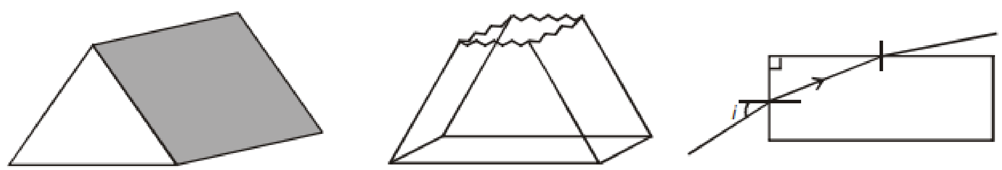
# RAY OPTICS AND OPTICAL INSTRUMENTS

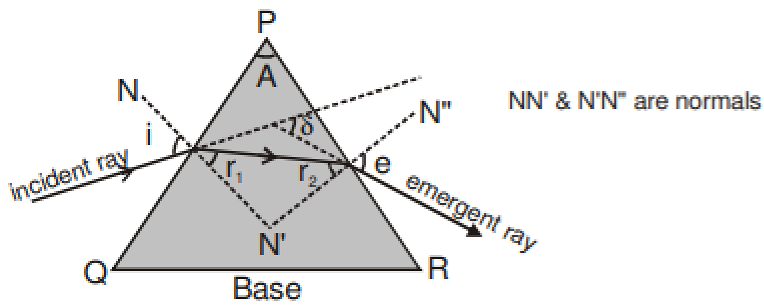
## REFRACTION THROUGH A PRISM

### PRISM

A prism is defined as a uniform and transparent solid medium with refractive properties, characterized by two planar surfaces that are inclined at an angle to each other. This geometric configuration provides a three-dimensional perspective when considering the structure and properties of a prism.




### Refraction through a prism:




**a.** Surfaces PQ and PR function as refracting surfaces.

**b.** The angle ∠QPR, denoted as A, is referred to as the refracting angle or the prism's angle (also recognized as the Apex angle).

**c.** The symbol δ represents the angle of deviation.

**d.** In the context of the refraction of a monochromatic (single wavelength) light ray through a prism, the relationship is expressed as follows:

$$\delta = (i+e) - (r_1 + r_2)$$

Where $r_1 + r_2 = A$

Therefore, the equation can be simplified as:

$$\delta = i + e - A$$

Observations:

**1.** When a ray traverses through two surfaces that are inclined to each other, the optical phenomenon is analyzed using the concept of a prism.

**2.** Alternatively, if a ray crosses two planar surfaces that are parallel to each other, the optical analysis involves the concept of a slab.

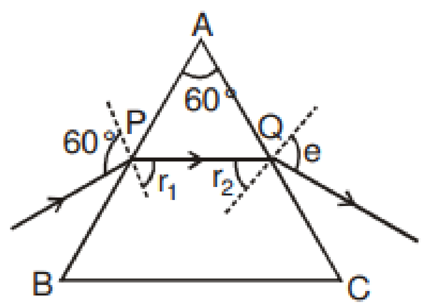
**Example.**

A light ray impinges upon a face of a prism (μ = 1.5) at an incident angle of 60°, matching the refracting angle of the prism. The task at hand is to determine the angles of emergence and deviation. Additionally, exploration is required to identify any alternative angle of incidence that would yield an equivalent deviation.

**Solution.**

Angle of incidence = i = 60°

At point P, $\frac{\sin 60^\circ}{\sin r_1} = \frac{15}{1} \Rightarrow \sin r_1 = \frac{1}{\sqrt{3}}$




Or, r1 ≈ 35°6

Using $r_1 + r_2 = A$, we get

$r_2 = A - r_1 = 60^\circ - 35^\circ 6' = 24^\circ 44'$

At point Q $\frac{\sin_2}{\sin e} = \frac{1}{1.5}$

$\Rightarrow \sin e = 1.5 \sin 24^\circ 44' \Rightarrow \sin e = 0.63$

$\Rightarrow e = 39^\circ$

∴ Deviation = $\delta = (i + e) - A = 60^\circ + 39^\circ - 60^\circ = 39^\circ$

If i and e are interchanged, deviation remains the same. Hence same deviation is obtained for angles of incidence 60° and 39°.

## Graph between ∠δ and ∠i

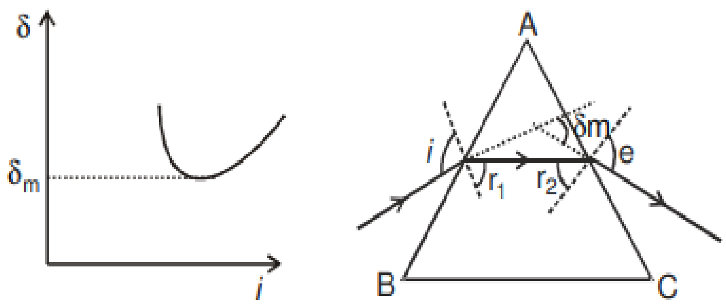
**1.** The diagram illustrates the relationship between the angle of deviation ($\delta$) and the angle of incidence($i$). Notably, for a given angle of deviation (excluding the minimum deviation $\delta_{min}$ ), there exist two corresponding values of the angle of incidence. The interchange of angles $i$ and $e$ yields an identical value of ($\delta$), a phenomenon attributed to the reversibility principle of light.

**2.** There exists a singular angle of incidence that minimizes the angle of deviation, denoted as $\delta_{min}$.

**3.** On the graphical representation, the right-hand side segment of the curve exhibits a steeper inclination compared to the left-hand side.

## Minimum Deviation and Condition for Minimum Deviation:

The angle of deviation is intricately linked to the angle of incidence, demonstrating a specific relationship. When the angle of incidence is minimal, the corresponding deviation is notably substantial. As the angle of incidence experiences an increment, the angle of deviation undergoes a rapid reduction, reaching a minimum value before gradually ascending with further increases in the angle of incidence. This minimum value of deviation, denoted as $\delta_{min}$, is termed the minimum deviation.




Theoretical analysis and experimental observations converge to reveal that the angle of deviation $\delta$ attains its minimum value when the trajectory of the light ray passing through the prism exhibits symmetry.

i.e., angle of incidence = angle of emergence

Or, $\angle i = \angle e$

For the refraction at the face AB, we have

$$\frac{\sin i}{\sin r_1} = \mu(snell's\ law)\ or\ \sin i = \mu \sin r_2$$

$$\frac{\sin e}{\sin r_2} = \mu \text{ Or, } \sin e = \mu \sin r_2$$

$$\mu \sin r_1 = \mu \sin r_2$$

$$r_1 = r_2$$

Hence, the condition for minimum deviation is

$$i = e \text{ and } r_1 = r_2 \qquad .... (19)$$

Relation between Refractive index and the angle of Minimum Deviation

When $\delta = \delta m$, we have

$$e = i \text{ and } r_1 = r_2 = r \text{ (say)}$$

We know

$$A = r_1 + r_2 = r + r = 2r \qquad \text{or,} \qquad r = \frac{A}{2}$$

Also, $A + \delta = i + e$

Or, $A + \delta_m = i + i$

Or, $i = \frac{A+\delta_m}{2}$

The refractive index of the material of the prism is given by

$$\mu = \frac{\sin i}{\sin r}(snell's\ law)\ or\ \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

If surrounding medium has refractive index = $n_s$

Then $$\frac{n_p}{n_s} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

**Example.**

A light ray strikes the face of an equilateral prism at an angle of 49° and proceeds through the prism symmetrically. The task at hand is to compute the refractive index of the material constituting the prism.

**Solution.**

As the prism is an equilateral one, A = 60°. As the ray of light passes symmetrically, the prism is in the position of minimum deviation.

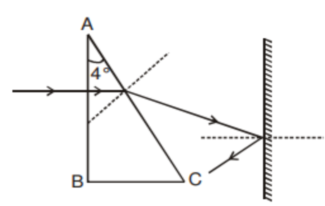
$$r = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$$

Also, i = 49°

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 40^\circ}{\sin 30^\circ} = \frac{0.7547}{0.5} = 1.5$$

**Example.**

A light ray strikes the face of an equilateral prism at an angle of 49° and proceeds through the prism symmetrically. The task at hand is to compute the refractive index of the material constituting the prism.



**Solution.**

The deviation suffered by refraction through the small angled prism is given by

$\delta = (\mu - 1) A = (1.5 - 1) \times 4° = 2°$

This gives the angle of incidence 2° at the mirror.