RAY OPTICS AND OPTICAL INSTRUMENTS

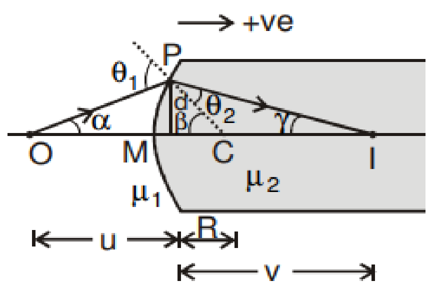
REFRACTION AT SPHERICAL SURFACES AND BY LENSES

REFRACTION FROM A SPHERICAL SURFACE

Let's examine two transparent media characterized by indices of refraction μ_1 and μ_2, with the interface between them forming a spherical surface of radius R. It is presumed that $\mu_1 < \mu_2$. Envision a single ray originating from point O and converging at point I. The application of Snell's law to this refracted ray is expressed as:

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

Where θ_1 and θ_2 denote the angles of incidence and refraction, respectively.



Given the assumption that θ_1 and θ_2 are of small magnitude, we can employ the small angle approximation, expressed as $\sin\theta \approx \theta$.

(Angles in radians) and say that

$\mu_1 \theta_1 = \mu_2 \theta_2$ ... (1)

From the geometry shown in the figure.

$\theta 1 = \alpha + \beta$ ... (2)

And $\beta = \theta 2 + \gamma$ ... (3)

Equations (1) and (3) can be integrated to formulate an expression for θ_2 in relation to the angles α and β. Subsequently, by substituting this derived expression into Equation (2), we obtain the following result.

$$\beta = \frac{\mu_1}{\mu_2}(\alpha + \beta) + \Upsilon$$

So,

$$\mu_1\alpha + \mu_2\Upsilon = (\mu_2 - \mu_1)\beta \quad \text{... (4)}$$

Considering that the arc PM, with a length denoted as S, creates an angle β at the center of curvature,

$$\beta = \frac{S}{R}$$

Also, in the paraxial approximation

$$\alpha = \frac{S}{U} \ and \ \Upsilon = \frac{S}{V}$$

Using these expressions in Eq. (4) with proper signs, we are left with

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

Or

$$\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

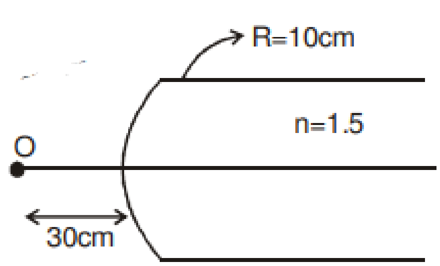
While the formula (5) has been derived for a specific scenario, it holds true for all other cases involving refraction at a solitary spherical surface.

Important point for above formula

- The formula presented above is applicable solely to paraxial rays.
- When employing the formula, it is essential to consider the signs of u, v, and R.
- μ_2 Represents the refractive index of the medium to which the rays are transitioning, while μ_1 denotes the refractive index of the medium from which the rays originate.

Example.

Determine the location of the resulting image and construct the suitable ray diagram given the following parameters:
u = – 30 cm
R = + 10 cm



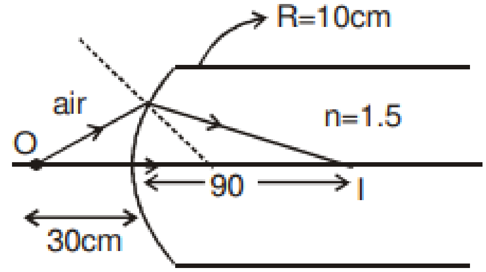
Solution.

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1.5}{v} = \frac{1}{30} = \frac{0.5}{10}$$

$$\frac{1.5}{v} = \frac{0.5}{10} - \frac{1}{30}$$

$$\frac{1.5}{v} = \frac{0.5}{30} \Rightarrow v = \frac{30 \times 1.5}{0.5} = 90cm$$

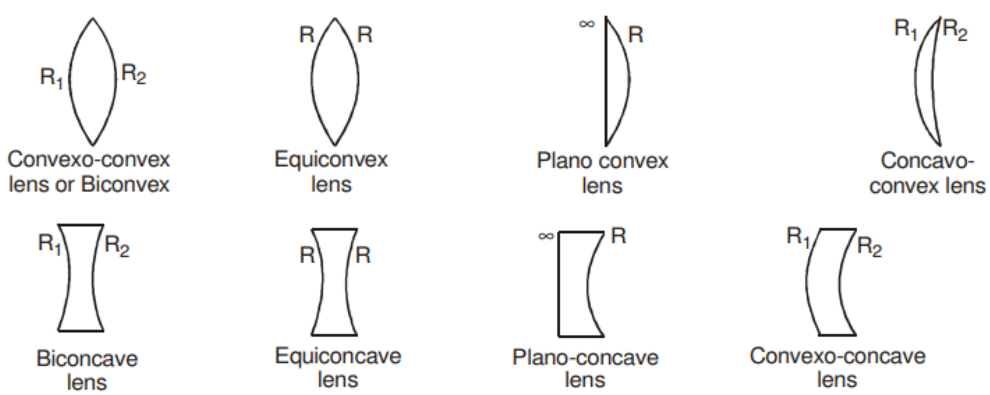


REFRACTION THROUGH THIN LENSES

Lens: A lens is defined as a transparent medium encompassed by two refracting surfaces, and it is stipulated that at least one of these refracting surfaces is curved, typically taking on a spherical shape.

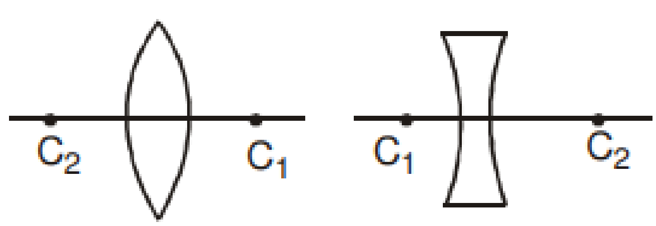
Types of lenses

Categories of Lenses: In a general classification, lenses can be categorized into the following types:



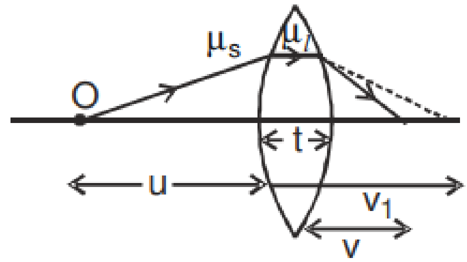
(i) Principal Axis:

The principal axis is defined as the line connecting the centers of curvature of the two bounding surfaces of the lens.



Lens Maker formula

For 1st refraction:



$$\frac{\mu_\ell}{v_1}+\frac{\mu_s}{u}=\frac{\mu_\ell-\mu_s}{R_1}\text{ b} \qquad \dots (1)$$

For 2nd refraction:

$$\frac{\mu_s}{v}+\frac{\mu_\ell}{v_1}=\frac{\mu_s-\mu_\ell}{R_2} \qquad \dots (2)$$

Adding (1) and (2) equation we get

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_\ell}{\mu_s}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

POWER OF A LENS.

$$Power=\frac{1}{f}(diopter)$$

Where f → meter and f should be put with sign
Power of converging lens. = + ve
Power of diverging lens. = – ve

Important points for the above formula:

1. It is essential that the rays are paraxial.

2. When working with the formula, it is imperative to include the appropriate signs for v, u, R_1 and R_2.

3. R_1 Corresponds to the radius of curvature of the surface initially encountered by the ray.

4. The lens should exhibit thin characteristics.

5. The medium on both sides of the lens must be identical.

Sign Convention (consider pole as origin)

1. Whenever it is feasible, light rays are assumed to propagate from left to right.

2. When evaluating distances, they are measured along the principal axis starting from the optical center of the lens.

3. Distances along the principal axis in the direction of incident rays are considered positive, whereas those measured against the direction of incident rays are deemed negative.

4. Distances above the principal axis are assigned positive values, while those below the principal axis are assigned negative values.

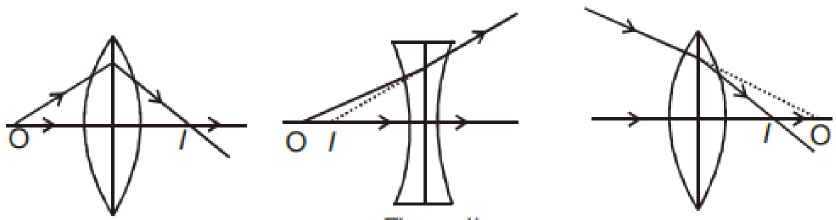
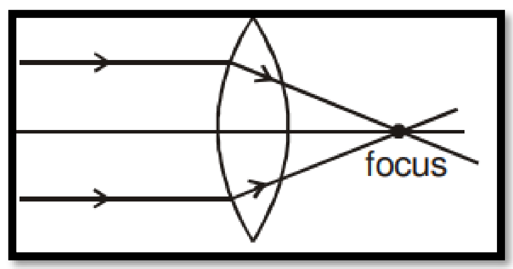


Figure I Figure II Figure III

Focus

In the realm of optical systems, the term "focus" denotes the specific point at which rays of light, characterized by their parallel alignment with the optical axis and their confinement within a small angle, come together or seem to come together. This focal point plays a crucial role in various optical phenomena and devices.



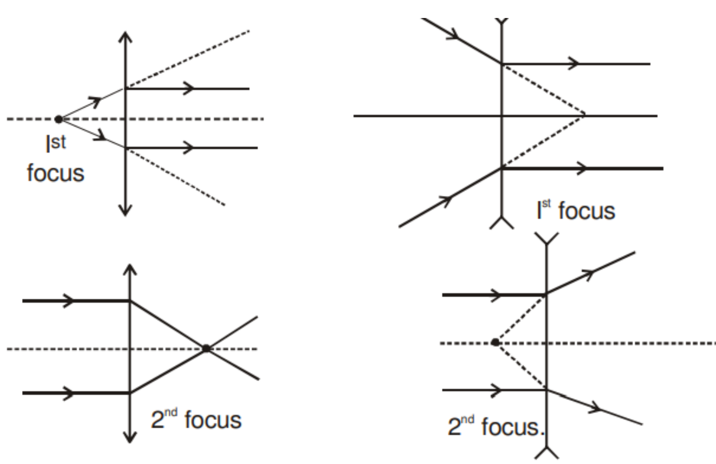
In the lens maker formula if u →∞, v = f

$$\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Substituting in the lens maker formula:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ (Lens formula)}$$

Lenses have two foci called first and second focus



Example.

Determine the focal length of a double-convex lens in a gaseous medium with radii of curvature measuring 60 cm and 15 cm. The lens is composed of glass with a refractive index of 1.5.

Solution.

Consider a light ray passing through the lens, impacting the convex side with a 60 cm radius and the concave side with a 15 cm radius upon exiting.

R1 = + 60 cm, R2 = – 15 cm

$$\therefore \frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\therefore \frac{1}{f} = \left(\frac{1.5}{1} - 1\right)\left[\frac{1}{60} - \frac{1}{15}\right]$$

f = + 24 cm

- Since for converging and diverging lenses
- Focal length of lens depends on surrounding medium.
- If f = + ve implies converging and if f = – ve implies diverging lens.