EXPONENTS AND POWERS

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EXPONENTS

The repeated addition of numbers can be written in short form (product form).

Examples :

S.No.	Statements	Repeated Addition	Products Form
(i)	4 times 2	2 + 2 + 2 + 2	4 × 2
(ii)	5 time – 1	(–1) + (–1) + (–1) + (–1) + (–1)	5 × (–1)
(iii)	3 times $\frac{-2}{3}$	$\left(\frac{-2}{3}\right)+\left(\frac{-2}{3}\right)+\left(\frac{-2}{3}\right)$	$3\times\left(\frac{-2}{3}\right)$
(iv)	2 times 1	1 + 1	2 × 1

Also, we can write the repeated multiplication of numbers in a short form known as exponential form.

For example, when 5 is multiplied by itself for two times, we write the product 5 × 5 in exponential form as 5^2 which is read as 5 raised to the power two.

Similarly, if we multiply 5 by itself for 6 times, the product 5 × 5 × 5 × 5 × 5 × 5 is written in exponential form as 5^6 which is read as 5 raised to the power 6.

In 5^6, the number 5 is called the **base** of 5^6 and 6 is called the **exponent** of the base.

In general, we write,

An exponential number as b^a, where b is the base and a is the exponent.

The notation of writing the multiplication of a number by itself several times is called the exponential notation or power notation.

Thus, in general we find that :

If 'a' is a rational number then 'n' times the product of 'a' by itself is given as a × a × a × a ..... , n times and is denoted by a^n, where 'a' is called the base and n is called the exponent of a^n.

❖ EXAMPLES ❖

Ex.1 Write the following statements as repeated multiplication and complete the table :

S. No.	Statements	Repeated Multiplication	Short form
(i)	3 multiplied by 3 for 6 times	3 × 3 × 3 × 3 × 3 × 3 = 729	3^6
(ii)	2 multiplied by 2 for 3 times	2 × 2 × 2	2^3
(iii)	1 multiplied by 1 for 7 times	1 × 1 × 1 × 1 × 1 × 1 × 1	1^7

Ex.2 Write the base and exponent of following numbers. And also write in expanded form :

S. No.	Numbers	Base	Exponent	Expanded Form	Value
(i)	3^4	3	4	3 × 3 × 3 × 3	81
(ii)	2^5	2	5	2×2×2×2×2	32
(iii)	3^3	3	3	3 × 3 × 3	27
(iv)	2^2	2	2	2 × 2	4
(v)	1^7	1	7	1 × 1 × 1 × 1 × 1 × 1 × 1	1

➤ EXPONENTS OF NEGATIVE INTEGERS

When the exponent of a negative integer is odd, the resultant is a negative number, and when the power of a negative number is even, the resultant is a positive number.

or (a negative integer)$^{\text{an odd number}}$ = a negative integer.

(a negative integer)$^{\text{an even number}}$ = a positive integer.

❖ EXAMPLES ❖

Ex.3 Express 144 in the powers of prime factors.

Sol. $144 = 16 \times 9 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Here 2 is multiplied four times and 3 is multiplied 2 times to get 144.

$\therefore \quad 144 = 2^4 \times 3^2$

Ex.4 Which one is greater : 3^5 or 5^3 ?

Sol. $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 9 \times 9 \times 3$

$= 81 \times 3 = 243$

and $5^3 = 5 \times 5 \times 5 = 25 \times 5 = 125$

Clearly, $243 > 125 \quad \therefore \quad 3^5 > 5^3$

➤ LAWS OF EXPONENTS

Law-1 : If a is any non-zero integer and m and n are whole numbers, then

$$a^m \times a^n = a^{m+n}$$

Eg :

(i) $$3^4 \times 3^2 = \underbrace{(3\times3\times3\times3)}_{\substack{\text{4 times multiplication}\\ \text{of 3 by itself}}} \times \underbrace{(3\times3)}_{\substack{\text{2 times multiplication}\\ \text{of 3 by itself}}}$$

$$= \underbrace{3\times3\times3\times3\times3\times3}_{\substack{\text{6 times multiplication}\\ \text{of 3 by itself}}} = 3^6 = 3^{4+2}$$

Thus, $3^4 \times 3^2 = 3^{4+2}$

(ii) $$2^3 \times 2^5 = \underbrace{(2\times2\times2)}_{\substack{\text{3 times multiplication}\\ \text{of 2 by itself}}} \times \underbrace{(2\times2\times2\times2\times2)}_{\substack{\text{5 times multiplication}\\ \text{of 2 by itself}}}$$

$$= \underbrace{2\times2\times2\times2\times2\times2\times2\times2}_{\substack{\text{8 times multiplication}\\ \text{of 2 by itself}}}$$

$= 2^8 = 2^{3+5}$

Thus, $2^3 \times 2^5 = 2^{3+5}$

Therefore, in general, we write,

$$a^m \times a^n = \underbrace{(a\times a\times a\times a\times\ldots)}_{\substack{\text{m times multiplication}\\ \text{of 'a' by itself}}} \times \underbrace{(a\times a\times a\times a\times\ldots)}_{\substack{\text{n times multiplication}\\ \text{of 'a' by itself}}}$$

$= a \times a \times a \times a \times a \times \ldots$, (m + n) times $= a^{m+n}$.

Law-2 :

If a and b are non-zero integers and m is a positive integer, then

$$a^m \times b^m = (a \times b)^m$$

Eg :

$5^3 \times 3^3 = (5 \times 5 \times 5) \times (3 \times 3 \times 3)$

$= (5 \times 3) \times (5 \times 3) \times (5 \times 3)$

$= 15 \times 15 \times 15 = (15)^3$

So, $5^3 \times 3^3 = (5 \times 3)^3 = (15)^3$

Here, we find that 15 is the product of bases 5 and 3.

Also, if a and b are non-zero integers, then

$a^5 \times b^5 = (a \times a \times a \times a \times a) \times (b \times b \times b \times b \times b)$

$= (a \times b)\,(a \times b)\,(a \times b)\,(a \times b)\,(a \times b) = (ab)^5$

Ex.5 Write in exponential form :

(i) $(5 \times 7)^6$ (ii) $(-7n)^5$

Sol. (i) $(5 \times 7)^6$

$= (5 \times 7)\,(5 \times 7)\,(5 \times 7)\,(5 \times 7)\,(5 \times 7)\,(5 \times 7)$

$= (5\times5\times5\times5\times5\times5)(7\times7\times7\times7\times7\times7) = 5^6\times7^6$

Hence, $(5 \times 7)^6 = 5^6 \times 7^6$

(ii) $(-7n)^5 = (-7n)\,(-7n)\,(-7n)\,(-7n)\,(-7n)$

$= (-7\times-7\times-7\times-7\times-7)\,(n\times n\times n\times n\times n)$

$= (-7)^5 \times (n)^5$

Law-3 :

If a is a non-zero integer and m and n are two whole numbers such that m > n, then

$$a^m \div a^n = a^{m-n}$$

and for m < n

$$a^m \div a^n = (a)^{m-n} = \frac{1}{a^{n-m}}$$

For example, $2^5 \div 2^7 = \dfrac{2\times2\times2\times2\times2}{2\times2\times2\times2\times2\times2\times2}$

$$= \frac{1}{2\times2} = \frac{1}{2^2} = \frac{1}{2^{7-5}}$$

When an exponential form is divided by another exponential form whose bases are same, then the resultant is an exponential form with same base but the exponent is the difference of the exponent of the divisor from the exponent of the dividend.

Law-4 :

Division of exponential forms with the same exponents and different base :

If a and b are any two non-zero integers, have same exponent m then for $a^m \div b^m$, we write

$$\frac{a^m}{b^m} = \frac{a \times a \times a \times ..., m \text{ times}}{b \times b \times b \times ..., m \text{ times}}$$

$$= \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times ..., m \text{ times} = \left(\frac{a}{b}\right)^m$$

For examples

(i) $2^6 \div 3^6 = \frac{2^6}{3^6} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^6$$

Hence, $2^6 \div 3^6 = \left(\frac{2}{3}\right)^6$

(ii) $(-2)^4 \div b^4 = \frac{(-2)^4}{b^4} = \frac{-2 \times -2 \times -2 \times -2}{b \times b \times b \times b}$

$$= \frac{-2}{b} \times \frac{-2}{b} \times \frac{-2}{b} \times \frac{-2}{b} = \left(-\frac{2}{b}\right)^4$$

Hence, $(-2)^4 \div b^4 = \left(-\frac{2}{b}\right)^4$

Ex.6 Write the following in expanded form :

(i) $\left(-\frac{7}{9}\right)^3$

(ii) $\left(\frac{5}{8}\right)^6$

Sol. (i) $\left(-\frac{7}{9}\right)^3 = \frac{-7}{9} \times \frac{-7}{9} \times \frac{-7}{9}$

$$= \frac{-7 \times -7 \times -7}{9 \times 9 \times 9} = \frac{(-7)^3}{9^3}$$

(ii) $\left(\frac{5}{8}\right)^6 = \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}$

$$= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{8 \times 8 \times 8 \times 8 \times 8 \times 8} = \frac{5^6}{8^6}$$

Law-5 :

If 'a' be any non-zero integer and m and n any two positive integers then

$$[(a)^m]^n = a^{mn}$$

Eg : $(2^2)^3 = 2^2 \times 2^2 \times 2^2 = 2^{2+2+2} = 2^6 = 2^{2 \times 3}$

$(2^7)^2 = 2^7 \times 2^7 = 2^{7+7} = 2^{14} = 2^{7 \times 2}$

Law-6 :

Law of zero Exponent

We know that

$$2^6 \div 2^6 = \frac{2^6}{2^6} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 1$$

By using Law-3 of exponents, we have

$2^6 \div 2^6 = 2^{6-6} = 2^0$

Thus, $2^0 = 1$

In general $a^m \div a^m = a^{m-m} = a^0$ and also

$$\frac{a^m}{a^m} = \frac{a \times a \times a \times a \times a \times a \times ...., m \text{ times}}{a \times a \times a \times a \times a \times a \times ...., m \text{ times}}$$

$= 1$

Hence, $\boxed{a^0 = 1}$

∴ Any non-zero integer raised to the power 0 always results into 1.

Ex.7 Find the value of :

(i) $(3^0 - 2^0) \times 5^0$

(ii) $2^0 \times 3^0 \times 4^0$

(iii) $(6^0 - 2^0) \times (6^0 + 2^0)$

Sol. (i) We have, $(3^0 - 2^0) \times 5^0$

Therefore, $(1 - 1) \times 1 = 0 \times 1 = 0$

[Since $3^0 = 1$, $2^0 = 1$]

(ii) We have, $2^0 \times 3^0 \times 4^0 = (1 \times 1 \times 1) = 1$

(iii) We have, $(6^0 - 2^0) \times (6^0 + 2^0)$

$= (1 - 1) \times (1 + 1)$

$= 0 \times 2 = 0.$

➢ USE OF EXPONENTS IN EXPRESSING LARGE NUMBERS

We know that

$100 = 10 \times 10 = 10^2$,

$1000 = 10 \times 10 \times 10 = 10^3$,

$10000 = 10 \times 10 \times 10 \times 10 = 10^4$

We can write a number followed by large number of zeroes in powers of 10.

For example, we can write the speed of light in vacuum = 300,000,000 m/s

$= 3 \times 1,00,000,000$ m/s $= 3 \times 10^8$ m/s

$= 30 \times 10^7$ m/s $= 300 \times 10^6$ m/s

Similarly,

the age of universe = 8,000,000,000 years (app.)

$= 8 \times 10^9$ years (app.)

We can also express the age of universe as 80×10^8 years or 800×10^7 years, etc.

But generally the number which preceded the power of 10 should be less than 10. Such a notation is called **standard or scientific** notation.

So 8×10^9 years is the standard form of the age of the universe.

Similarly, the standard form of the speed of light is 3×10^8 m/s.

❖ EXAMPLES ❖

Ex.8 Write the following numbers in standard form :

(i) 4340000

(ii) 173000

(iii) 140000

Sol. (i) It is clear that $4340000 = 434 \times 10000$

Also, $4340000 = 4.34 \times 10^6$

$\because \quad 434 = 4.34 \times 100 = 4.34 \times 10^2$

(ii) Also, $173000 = 1.73 \times 10^5$

(iii) Also, $140000 = 1.4 \times 10^5$

Ex.9 Express the following numbers in standard form :

(i) 980000000

(ii) 602000000000000000000000

Sol. We have,

(i) $980000000 = 9.8 \times 10^8$

(ii) 602000000000000000000000

$= 6.02 \times 10^{23}$

Ex.10 Write number in usual form :

(i) 1.001×10^9

(ii) 6.678×10^8

Sol. (i) We have $1.001 \times 10^9 = 1001000000$

(ii) We have, $6.678 \times 10^8 = 667800000$

Ex.11 Express the number appearing in the following statements in the form $K \times 10^n$, where $1 < K < 10$ and n is an integer.

(i) Every day about 1050000 kg pollutants are emitted in the capital of India.

(ii) The Earth has about 1,353000000 cubic km of sea water and this sea water contains around 1,361000,000 kg of gold.

Sol. (i) We have, $1050000 = 105 \times 10^4 = 10.5 \times 10^5$

$= 1.05 \times 10^6$

(ii) We have, $1,353,000,000 = 1,353 \times 10^6$

$= 135.3 \times 10^7$

$= 13.53 \times 10^8$

$= 1.353 \times 10^9$

and $1,361,000,000 = 1,361 \times 10^6$

$= 136.1 \times 10^7$

$= 13.61 \times 10^8$

$= 1.361 \times 10^9$

Ex.12 Write the following number in the usual form :

(i) 3.49×10^4 (ii) 1.11×10^6

Sol. (i) $3.49 \times 10^4 = 34900$

(ii) $1.11 \times 10^6 = 1110000$

Ex.13 Express the following number in the form $K \times 10^n$, where K is a number and n is an integer : 4176300000

Sol. We can write

4176300000 as 41763×10^5

or 4176.3×10^6 or 417.63×10^7

or 41.763×10^8 or 4.1763×10^9

Ex.14 Express 128 as power of 2 and also write its base and exponent.

Sol.

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

i.e., $128 = 2^7$

In 2^7, Base = 2

Exponent (power) = 7

Ex.15 Express 625 as a power of 5 and also write its base and exponent.

Sol.

5	625
5	125
5	25
5	5
	1

$625 = 5 \times 5 \times 5 \times 5$

i.e., $625 = 5^4$

In 5^4, Base = 5,

Exponent = 4

Ex.16 Which is smaller : 5^2 or 2^5 ?

Sol. $5^2 = 5 \times 5 = 25$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

$5^2 < 2^5$ $(\because \ 25 < 32)$

Ex.17 Which is greater : 2^7 or 7^2 ?

Sol. $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128.$

$7^2 = 7 \times 7 = 49 \Rightarrow 2^7 > 7^2$

Ex.18 Expand $y^3x^2, y^2x^3, x^2y^3, x^3y^2$. Are they same ?

Sol. $y^3x^2 = y \times y \times y \times x \times x$

$y^2x^3 = y \times y \times x \times x \times x$

$x^2y^3 = x \times x \times y \times y \times y$

$x^3y^2 = x \times x \times x \times y \times y$

In the case of x^3y^2 and x^2y^3 the powers of x and y are different. Thus x^3y^2 and x^2y^2 are different.

On the other hand, x^3y^2 and y^2x^3 are same, as the powers of x and y in these two terms are the same. The order of factors does not matter.

$x^3y^2 = x^3 \times y^2 = y^2 \times x^3 = y^2x^3.$

Similarly, x^2y^3 and y^3x^2 are same.

Ex.19 Express the following numbers as a product of powers of prime factors :

(i) 27 (ii) 512 (iii) 343

(iv) 729 (v) 3125

Sol. (i) $27 = 3 \times 3 \times 3 = 3^3 \Rightarrow 27 = 3^3$

(ii) $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$

$\Rightarrow 512 = 2^8$

(iii) $343 = 7 \times 7 \times 7 = 7^3 \Rightarrow 343 = 7^3$

(iv) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

$\Rightarrow 729 = 3^6$

(v) $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

$\Rightarrow 3125 = 5^5$

Ex.20 Simplify :

(i) $(-4)^3$ (ii) $-3 \times (-2)^3$

(iii) $(-4)^2 \times (-5)^2$ (iv) $(-2)^3 \times (-10)^3$

Sol. (i) $(-4)^3 = (-4) \times (-4) \times (-4) = 64$

$= 4 \times 4 \times 4 \times (-1)^3$

$(\because \ (-1)^{\text{odd number}} = \text{negative})$

(ii) $-3 \times (-2)^3 = (-3) \times (-2) \times (-2) \times (-2)$

$= 3 \times 2 \times 2 \times 2 \times (-1)^4$

$= 24 \times 1$

$(\because \ (-1)^{\text{even number}} = \text{positive})$

$= 24$

(iii) $(-4)^2 \times (-5)^2 = (-4) \times (-4) \times (-5) \times (-5)$

$= 4 \times 4 \times 5 \times 5 \times (-1)^4$

$= 16 \times 25 \times 1 = 400$

(iv) $(-2)^3 \times (-10)^3$

$= (-2) \times (-2) \times (-2) \times (-10) \times (-10) \times (-10)$

$= 2 \times 2 \times 2 \times 10 \times 10 \times 10 \times (-1)^6 = 8000$

Ex.21 Compare : 3.7×10^{12}, 2.5×10^{8}.

Sol. $3.7 \times 10^{12} = 3.7 \times 1000000000000$

$= 3700000000000$

$2.5 \times 10^{8} = 2.5 \times 100000000$

$= 25 \times 10000000$

$3.7 \times 10^{12} > 2.5 \times 10^{8}$

($\because$ place value of 10^{12} is greater than the place value of 10^{8})

Ex.22 Calculate : $3^2 \times 3^4$.

Sol. $3^2 \times 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^6 = 3^{2+4}$

Ex.23 Calculate : $(-2)^3 \times (-2)^4$.

Sol. $(-2)^3 \times (-2)^4$

$= (-2 \times -2 \times -2) \times (-2 \times -2 \times -2 \times -2)$

$= (-2)^7 = (-2)^{3+4}$.

Ex.24 Evaluate : $4^8 \div 4^3$

Sol. $4^8 \div 4^3 = \frac{4^8}{4^3} = \frac{4\times4\times4\times4\times4\times4\times4\times4}{4\times4\times4}$

$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 4^{8-3}$

Ex.25 Evaluate : $(4^2)^3$, $(2^4)^3$.

Sol.

$(4^2)^3 = 4^2 \times 4^2 \times 4^2$	&	$(2^4)^3 = 2^4 \times 2^4 \times 2^4$
$= 4^{2+2+2}$		$= 2^{4+4+4}$
$= 4^6$		$= 2^{12}$
($\because$ $a^m \times a^n \times a^p = a^{m+n+p}$)		($\because$ $a^m \times a^n \times a^p = a^{m+n+p}$)
$(4^2)^3 = 4^{2\times3}$		$(2^4)^3 = 2^{4\times3}$.

Ex.26 Evaluate : $(3^2 \times 4^2)$

Sol. $3^2 \times 4^2 = 3 \times 3 \times 4 \times 4 = (3 \times 4) \times (3 \times 4)$

$= 12 \times 12 = 12^2$

Ex.27 Evaluate : $(5^3 \times 2^3)$

Sol. $5^3 \times 2^3 = 5 \times 5 \times 5 \times 2 \times 2 \times 2$

$= 5 \times 2 \times 5 \times 2 \times 5 \times 2$

$= 10 \times 10 \times 10$

$\therefore$ $5^3 \times 2^3 = 10^3$

Ex.28 Evaluate : $\frac{3^2}{4^2}$, $\frac{4^4}{7^5}$.

Sol. $\frac{3^2}{4^2} = \frac{3\times3}{4\times4} = \frac{3}{4}\times\frac{3}{4} = \left(\frac{3}{4}\right)^2$

$\frac{4^5}{7^5} = \frac{4\times4\times4\times4\times4}{7\times7\times7\times7\times7} = \left(\frac{4}{7}\right)^5$

Ex.29 Evaluate : $\frac{x^3}{y^3}$.

Sol. $\frac{x^3}{y^3} = \frac{x\times x\times x}{y\times y\times y} = \left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right) = \left(\frac{x}{y}\right)^3$

Ex.30 Find the value of $\frac{4^3}{4^3}$.

Sol. **First method :** $\frac{4^3}{4^3} = \frac{4\times4\times4}{4\times4\times4} = 1$

Second method : Using laws of exponent.

$\frac{4^3}{4^3} = 4^{3-3}$ ($\because$ $a^m \div a^n = a^{m-n}$)

$= 4^0 = 1$.

Ex.31 Write exponential form for $9 \times 9 \times 9 \times 9 \times 9$ taking base as 3.

Sol. We have, $9 \times 9 \times 9 \times 9 \times 9 = 9^5$

$= (3 \times 3)^5$ ($\because$ $9 = 3 \times 3$)

($\because$ $a^m \times a^n = a^{m+n}$)

$= (3^2)^5$

$= 3^{2\times5} = 3^{10}$ [$\because$ $(a^m)^n = a^{mn}$]

Ex.32 Using laws of exponents, simplify and write the answer in exponential form :

(i) $2^3 \times 2^4 \times 2^7$ (ii) $4^{13} \div 4^8$

(iii) $5^2 \times 2^2$ (iv) $x^3 \times x^2$

(v) $6^x \times 6^2$ (vi) $(5^2)^3 \div 5^3$

(vii) $(3^4)^3$ (viii) $(2^{20} \div 2^{15}) \times 2^3$

(ix) $8^x \div 8^2$ (x) $a^5 \times b^5$

Sol. (i) $2^3 \times 2^4 \times 2^7 = 2^{3+4+7} = 2^{14}$

$(\because a^m \times a^n \times a^p = a^{m+n+p})$

(ii) $4^{13} \div 4^8 = 4^{13-8} = 4^5 = (2^2)^5$

$= 2^{2\times 5} = 2^{10}$

$(\because a^m \times a^n = a^{m+n}, (a^m)^n = a^{mn})$

(iii) $5^2 \times 2^2 = (5 \times 2)^2 = 10^2$

$(\because a^m \times b^m = (a \times b)^m)$

(iv) $x^3 \times x^2 = x^{3+2} = x^5 \quad (\because a^m \times a^n = a^{m+n})$

(v) $6^x \times 6^2 = 6^{x+2} \quad (\because a^m \times a^n = a^{m+n})$

(vi) $(5^2)^3 \div 5^3 = 5^{2\times 3} \div 5^3$

$= 5^6 \div 5^3 \quad [\because (a^m)^n = a^{mn}]$

$= 5^{6-3}$

$= 5^3$

(vii) $(3^4)^3 = 3^{4\times 3} = 3^{12} \quad [\because (a^m)^n = a^{mn}]$

(viii) $(2^{20} \div 2^{15}) \times 2^3 = (2^{20-15}) \times 2^3$

$(\because a^m \div a^n = a^{m-n})$

$= 2^5 \times 2^3 = 2^{5+3} = 2^8$

(ix) $8^x \div 8^2 = 8^{x-2} \quad (\because a^m \div a^n = a^{m-n})$

(x) $a^5 \times b^5 = (a \times b)^5$

Ex.33 Say true or false and justify your answer :

(i) $10 \times 10^{11} = 100^{11}$

(ii) $2^3 > 5^2$

(iii) $6^0 = (400)^0$

Sol. (i) $10 \times 10^{11} = 10^1 \times 10^{11}$

$= 10^{1+11} \quad (\because a^m \times a^n = a^{m+n})$

$\therefore \quad 10 \times 10^{11} = 10^{12}$

Also $100^{11} = (10 \times 10)^{11}$

$= (10^2)^{11} = 10^{2\times 11}$

$\therefore \quad 100^{11} = 10^{22}$

So, $10 \times 10^{11} = 100^{11} \rightarrow$ False

$(\because 10^{12} \neq 10^{22})$

(ii) $2^3 = 2 \times 2 \times 2 = 8$

$5^2 = 5 \times 5 = 25.$

$2^3 = 5^2 \rightarrow$ False $\quad (\because 8 \neq 25)$

(iii) $6^0 = 1 \quad (\because a^0 = 1)$

and $(400)^0 = 1$

$6^0 = 400^0 \rightarrow$ True $\quad (\because 1 = 1)$

Ex.34 Express each of the following as product of prime factors only in exponential form :

(i) 729×64 (ii) 270

(iii) 108×192 (iv) 512×216

Sol. (i) 729×64

$= (3\times 3\times 3\times 3\times 3\times 3)\times(2\times 2\times 2\times 2\times 2\times 2)$

$(\because 729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3)$

$729 \times 64 = 3^6 \times 2^6 = (3 \times 2)^6 = 6^6$

$279 \times 64 = 3^6 \times 2^6 = (3 \times 2)^6 = 6^6$

(ii)

2	270
3	135
3	45
3	15
5	5
	1

$270 = 2 \times 3 \times 3 \times 3 \times 5$

$270 = 2 \times 3^3 \times 5$

(iii)

2	108
2	54
3	27
3	9
3	3
	1

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$108 \times 192 = (2 \times 2 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3)$

$= (2^2 \times 3^3) \times (2^6 \times 3)$

$= 2^{2+6} \times 3^{3+1}$

$= 2^2 \times 2^6 \times 3^3 \times 3 = 2^8 \times 3^4$

(iv)

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2	216
2	108
2	54
3	27
3	9
3	3
	1

$512 \times 216 = (2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2) \times (2 \times 2 \times 2 \times 3 \times 3 \times 3)$

$= 2^9 \times 2^3 \times 3^3 \quad = 2^{9+3} \times 3^3 \quad = 2^{12} \times 3^3$

Ex.35 Simplify :

(i) $\dfrac{(2^5)^2 \times 7^3}{8^3 \times 7}$

(ii) $\frac{25 \times 5^2 \times x^8}{10^3 \times x^4}$

(iii) $\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

(iv) $\frac{4^7 \times 3^4}{4^4 \times 4^3 \times (3^2)^2}$

Sol. (i) $\frac{(2^5)^2 \times 7^3}{8^3 \times 7} = \frac{2^{5\times2} \times 7^3}{(2^3)^3 \times 7}$ [$\because$ $(a^m)^n = a^{mn}$]

$= \frac{2^{10} \times 7^3}{2^9 \times 7}$

$= 2^{10-9} \times 7^{3-1}$ ($\because$ $a^m \div a^n = a^{m-n}$)

$= 2^1 \times 7^2$

$= 2 \times 49$

$= 98$

(ii) $\frac{25 \times 5^2 \times x^8}{10^3 \times x^4} = \frac{5^2 \times 5^2 \times x^8}{(5 \times 2)^3 \times x^4}$

$= \frac{5^2 \times 5^2 \times x^8}{5^3 \times 2^3 \times x^4}$ [$\because$ $(a \times b)^m = a^m \times b^m$]

$= \frac{5^4 \times x^8}{5^3 \times 2^3 \times x^4}$

$= \frac{5^{4-3} \times x^{8-4}}{2^3}$ ($\because$ $a^m \div a^n = a^{m-n}$)

$= \frac{5 \times x^4}{2^3} = \frac{5}{8}x^4$

(iii) $\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5} = \frac{3^5 \times (5 \times 2)^5 \times 5^2}{5^7 \times (3 \times 2)^5}$

[$\because$ $(a \times b)^m = a^m \times b^m$]

$= \frac{3^5 \times 5^5 \times 2^5 \times 5^2}{5^7 \times 3^5 \times 2^5}$ ($\because$ $a^m \div b^n = a^{m-n}$)

$= 3^{5-5} \times 5^{5+2-7} \times 2^{5-5}$ ($\because$ $a^0 = 1$)

$= 1 \times 1 \times 1 = 1$

(iv) $\frac{4^7 \times 3^4}{4^4 \times 4^3 \times (3^2)^2}$

$= \frac{4^7 \times 3^4}{4^4 \times 4^3 \times 3^{2\times2}}$ [$\because$ $(a^m)^n = a^{mn}$]

$= \frac{4^7 \times 3^4}{4^7 \times 3^4}$ ($\because$ $a^m \div a^n = a^{m-n}$)

$= 4^{7-7} \times 3^{4-4}$

$= 4^0 \times 3^0$

$= 1 \times 1 = 1$ ($\because$ $a^0 = 1$)

Ex.36 Simplify and express each of the following in exponential form :

(i) $25^4 \div 5^3$

(ii) $2^0 \times 3^0 \times 4^0$

(iii) $2^0 + 3^0 + 4^0$

(iv) $\frac{2^8 \times a^5}{4^3 \times a^3}$

(v) $(3^0 + 2^0) \times 5^0$

Sol. (i) $25^4 \div 5^3 = (5^2)^4 \div 5^3$

$= 5^8 \div 5^3$ ($\because$ $a^m \div a^n = a^{m-n}$)

$= (5)^{8-3} = 5^5$.

(ii) $2^0 \times 3^0 \times 4^0 = 1 \times 1 \times 1$ ($\because$ $a^0 = 1$)

$= 1$

(iii) $2^0 + 3^0 + 4^0 = 1 + 1 + 1$ ($\because$ $a^0 = 1$)

$= 3$

(iv) $\frac{2^8 \times a^5}{4^3 \times a^3} = \frac{2^8 \times a^5}{(2^2)^3 \times a^3} = \frac{2^8 \times a^5}{2^6 \times a^3}$ [$\because$ $(a^m)^n = a^{mn}$]

$= 2^{8-6} \times a^{5-3}$ ($\because$ $a^m \div a^n = a^{m-n}$)

$= 2^2 \times a^2 = (2 \times a)^2 = (2a)^2$

(v) $(3^0 + 2^0) \times 5^0$

$= (1 + 1) \times 1$ ($\because$ $a^0 = 1$)

$= 2 \times 1$

Ex.37 Express the following numbers in the standard form :

(i) 35794.8 (ii) 78,640

(iii) 2,160,000 (iv) 60,090,000,000,000

Sol. (i) Four digit shifted so multiply by 1 followed by 4 zeroes.

$35794.8 = 3.57948 \times 10000$

$= 3.57948 \times 10^4$.

(ii) $78,640 = 78640.0$

$= 78640 \times 10000 = 7.864 \times 10^4$

(iii) $2,160,000 = 2160000.0$

$= 2.160,000 \times 10^6 = 2.16 \times 10^6$

(iv) 60,090,000,000,000

$= 60,090,000,000,000.0$

$= 6.0090,000,000,000 \times 10^{13}$

$= 6.009 \times 10^{13}$.

Ex.38 Write the following in expanded form using exponent :

(i) 279404 (ii) 3006194 (iii) 20068

Sol. (i) $279404 = 2 \times 100000 + 7 \times 10000 + 9 \times 1000 + 4 \times 100 + 00 + 4 \times 1$

$= 2 \times 10^5 + 7 \times 10^4 + 9 \times 10^3 + 4 \times 10^2 + 4 \times 10^0$

(ii) 3006194

$= 3 \times 1000000 + 0 \times 100000 + 0 \times 10000 + 6 \times 1000 + 1 \times 100 + 9 \times 10 + 4 \times 1$

$= 3 \times 10^6 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$

(iii) $20068 = 2 \times 10000 + 6 \times 10 + 8 \times 1$

$= 2 \times 10^4 + 6 \times 10^1 + 8 \times 10^0$

Ex.39 Find the number from each of the following expanded forms :

(i) $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

(ii) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

(iii) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

Sol. (i) $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

$= 8 \times 10000 + 6 \times 1000 + 0 \times 100 + 4 \times 10 + 5 \times 1$

$= 80000 + 6000 + 0 + 40 + 5 = 86045$.

(ii) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

$= 9 \times 100000 + 2 \times 100 + 3 \times 10$

$= 900000 + 200 + 30 = 900230$

(iii) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

$= 4 \times 100000 + 5 \times 1000 + 3 \times 100 + 2 \times 1$

$= 400000 + 5000 + 300 + 2 = 405302$