 0.1 (ii) 0.2

31. $\frac{5}{6}$ **32.** 0.3 **33.** $\frac{4}{13}$ **34.** $\frac{11}{15}$ **35.** $\frac{2}{13}$ **36.** $\frac{4}{13}$

37. $\frac{4}{13}$ **38.** $\frac{33}{100}$ **39.** $\frac{11}{36}$ **40.** $\frac{5}{9}$ **41.** $\frac{4}{9}$ **42.** $\frac{2}{5}$

43. $\frac{37}{45}$ **44.** $\frac{17}{105}$ **45.** 0.03 **46.** $\frac{13}{30}$