ELECTROMAGNETIC WAVES

DISPLACEMENT CURRENT

INTRODUCTION

Maxwell formulated a set of equations that encompassed electric and magnetic fields, as well as their sources, which are the charge and current densities. These equations are renowned as Maxwell's equations and, when combined with the Lorentz force formula, they provide a mathematical foundation for all the fundamental principles of electromagnetism.

One of the most significant insights resulting from Maxwell's equations is the prediction of electromagnetic waves. These waves consist of time-varying electric and magnetic fields that propagate through space. Remarkably, these equations also revealed that the speed of these waves is nearly identical to the speed of light, approximately 3×10^8 m/s, as determined through optical measurements. This remarkable discovery led to the conclusion that light itself is an electromagnetic wave.

Maxwell's groundbreaking work unified the realms of electricity, magnetism, and light. In 1885, Hertz conducted experiments that experimentally validated the existence of electromagnetic waves. The practical utilization of this knowledge by inventors like Marconi subsequently triggered a revolution in communication technology, which continues to shape the modern world.

In this chapter, we first explore the concept of displacement current and its ramifications. Subsequently, we provide a comprehensive overview of electromagnetic waves, covering a broad spectrum that spans from γ rays with very short wavelengths ($\sim 10^{-12}$ m) to long radio waves with considerably larger wavelengths (~106 m).

DISPLACEMENT CURRENT

We've observed that an electrical current generates a magnetic field in its vicinity. However, Maxwell demonstrated that for the sake of logical consistency, a changing electric field must also give rise to a magnetic field. This insight is profoundly significant as it elucidates the existence of a wide range of electromagnetic waves, including radio waves, gamma rays, and visible light.

To understand how a changing electric field leads to the emergence of a magnetic field, let's examine the process of charging a capacitor and apply Ampere's circuital law, which is articulated as

$$\int B.dl = \mu_0 it \qquad \text{..... (1)}$$

To determine the magnetic field at a point located outside the capacitor. Consider Figure 1(a), which illustrates a parallel plate capacitor denoted as C, forming part of a circuit through which a time-dependent current, i (t), is flowing. Our objective is to ascertain the magnetic field at a point like P, situated in a region external to the parallel plate capacitor.

To achieve this, we contemplate a planar circular loop with a radius r. The plane of this loop is oriented perpendicular to the direction of the current-carrying wire, and it is symmetrically centered concerning the wire.

Due to the inherent symmetry of this configuration, the magnetic field encircles the circular loop, and its magnitude remains consistent at all points along the loop's circumference. Thus, if B represents the field's magnitude, the left side of the equation reads as B (2πr). Consequently, we can express it as:

$B(2\pi r) = \mu_0 i(t)$ ..... (2)

This equation signifies the relationship between the magnetic field, the loop's circumference, and the time-dependent current flowing through the wire.

Now, let's contemplate a different type of surface—one that shares the same boundary as the one previously discussed. This new surface takes on a pot-like shape (as shown in Figure 1(b)). Importantly, this surface doesn't intersect with the current path at any point, but its base is positioned between the plates of the capacitor, and its mouth corresponds to the circular loop we mentioned earlier.

Similarly, we can envision another surface with the shape of a container, resembling a tiffin box without the lid (as depicted in Figure 1(c)). When we apply Ampere's circuital law to these surfaces, both of which possess the same perimeter as the previous one, a peculiar observation emerges. On the left-hand side of Equation (1), nothing changes—it remains the same. However, on the right-hand side, we now find that it is zero, not μ0i, because no current traverses through the surfaces (b) and (c).

This situation presents us with a paradox. When we calculate it one way, it suggests the existence of a magnetic field at a point P. Yet, when calculated from a different perspective, the magnetic field at P appears to be non-existent. This incongruity arises from our utilization of Ampere's circuital law, indicating that there might be something missing in the law. This missing component must be such that it yields the same magnetic field at point P, irrespective of the chosen surface for calculation.

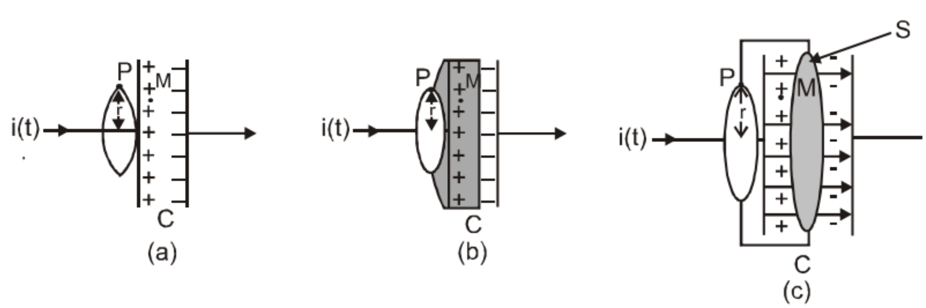

Figure 1

To identify the missing term, we can focus on the surface (c) and observe closely. Is there anything passing through the surface S positioned between the plates of the capacitor? Indeed, there is—the electric field. If the plates of the capacitor have a specific area denoted as A and carry a total charge Q, the magnitude of the electric field E between these plates can be calculated as (Q/A)/ε0. This electric field is oriented perpendicularly to the surface S of the capacitor.

The electric field's magnitude remains uniform over the entire area A of the capacitor plates and becomes negligible beyond this region. Consequently, we can calculate the electric flux ΦE through the surface S using Gauss's law as follows:

$$\Phi_E = |E|A = \frac{1}{\varepsilon_0}\frac{Q}{A}A = \frac{Q}{\varepsilon_0} \qquad \text{..... (3)}$$

This equation quantifies the electric flux ΦE, illustrating that it equals Q/ε0, taking into account the magnitude of the electric field and the surface area A of the capacitor plates.

Now, in the event that the charge Q on the plates of the capacitor experiences changes over time, it gives rise to a current, which can be expressed as i = (dQ / dt). Thus, by applying Equation (3), we can write:

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt}$$

This implies that for consistency

$$\varepsilon_0\left(\frac{d\Phi_E}{dt}\right) = i \qquad \text{..... (4)}$$

This corresponds to the missing component in Ampere's circuital law. To rectify this, we can generalize the law by introducing an additional term. This term is ε0 times the rate at which the electric flux through the surface changes over time. By augmenting the law in this manner, the total value of the current i remains consistent for all surfaces. With this adjustment, there is no longer any discrepancy in the magnetic field B value obtained at any location when using the generalized Ampere's law.

Now, at a specific point P, the magnetic field B is non-zero, regardless of the chosen surface for calculation. In the case of a point P located outside the plates (as shown in (a)), the magnetic field value is the same as that at a point M situated just inside the plates, in accordance with expectations.

The current generated by conductors due to the flow of electric charges is referred to as the conduction current. In addition to this conduction current, a new term emerges, which arises from the changing electric field or electric displacement. This new term is known as displacement current or Maxwell's displacement current. It plays a crucial role in understanding the electric and magnetic fields within the parallel plate capacitor discussed previously.

The generalization introduced by Maxwell can be summarized as follows: The source of a magnetic field is not solely the conduction current arising from the flow of electric charges but also the time rate of change of the electric field. More precisely, the total current i encompasses both the conduction current denoted as i_C and the displacement current denoted as i_d $(=\varepsilon_0(d\Phi_E)/dt)$. Therefore, we can express this as:

$$i = i_e + i_d = i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{..... (5)}$$

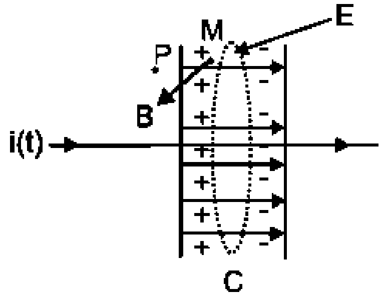

Figure 2 (a)

To elaborate further, let's clarify that beyond the plates of the capacitor, we solely encounter conduction current, which can be represented as $i_c = i$. There is no presence of displacement current in this region, meaning $i_d = 0$.

Conversely, inside the capacitor, there is an absence of conduction current. Consequently, we observe the following:

The generalized Ampere's circuital law maintains the same structure as the original law with one critical distinction: it considers the total current passing through any surface, where the closed loop constitutes its perimeter. This total current is the amalgamation of the conduction current and the displacement current. The generalized law can be expressed as:

$$\int B \cdot d\ell = \mu_0 i_0 + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{..... (6)}$$

This modified form of Ampere's law encapsulates both types of current, offering a more comprehensive perspective on the generation of magnetic fields.

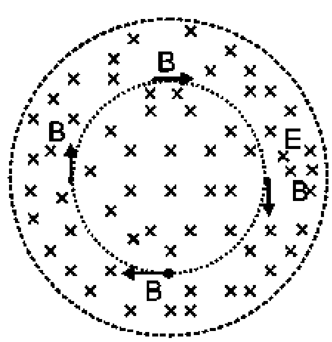

Figure 2(b)

This modified version of Ampere's law, referred to as the Ampere-Maxwell law, incorporates the concept of displacement current. Remarkably, the displacement current yields the same physical consequences as conduction current in all respects. However, there are situations, such as when dealing with steady electric fields within a conducting wire, where the displacement current may be negligible because the electric field E remains constant over time.

In alternative scenarios, such as the charging capacitor described earlier, both conduction and displacement currents may coexist in different regions of space. In many cases, they can even both be present within the same space, as perfect conductors or insulators are rare in reality. What's intriguing is that there can be substantial regions of space where conduction current is absent, but displacement current due to time-varying electric fields is present. In such regions, we anticipate the existence of a magnetic field, even in the absence of a nearby (conduction) current source. This anticipation of displacement current can be experimentally confirmed. For instance, measuring the magnetic field, like at point M between the plates of the capacitor in scenario (a), shows that it is identical to the field just outside the plates at point P.

The concept of displacement current has profound implications. One immediate observation is that the laws governing electricity and magnetism are now more symmetrical. Faraday's law of electromagnetic induction asserts the existence of an induced emf equal to the rate of change of magnetic flux. Emf implies the presence of an electric field. Hence, we can rephrase Faraday's law by stating that a changing magnetic field generates an electric field. Conversely, the idea that a time-varying electric field produces a magnetic field represents the symmetrical counterpart. This is a consequence of the displacement current serving as a source of magnetic fields. Consequently, time-dependent electric and magnetic fields are interrelated—each one gives rise to the other. Faraday's law of electromagnetic induction and the Ampere-Maxwell law provide a quantitative expression of this notion, with the total current, as described in Equation (5).

One particularly significant outcome of this symmetry is the existence of electromagnetic waves. We will explore this topic qualitatively in the following section.

Maxwell's Equations

1. $\int E \cdot dA = Q/\varepsilon_0$ (Gauss's Law for electricity)
2. $\int B \cdot dA = 0$ (Gauss's Law for magnetism)
3. $\int E \cdot d\ell = \frac{-d\Phi_E}{dt}$ (Faraday's Law)
4. $\int B \cdot d\ell = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ (Ampere - Maxwell Law)