SURFACE AREA & VOLUME

FORMULAE

1. If ℓ, b and h denote respectively the length, breadth and height of a cuboid, then -

(i) total surface area of the cuboid = $2(\ell b + bh + \ell h)$ square units.

(ii) Volume of the cuboid

= Area of the base × height = ℓbh cubic units.

(iii) Diagonal of the cuboid or longest rod

$= \sqrt{\ell^2 + b^2 + h^2}$ units.

(iv) Area of four walls of a room

$= 2(\ell + b)\,h$ sq. units.

2. If the length of each edge of a cube is 'a' units, then-

(i) Total surface area of the cube = $6a^2$ sq. units.

(ii) Volume of the cube = a^3 cubic units

(iii) Diagonal of the cube = $\sqrt{3}$ a units.

3. If r and h denote respectively the radius of the base and height of a right circular cylinder, then -

(i) Area of each end = πr^2

(ii) Curved surface area = $2\pi rh$

= (circumference) height

(iii) Total surface area = $2\pi r(h + r)$ sq. units.

(iv) Volume = $\pi r^2 h$ = Area of the base × height

4. If R and r (R > r) denote respectively the external and internal radii of a hollow right circular cylinder, then -

(i) Area of each end = $\pi(R^2 - r^2)$

(ii) Curved surface area of hollow cylinder $= 2\pi(R + r)\,h$

(iii) Total surface area = $2\pi(R + r)(R + h - r)$

(iv) Volume of material = $\pi h(R^2 - r^2)$

5. If r, h and ℓ denote respectively the radius of base, height and slant height of a right circular cone, then-

(i) $\ell^2 = r^2 + h^2$

(ii) Curved surface area = $\pi r\ell$

(iii) Total surface area = $\pi r^2 + \pi r\ell$

(iv) Volume = $\frac{1}{3}\pi r^2 h$

6. For a sphere of radius r, we have

(i) Surface area = $4\pi r^2$

(ii) Volume = $\frac{4}{3}\pi r^3$

7. If h is the height, ℓ the slant height and r_1 and r_2 the radii of the circular bases of a frustum of a cone then -

(i) Volume of the frustum $= \frac{\pi}{3}(r_1^2 + r_1r_2 + r_2^2)\,h$

(ii) Lateral surface area = $\pi(r_1 + r_2)\,\ell$

(iii) Total surface area = $\pi\{(r_1 + r_2)\,\ell + r_1^2 + r_2^2\}$

(iv) Slant height of the frustum = $\sqrt{h^2 + (r_1 - r_2)^2}$

(v) Height of the cone of which the frustum is a part = $\frac{hr_1}{r_1 - r_2}$

(vi) Slant height of the cone of which the frustum is a part = $\frac{\ell r_1}{r_1 - r_2}$

(vii) Volume of the frustum

$= \frac{h}{3}\left\{A_1 + A_2 + \sqrt{A_1A_2}\right\}$, where A_1 and A_2 denote the areas of circular bases of the frustum.

❖ EXAMPLES ❖

Ex.1 A circus tent is in the shape of a cylinder, upto a height of 8 m, surmounted by a cone of the same radius 28 m. If the total height of the tent is 13 m, find:

(i) total inner curved surface area of the tent.

(ii) cost of painting its inner surface at the rate of ₹ 3.50 per m^2.

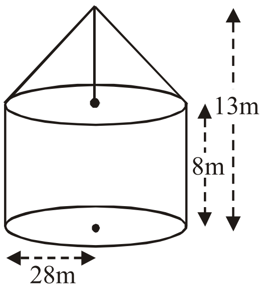
Sol. According to the given statement, the rough sketch of the circus tent will be as shown:

(i) For the cylindrical portion :

$r = 28$ and $h = 8$ m

$\therefore$ Curved surface area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 28 \times 8 \text{ m}^2 = 1408 \text{ m}^2$



For conical portion :

$r = 28$ m and $h = 13$ m $- 8$ m $= 5$ m

$\therefore \ \ell^2 = h^2 + r^2 \Rightarrow \ell^2 = 5^2 + 28^2 = 809$

$\Rightarrow \ \ell = \sqrt{809}$ m $= 28.4$ m

$\therefore$ Curved surface area $= \pi r\ell$

$= \frac{22}{7} \times 28 \times 28.4 \text{ m}^2 = 2499.2 \text{ m}^2$

$\therefore$ Total inner curved surface area of the tent.

= C.S.A. of cylindrical portion + C.S.A. of the conical portion

$1408 \text{ m}^2 + 2499.2 \text{ m}^2 = 3907.2 \text{ m}^2$

(ii) Cost of painting the inner surface

$= 3907.2 \times$ ₹ 3.50

$=$ ₹ 13675.20

Ex.2 A cylinder and a cone have same base area. But the volume of cylinder is twice the volume of cone. Find the ratio between their heights.

Sol. Since, the base areas of the cylinder and the cone are the same.

$\Rightarrow$ their radius are equal (same).

Let the radius of their base be r and their heights be h_1 and h_2 respectively.

Clearly, volume of the cylinder $= \pi r^2h_1$

and, volume of the cone $= \frac{1}{3}\pi r^2h_2$

Given :

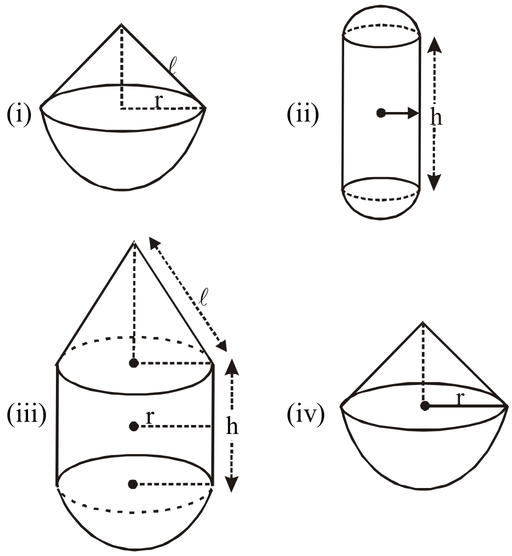
Volume of cylinder = 2 × volume of cone

$\Rightarrow \ \pi r^2h_1 = 2 \times \frac{1}{3}\pi r^2h_2$

$\Rightarrow \ h_1 = \frac{2}{3}h_2 \Rightarrow \frac{h_1}{h_2} = \frac{2}{3}$

i.e., $h_1 : h_2 = 2 : 3$

Ex.3 Find the formula for the total surface area of each figure given bellow :



Sol. (i) Required surface area

= C.S.A. of the hemisphere + C.S.A. of the cone

$= 2\pi r^2 + \pi r\ell = \pi r\,(2r + \ell)$

(ii) Required surface area

= 2 × C.S.A. of a hemisphere + C.S.A. of the cylinder

$= 2 \times 2\pi r^2 + 2\pi rh = 2\pi r\,(2r + h)$

(iii) Required surface area

= C.S.A. of the hemisphere

+ C.S.A. of the cylinder + C.S.A. of the cone

$= 2\pi r^2 + 2\pi rh + \pi r\ell = \pi r\,(2r + 2h + \ell)$

(iv) If slant height of the given cone be ℓ

$= \ell^2 = h^2 + r^2 \quad \Rightarrow \quad \ell = \sqrt{h^2 + r^2}$

And, required surface area

$= 2\pi r^2 + \pi r\ell = \pi r\,(2r + \ell)$

$= \pi r\left(2r + \sqrt{h^2 + r^2}\right)$

Ex.4 The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

Sol. Let the original radius be r.

$\Rightarrow$ Original surface area of the sphere $= 4\pi r^2$

Increase radius = r + 25% of r

$= r + \frac{25}{100}r = \frac{5r}{4}$

$\Rightarrow$ Increased surface area

$= 4\pi\left(\frac{5r}{4}\right)^2 = \frac{25\pi r^2}{4}$

Increased in surface area

$= \frac{25\pi r^2}{4} - 4\pi r^2 = \frac{25\pi r^2 - 16\pi r^2}{4} = \frac{9\pi r^2}{4}$

and, percentage increase in surface area

$= \frac{\text{Increase in area}}{\text{Original aera}} \times 100\%$

$= \frac{\frac{9\pi r^2}{4}}{4\pi r^2} \times 100\% = \frac{9}{16} \times 100\%$

= 56.25%

Alternative Method :

Let original radius = 100

$\Rightarrow$ Original C.S.A. $= \pi(100)^2 = 10000\pi$

Increased radius = 100 + 25% of 100 = 125

$\Rightarrow$ Increased C.S.A. $= \pi(125)^2 = 15625\pi$

Increase in C.S.A. $= 15625\pi - 10000\pi$

$= 5625\pi$

$\therefore$ Percentage increase in C.S.A.

$= \frac{\text{Increase in C.S.A.}}{\text{Original C.S.A}} \times 100\%$

$= \frac{5625\pi}{1000\pi} \times 100\% = 56.25\%$

Conversely, if diameter decreases by 20%, the radius also decreases by 20%.

Ex.5 Three solid spheres of radii 1 cm, 6 cm and 8 cm are melted and recasted into a single sphere. Find the radius of the sphere obtained.

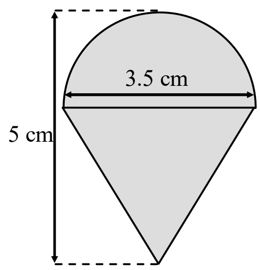
Sol. Let radius of the sphere obtained = R cm.

$\therefore \frac{4}{3} \times \pi R^3 = \frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3.$

$R^3 = 1 + 216 + 512 = 729$

$\therefore R = (9^3)^{1/3} = 9$ cm **Ans.**

Ex.6 In given figure the top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area it has to colour. (Take $\pi = \frac{22}{7}$)



Sol. TSA of the top = CSA of hemisphere + CSA of cone

Now, the curved surface of the hemisphere

$= \frac{1}{2}(4\pi r^2) = 2\pi r^2 = \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) cm^2$

Also, the height of the cone

= height of the top – height (radius) of the hemispherical part

$= \left(5 - \frac{3.5}{2}\right)$ cm = 3.25 cm

So, the slant height of the cone (l)

$= \sqrt{r^2 + h^2} = \sqrt{\left(\frac{3.5}{2}\right)^2 + (3.25)^2}$ cm

= 3.7 cm (approx.)

Therefore, CSA of cone = $\pi r l$

$= \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right) cm^2$

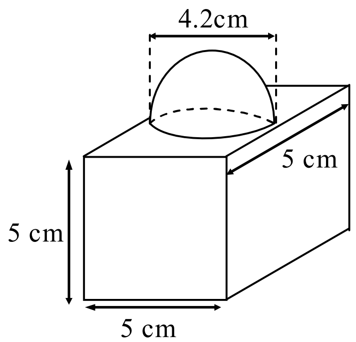
This gives the surface area of the top as

$\left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) cm^2 + \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right) cm^2$

$= \frac{22}{7} \times \frac{3.5}{2}(3.5 + 3.7) cm^2$

$= \frac{11}{2} \times (3.5 + 3.7)\ cm^2 = 39.6\ cm^2$ (approx)

Ex.7 The decorative block shown in figure is made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block. (Take $\pi = \frac{22}{7}$)



Sol. The total surface area of the cube $= 6 \times (\text{edge})^2 = 6 \times 5 \times 5\ cm^2 = 150\ cm^2$.

Note that the part of the cube where the hemisphere is attached is not included in the surface area.

So, the surface area of the block

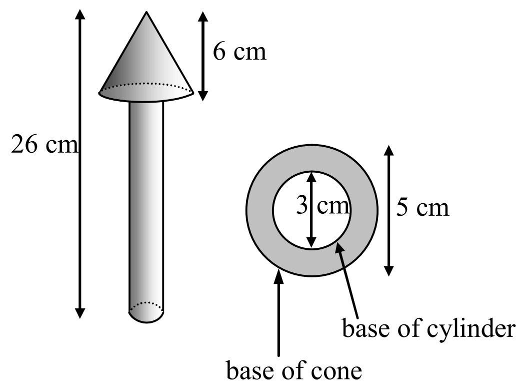
= TSA of cube – base area of hemisphere + CSA of hemisphere

$= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2)\ cm^2$

$= 150\ cm^2 + \left(\frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2}\right) cm^2$

$= (150 + 13.86)\ cm^2 = 163.86\ cm^2$

Ex.8 A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the best diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take $\pi = 3.14$)



Sol. Denote radius of cone by r, slant height of cone by ℓ, height of cone by h, radius of cylinder by r' and height of cylinder by h'. Then r = 2.5 cm, h = 6 cm, r' = 1.5 cm, h' = 26 – 6 = 20 cm and

$l = \sqrt{r^2 + h^2} = \sqrt{2.5^2 + 6^2}$ cm = 6.5 cm

Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So a part of the base of the cone (a ring) is to be painted.

So, the area to be painted orange

= CSA of the cone + base area of the cone – base area of the cylinder

$= \pi r l + \pi r^2 - \pi (r')^2$

$= \pi[(2.5 \times 6.5) + (2.5)^2 - (1.5)^2]\ \text{cm}^2$

$= \pi[20.25]\ \text{cm}^2 = 3.14 \times 20.25\ \text{cm}^2$

$= 63.585\ \text{cm}^2$

Now, the area to be painted yellow

= CSA of the cylinder + area of one base of the cylinder

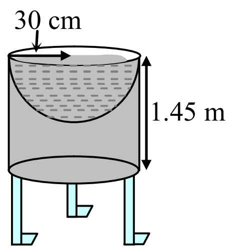
$= 2\pi r'h' + \pi (r')^2$

$= \pi r' (2h' + r')$

$= (3.14 \times 1.5)(2 \times 20 + 1.5)\ \text{cm}^2 = 4.71 \times 41.5\ \text{cm}^2$

$= 195.465\ \text{cm}^2$

Ex.9 A bird bath for garden in the shape of a cylinder with a hemispherical depression at one end (see figure). The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)



Sol. Let h be height of the cylinder and r the common radius of the cylinder and hemisphere. Then, the total surface area of the bird-bath

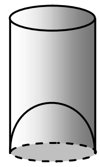
= CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r^2 = 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 30\ (145 + 30)\ \text{cm}^2$

$= 33000\ \text{cm}^2 = 3.3\ \text{m}^2$

Ex.10 A juice seller was serving his customers using glasses as shown in figure. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm. Find the apparent capacity of the glass and its actual capacity. (Use $\pi = 3.14$)



Sol. Since the inner diameter of the glass = 5 cm and height = 10 cm, the apparent capacity of the glass = $\pi r^2 h$

$= 3.14 \times 2.5 \times 2.5 \times 10\ \text{cm}^3 = 196.25\ \text{cm}^3$

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e. it is less by $\frac{2}{3}\pi r^3$

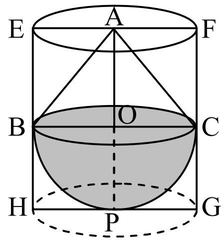
$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5\ \text{cm}^3 = 32.71\ \text{cm}^3$

So, the actual capacity of the glass = apparent capacity of glass – volume of the hemisphere

$= (196.25 - 32.71)\ \text{cm}^3$

$= 163.54\ \text{cm}^2$

Ex.11 A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and the toy. (Take $\pi = 3.14$)



Sol. Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere (see figure). The radius BO of the hemisphere (as well as of the cone) $= \frac{1}{2} \times 4\ \text{cm} = 2\ \text{cm}$

So, volume of the toy $= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$$= \left[\frac{2}{3} \times 3.14 \times (2)^3 + \frac{1}{3} \times 3.14 \times (2)^2 \times 2\right] \text{cm}^3$$

$= 25.12 \text{ cm}^3$

Now, let the right circular cylinder EFGH circumscribe the given solid. The radius of the base of the right circular cylinder

= HP = BO = 2 cm, and its height is

EH = AO + OP = (2 + 2) cm = 4 cm

So, the volume required

= Volume of the right circular cylinder – volume of the toy

$= (3.14 \times 2^2 \times 4 - 25.12) \text{ cm}^3$

$= 25.12 \text{ cm}^3$

$= 25.12 \text{ cm}^3$

Hence, the required difference of the two volumes = 25.12 cm^3.

Ex.12 A cone of height 24 cm and radius of base 6 cm is made up of modeling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Sol. Volume of cone = $\frac{1}{3} \times \pi \times 6 \times 6 \times 24 \text{ cm}^3$

If r is the radius of the sphere, then its volume is $\frac{4}{3}\pi r^3$.

Since the volume of clay in the form of the cone and the sphere remains the same, we have.

$$\frac{4}{3} \times \pi \times r^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24$$

$$r^3 = 3 \times 3 \times 24 = 3^3 \times 2^3$$

$$r = 3 \times 2 = 6$$

Therefore, the radius of the sphere is 6 cm.

Ex.13 Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (an underground tank) which is in the shape of a cuboid. The sump has dimensions 1.57 m × 1.44 m × 95 cm. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the sump after the overhead tank has been completely filled with water from the sump which had been full. Compare the capacity of the tank with that of the sump. (Use $\pi = 3.14$)

Sol. The volume of water in the overhead tank equals the volume of the water removed from the shump.

Now the volume of water in the overhead tank (cylinder) = $\pi r^2 h$

$= 3.14 \times 0.6 \times 0.6 \times 0.95 \text{ m}^3$

The volume of water in the sump when full

$= l \times b \times h = 1.57 \times 1.44 \times 0.95 \text{ m}^3$

The volume of water left in the sump after filling the tank

$= (1.57 \times 1.44 \times 0.95) - (3.14 \times 0.6 \times 0.6 \times 0.95)] \text{ m}^3$

$= (1.57 \times 0.6 \times 0.6 \times 0.95 \times 2) \text{ m}^3$

So, the height of the water left in the sump

$$= \frac{\text{volume of water left in the sump}}{\ell \times b}$$

$$= \frac{1.57 \times 0.6 \times 0.6 \times 0.95 \times 2}{1.57 \times 1.44} \text{ m}$$

= 0.475 m = 47.5 cm

Also, $\frac{\text{Capacity of tank}}{\text{Capacity of sump}}$

$$= \frac{3.14 \times 0.6 \times 0.6 \times 0.95}{1.57 \times 1.44 \times 0.95} = \frac{1}{2}$$

Therefore, the capacity of the tank is half the capacity of the sump.

Ex.14 A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Sol. The volume of the rod = $\pi \times \left(\frac{1}{2}\right)^2 \times 8 \text{ cm}^3$

$= 2\pi \text{ cm}^3$.

The length of the new wire of the same volume = 18 m = 1800 cm

If r is the radius (in cm) of cross section of the wire, its volume = $\pi \times r^2 \times 1800 \text{ cm}^3$

Therefore, $\pi \times r^2 \times 1800 = 2\pi$

i.e. $$r^2 = \frac{1}{900}$$

i.e. $$r = \frac{1}{30}$$

So, The diameter of the cross section i.e. the thickness of the wire is $\frac{1}{15}$ cm, i.e. 0.67 mm (approx.)