# ALTERNATING CURRENT

## POWER IN AC CIRCUIT: THE POWER FACTOR

### POWER IN AN AC CIRCUIT

When dealing with a constant electric current, the measure of the rate at which work is being performed can be expressed as follows:

$P = Vi$

Within an alternating circuit, where both current and voltage exhibit variations over time, it becomes necessary to define the work accomplished by the energy source during a small time interval, denoted as "dt." This can be articulated as follows:

$dW = Vidt$

Consider a scenario in an alternating current (AC) system where the current is leading the voltage by a specific angle denoted as "ϕ." In this circumstance, we can express this relationship as follows:

$V = V_0 \sin\omega t$
$i = i_0 \sin(\omega t + \phi)$
$dW = V_0\, i_0 \sin \omega t \sin (\omega t + \phi)\, dt$
$V_0\, i_0 (\sin 2\, \omega t \cos \phi + \sin\omega t \cos \omega t \sin\phi)\, dt$

The cumulative work performed over an entire cycle is represented as:

$$W = V_0 i_0 \cos \phi \int_0^T \sin^2 \omega t dt + V_0 i_0 \sin \phi \int_0^T \sin \omega t \cos \omega t dt$$

$$\frac{1}{2} V_0 i_0 \cos \phi \int_0^T (1 - \cos 2\omega t) dt + \frac{1}{2} V_0 i_0 \sin \phi \int_0^T \sin 2\omega t dt = \frac{1}{2} V_0 i_0 T \cos \phi$$

Hence, the mean power supplied by the source can be defined as:

$$P = \frac{W}{T} = \frac{1}{2} V_0 i_0 \cos \phi = (\frac{V_0}{\sqrt{2}})(\frac{i_0}{\sqrt{2}})(\cos \phi)$$

$V_{rnns} i_{rmns} \cos \phi$

$\langle P \rangle_{\text{one cycle}} = V_{rms} i_{rmscos\, \phi}$

In this context, the expression "cos ϕ" represents the power factor, a crucial parameter in electrical systems. It's characterized as leading when the current leads the voltage and lagging when the current lags behind the voltage. For instance, if the power factor is 0.5 lagging, it signifies that the current lags behind the voltage by 60° (since $\cos \tilde{n}\, 10.5 = 60°$). The product of the root mean square voltage ($V_{rms}$) and root mean square current ($i_{rms}$)provides the apparent power.

True power, on the other hand, is derived by multiplying the apparent power by the power factor, which is represented as cos ϕ. Therefore, we can express this relationship as:

True power = Apparent power × cos ϕ
And apparent power = $V_{rms} \times i_{rms}$

The equation for true power can be expressed as:

True power = Apparent power × Power factor

When the phase angle (ϕ) is 0°, it indicates that the current and voltage are perfectly in sync, or "in phase." In this condition, the power reaches its maximum value, which is $V_{rms} \times i_{rms}$.

On the other hand, when the phase angle ϕ is 90°, the power becomes zero. This situation is often referred to as "wattless," indicating that there is no real power being dissipated. This particular scenario occurs when the resistance in the circuit is negligible or zero, and the circuit is either purely inductive or capacitive.