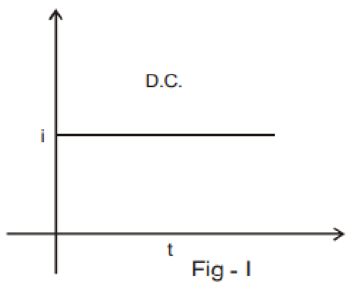
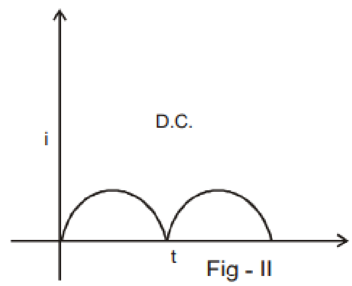
ALTERNATING CURRENT

REPRESENTATION OF AC CURRENT AND VOLTAGE

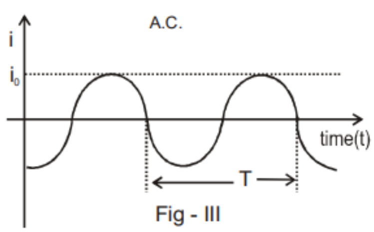
ALTERNATING CURRENT

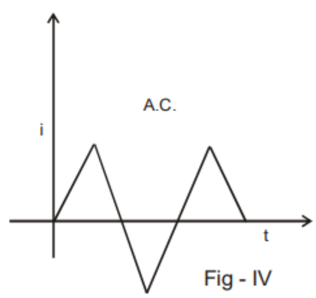
Up until now, our studies have focused on circuits with direct current (DC), which flows in a single consistent direction (illustrated in Figures I and II). In such circuits, the primary source of electromotive force (emf) is typically a battery. When a resistor is connected across the battery's terminals, a current is established in the circuit, flowing from the positive terminal to the negative terminal via the external resistance.





However, the majority of electric power generated and utilized worldwide is in the form of alternating current (AC). AC exhibits continuously changing magnitude over time and periodically reverses its direction (as depicted in Figures III and IV). It can be described mathematically as $i = i0 \sin(\omega t + \phi)$, where 'i' represents the instantaneous current value at any given moment, and 'i0' is the maximum current value, often referred to as the peak current or current amplitude.





This AC current repeats its values after specific time intervals, known as the time period. The angular frequency ω is equal to 2π times the frequency 'f,' where $\omega = 2\pi f$.

AC currents are positive for half of the time period and negative for the remaining half, indicating a reversal of current direction after each half-time period. In India, the standard frequency for AC is 50 Hz.

Similarly, an alternating voltage is expressed as $V = V0 \sin(\omega t + \phi)$, with V representing the instantaneous voltage and V0 indicating the peak voltage. AC voltages are generated by devices like AC generators, also known as AC dynamos.

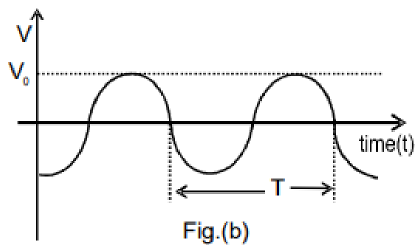
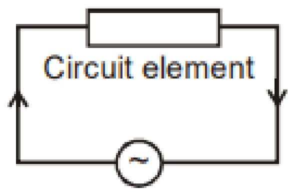


Fig.(b)

AC Circuit:

An AC circuit comprises various circuit elements, such as resistors, capacitors, inductors, or combinations of these components. These circuits are connected to a generator that provides the alternating current. In circuit diagrams, the AC source is represented by a specific symbol.



Average and RMS Value of Alternating Current

Average Current (Mean Current)

The formula for an alternating current is given by

$i = i_0 \sin(\omega t + \phi)$ (equation 1).

The mean or average value of AC over any time period 'T' is calculated as:

$$i_{avg} = \frac{\int_0^T i\,dt}{\int_0^T dt}$$

Using equation (1):

$$i_{avg} = \frac{\int_0^T i_0 \sin(\omega t + \phi)\,dt}{\int_0^T dt}$$

In one complete cycle, the average current is:

$$i_{avg} = -\frac{i_0}{T}\left[\frac{\cos(\omega t + \phi)}{\omega}\right]_0^T = -\frac{i_0}{T}\left[\frac{\cos(\omega t + \phi) - \cos\phi}{\omega}\right] = \frac{i_0}{T}\left[\frac{\cos(2\pi + \phi) - \cos\phi}{\omega}\right] = 0 \quad (as\ \omega T = 2\pi)$$

Since AC is positive during the first half cycle and negative during the second half cycle, the average current, iavg, is zero over a long period. Consequently, when a DC instrument is connected to a branch carrying AC current, it will indicate zero deflection. Therefore, the average current is defined for either the positive or negative half cycle.

$$i_{avg} = \frac{\int_0^T i_0 \sin(\omega t + \phi)}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} \approx 0.637 i_0$$

Similarly, $V \frac{2V_0}{\pi} \approx 0.637 V_0$

RMS Value of Alternating Current

The notation RMS refers to the square root of the mean of the square of the current.

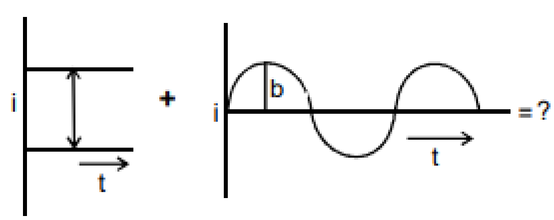
$$i_{rms} = \sqrt{i_{avg}{}^2}$$

$$i_{avg}^2 = \frac{\int_0^T i^2 dt}{\int_0^T dt} = \frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \phi) dt = \frac{i_0^2}{2T}\int_0^T 1 - \cos 2(\omega t + \phi) dt$$

$$i_{rms} = \sqrt{i_{avg}^2} = \frac{i_0}{\sqrt{2}}$$

Example.

In the context of a wire through which an alternating current i = b sin (ωt) is flowing, we consider the impact of adding a direct current with a value of 'a' amperes to the circuit. The objective is to determine the effective value of the resultant current in this composite circuit.



Solution.

As current at any instant in the circuit will be, $i = i_{dc} + i_{ac} = a + b \sin \omega t$

$$so,\ i_{ef} = \left[\frac{\int_0^T r^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T \left(a + b\sin\omega t\right)^2 dt\right]\left[\frac{1}{T}\int_0^T \left(a + b\sin\omega\right)^2 dt\right]^{1/2}$$

$$i_{ef} = \left[\frac{1}{T}\int_0^T \left(a^2 + 2b\sin\omega t + b^2 sn^{2\omega}\right) dt\right]^{1/2}$$

$$\frac{1}{T}\int_0^T \sin\omega dt = 0 \ and \ \frac{1}{T}\int_0^T \sin^2 \omega t dt = \frac{1}{2}$$

$$so,\ i_{eff} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$$

$$\frac{i_0^2}{2T}\left[t - \frac{\sin 2\left(\omega t + \phi\right)}{2\omega}\right]_0^T = \frac{i_0^2}{2T}\left[T - \frac{\sin\left(4\pi + 2\phi\right) - \sin 2\phi}{2\omega}\right] = \frac{i_0^2}{2}$$

$$i\,\frac{i_0}{\sqrt{2}} \approx 0.707 i_0$$

Similarly, the rms voltage is given by

$$V_{rms} = \frac{V_0}{\sqrt{2}} \approx 0.707 V_0$$

The importance of root mean square (rms) current and voltage can be demonstrated by examining a resistor with resistance 'R' through which a current 'i' flows, defined as $i = i_0 \sin(\omega t + \phi)$. The voltage across the resistor under these conditions can be described as follows:

$V_R = Ri = (i_0 R) \sin(\omega t + \phi)$

The thermal energy developed in the resistor during the time t to t + dt is

$i^2 R\, dt = i_0^2 R \sin^2(\omega t + \phi)\, dt$

The thermal energy developed in one time period is

$$U = \int_0^T r^2 R dt = R\int_0^T i_0^2 \sin^2(\omega t + \phi) dt = Rt\left[\frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \phi) dt\right] = i_{rms}^2 RT$$

The root mean square (rms) value of an alternating current (ac) signifies the steady current magnitude that, when flowing through a specific resistance for a given duration, would produce an equivalent amount of heat.

Therefore, in ac circuits, both current and voltage are quantified in relation to their rms values. For instance, when we state that household electrical supply is at 220 V ac, it implies that the rms value is 220 V, while the peak value is $220\sqrt{2} \approx 311$ V.

Example.

The expression describing the electric current in a circuit is i = i0 (t/T) for a specific duration. In this context, we aim to compute the root mean square (rms) current for the time interval spanning from t = 0 to t = T.

Solution.

The mean square current is

$$\left(i^2\right)_{avg} = \frac{1}{T}\int_0^T i_0^2 (t-T)^2 dt = \frac{i_0^2}{T^3}\frac{1}{T}\int_0^T t^2 dt = \frac{i_0^2}{3}$$

Thus, the rms current is

$$i_{rms} = \sqrt{i_{avg}^2} = \frac{i_0}{\sqrt{2}}$$