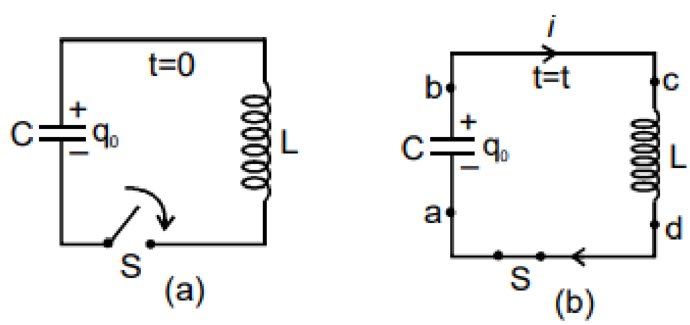
ALTERNATING CURRENT

LC OSCILLATIONS

OSCILLATIONS IN L-C CIRCUIT

In the event that a charged capacitor, denoted as "C," is promptly connected through an inductor, marked as "L," both the charge and the current within the circuit commence oscillating in a simple harmonic manner. In cases where the circuit's resistance is practically zero, no energy is dissipated in the form of heat. To maintain ideal conditions, assuming that no energy is radiated away from the circuit, these oscillations within the circuit endure indefinitely. During this perpetual oscillation, energy transfers back and forth between the electric field of the capacitor and the magnetic field of the inductor. Consequently, the overall energy linked with the circuit remains constant.

This phenomenon closely resembles the process observed in oscillating mechanical systems, where energy shifts between potential and kinetic forms, maintaining a constant total energy. An illustrative mechanical counterpart to this system is the spring-mass system, where energy transfers between potential and kinetic forms in a manner similar to the electrical oscillations described above.



We will proceed to formulate an equation that describes the oscillatory behavior of charge and current within an L-C (inductor-capacitor) circuit. Please consult the provided figure for reference.

a. Initially, the capacitor is charged to a voltage V, resulting in a charge on the capacitor given by $q0 = CV$.

Here, q0 represents the maximum charge that the capacitor can hold. At the precise moment when time t equals zero, it is then connected to an inductor via a switch denoted as "S." Consequently, as time reaches $t = 0$, the switch S is closed. Please refer to the accompanying figure for further clarification.

b. As the switch is closed, the process of discharging the capacitor begins. At any given time t, let the charge on the capacitor be denoted as q, where q ($< q0$). Since the charge is progressively decreasing, there is an associated current, represented as i, flowing in the circuit in the direction indicated in the accompanying figure.

The voltage across the capacitor is equivalent to the voltage across the inductor, which can be expressed as:

$$V_b - V_a = V_c - V_d$$

$$\frac{q}{c} = L(\frac{di}{dt}) \quad \text{... (1)}$$

At this point, since the charge is diminishing,

$$i = (\frac{-dq}{dt})$$

$$\frac{di}{dt} = -\frac{d^2q}{dt^2}$$

By replacing the values in equation (1), we obtain:

$$\frac{q}{c} = -L(\frac{d^2q}{dt^2})$$

$$\frac{d^2q}{dt^2} = -(\frac{1}{LC})q \quad \text{... (2)}$$

This equation is the conventional form of the equation for simple harmonic motion.

$$(\frac{d^2x}{dt^2} = -\omega^2x)$$

$$= \frac{1}{\sqrt{LC}} \quad \text{... (3)}$$

The comprehensive solution to equation (2) is as follows:

$$q = q_0 \cos(\omega t \pm \phi) \quad \text{... (4)}$$

In our case $\phi = 0$ as $q = q0$ at $t = 0$.

Hence, it can be concluded that the charge within the circuit experiences oscillations characterized by the angular frequency ω, as determined by equation (3). In L ñ C (inductor-capacitor) oscillations, the quantities q, i, and $\frac{di}{dt}$ all oscillate harmonically with the same angular frequency ω. However, there exists a phase difference of $\frac{\pi}{4}$ radians (or 45°) between q and i, as well as between i and $\frac{di}{dt}$. Their amplitudes are distinct from one another.

q_0 q_0 ω are ω^2 q_0 ... (4)

So,

$q = q_0 \cos\omega t$, then ... (5)

$i = \frac{dq}{dt} = q_0 \omega \sin \omega t$ ... (6)

$\frac{di}{dt} = q_0 \omega^2 \sin \omega t$ ... (7)

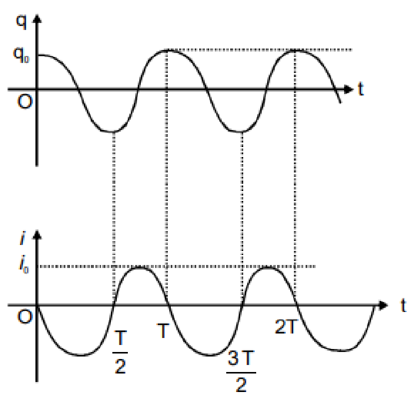
Potential energy in the capacitor

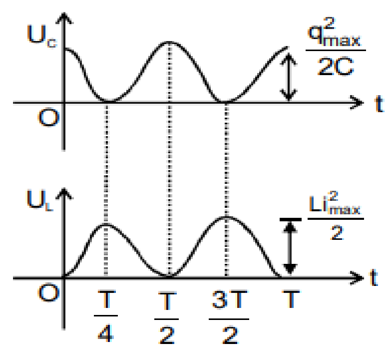
$U_c = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{q_0^2}{C}\cos^2 \omega t$... (8)

Potential energy in the inductor

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}\frac{q_0^2}{C}\sin^2 \omega t$ ... (9)

Consequently, the potential energy stored in both the capacitor and the inductor undergoes oscillations, fluctuating between their maximum values and zero, but these oscillations occur at twice the frequency. Visual representations of these behaviors can be observed in the subsequent figures.





Example.

Consider a scenario where a capacitor with a capacitance of 25 μF is initially charged to 300 volts and subsequently connected to a 10 mH inductor, with negligible resistance in the circuit.

(a) Determine the oscillation frequency of the circuit.

(b) Calculate the potential difference across the capacitor and the magnitude of the circuit current 1.2 milliseconds after the capacitor and inductor are connected.

(c) Find the magnetic energy and electric energy in the circuit at two specific instances, namely, at t = 0 and t = 1.2 milliseconds.

Solution.

(a) The oscillation frequency of the circuit is given by,

$f = \frac{1}{2\pi\sqrt{LC}}$

By replacing the provided numerical values, we obtain:

$f = \frac{1}{2\pi\sqrt{10\times10^{-3}(25\times10^{-6})}} = \frac{10^3}{\pi}\,\text{Hz}$

(b) Charge across the capacitor at time t will be,

q = q0 cos ωt

$i = -\,q_0\,\omega \sin \omega t$

Here $q_0 = CV_0$ = (25 × 10 -6) (300) = 7.5 × 10-3 C

Now, charge is the capacitor after

$t = 1.2 \times 10^{-3}$ s is,

$q = (7.5 \times 10^{-3}) \cos (2\pi \times 318.3) (1.2 \times 10^{-3})$ C

P.D. across capacitor,

$V = \frac{|q|}{c} = \frac{5.53\times10^{-3}}{25\times10^{-6}} = 221.2$ volt

The current's magnitude within the circuit at this particular moment,

$t = 1.2 \times 10^{-3}$ s is,

$|i| = q_0\, \omega \sin\omega t$

$(7.5 \times 10^{-3}) (2\pi) (318.3) \sin (2\pi \times 318.3) (1.2 \times 10^{-3})$ A = 10.13 A

(c) At t = 0: Current in the circuit is zero. Hence, UL = 0

Charge on the capacitor is maximum

Hence, $U_c = \frac{1}{2}\frac{q_0^2}{C}$

$U_c = \frac{1}{2} \times \frac{(7.5\times10^{-3})^2}{(25\times10^{-6})} = 1.125$ J

Total energy $E = U_L + U_C = 1.125$ J

At t = 1.2 ms

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(10 \times 10^{-3})(10.13)^2 = 0.513 \text{ J}$$

$$U_C = E - U_L = 1.125 - 0.513 = 0.612$$

Otherwise, U_C can be calculated as,

$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{(5.53 \times 10^{-3})}{(25 \times 10^{-6})} = 0.612 \text{ J}$$