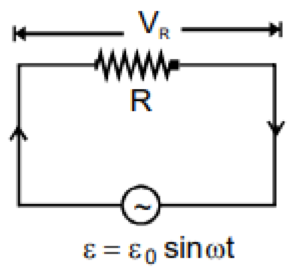
ALTERNATING CURRENT

AC VOLTAGE APPLIED TO A RESISTOR, INDUCTOR AND CAPACITOR

SERIES AC CIRCUIT

WHEN ONLY RESISTANCE IS IN AN AC CIRCUIT

Let's contemplate a straightforward alternating current (AC) circuit configuration, comprising a resistor with resistance R and an AC generator, as depicted in the diagram. In accordance with Kirchhoff's loop law, it is essential to recognize that at any given moment, the total algebraic sum of the voltage differences along a closed loop within the circuit must equate to zero.



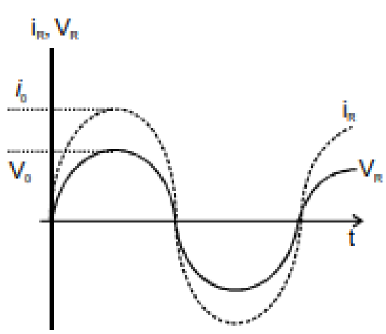
$$\varepsilon - V_R = 0$$
$$\varepsilon - i_R R = 0$$
$$\varepsilon 0 \sin \omega t - i_R = 0$$
$$i_R = \frac{\varepsilon_0}{R} \sin \omega t = i_0 \sin \omega t$$

Where i_0 is the maximum current $i_0 = \frac{\varepsilon_0}{R}$

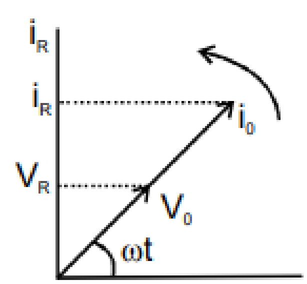
Based on the equations provided previously, we can observe that the immediate or instantaneous voltage decrease across the resistor is as follows:

$$V_R = i_0 R \sin \omega t..(ii)$$

In equations (i) and (ii), it becomes apparent that both i_R and V_R exhibit variations following a sine function with time, specifically as sin (ωt). These variations peak simultaneously, as depicted in Figure (a), and are considered to be in phase with each other. To represent these phase relationships visually, a phasor diagram is employed. In this diagram, the lengths of the arrows correspond to the maximum values, denoted as V_0 and i_0. This graphical representation aids in understanding and visualizing the phase relationship between current and voltage in the AC circuit.



Wave diagram
Fig.(a)



Phasor diagram
Fig.(b)

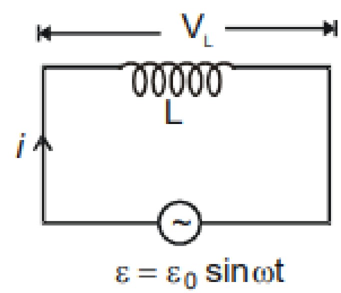
The vertical components of the arrows in the phasor diagram represent V_R and i_R in the context of a single-loop resistive circuit, where only resistance is present, the phasors representing current and voltage align along a single line, as illustrated in Figure (b). This alignment occurs because i_R and V_R are in phase with each other, meaning they reach their peak values at the same time during the alternating current cycle.

WHEN ONLY INDUCTOR IS IN AN AC

Let's examine an alternating current (AC) circuit configuration featuring solely an inductor with inductance L, connected to the terminals of an AC generator, as depicted in the diagram. The electromotive force (emf) induced across the inductor is expressed as Ldi/dt, where i is the current and "di/dt" represents the rate of change of current over time.

When we apply Kirchhoff's loop rule to this circuit, we can analyze its behavior and characteristics.

$$\varepsilon - V_L = 0 \Rightarrow \varepsilon - L\frac{di}{dt} = 0$$



Upon rearranging this equation and making the substitution

$$\varepsilon = \varepsilon_0 \sin\omega t$$

We get

$$L\frac{di}{dt} = \varepsilon_0 \sin\omega t \quad \text{... (iii)}$$

By performing the integration of this expression, we obtain the current as a function of time.

$$i_L = \frac{\varepsilon_0}{L}\int \sin\omega t dt = -\frac{\varepsilon_0}{\omega L}\int \cos\omega t + c$$

In order for the average value of the current over a complete time period to be equal to zero, C = 0

$$i_L = -\frac{\varepsilon_0}{\omega L}\cos\omega t$$

When we use the trigonometric identity

$\cos\omega t = -\sin(\omega t\tilde{n}\pi / 2)$, we can express equation as

$$i_L = \frac{\varepsilon_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{... (iv)}$$

From equation (iv), it becomes evident that the current attains its peak values when cos ωt = 1.

$$i_0 = \frac{\varepsilon_0}{\omega L} = \frac{\varepsilon_0}{X_1}$$

In this context, the term X_L, referred to as inductive reactance, can be defined as follows:

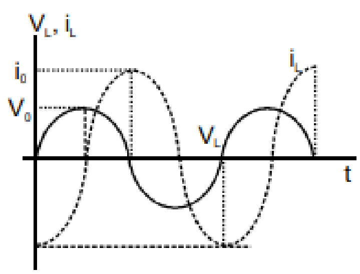
$X_L = \omega L$

The formula for the root mean square (rms) current resembles equation (v), with the substitution of ε_0 by ε_{rms} it's important to note that inductive reactance, similar to resistance, and is measured in ohms.

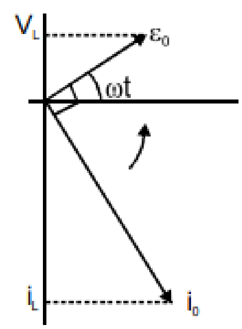
$$V_L = L\frac{di}{dt} = \varepsilon_0 \sin \omega t = i_0 X_L \sin \omega t$$

Equation (v) can be conceptualized as Ohm's law specifically tailored for an inductive circuit. When we compare the results of equation (iv) with equation (iii), it becomes evident that the current and voltage exhibit a phase difference of π/2 rad, which is equivalent to 90º.

A graphical representation of voltage and current variations over time can be found in Figure (a). In this representation, the voltage reaches its maximum value one-quarter of a full oscillation period prior to the current reaching its peak. A corresponding phasor diagram for this circuit is presented in Figure (b). Consequently, it is clear that when a sinusoidal voltage is applied, the current in an inductor consistently lags behind the voltage across the inductor by 90º.



Wave diagram
Fig.(a)



Phasor diagram
Fig.(b)

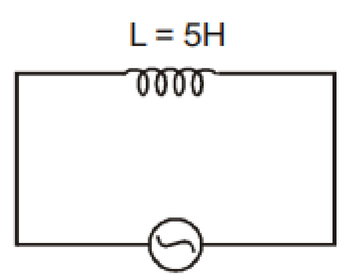
Example.

Determine the following quantities for an inductor with inductance L = 5 H connected to an AC source with a voltage given by

$$V = 10\sin\left(10t + \frac{\pi}{6}\right)$$

Find

(i) Calculate the inductive reactance (x_L).



(ii) Find the peak voltage (V_0) and the root mean square (rms) voltage (V_{rms}).
(iii) Determine the peak current (I_0) and the rms current (I_{rms}).
(iv) Obtain the expression for the instantaneous current (I_t).

Solution.

(i) $x_L = \omega L = 10 \times 5 = 50$

(ii) $v_0 = 10$

$$V_{rms} = \frac{10}{\sqrt{2}}$$

(iii) $i_0 \frac{v_0}{x_L} = \frac{1}{5}$

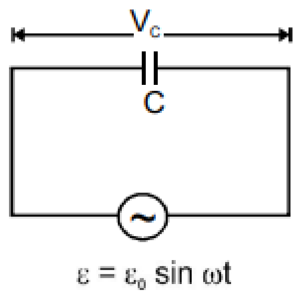
$$I_{rms} = \frac{1}{5\sqrt{2}}$$

(iv) $I(t) = \frac{1}{5}\sin\left(10t + \frac{\pi}{6} - \frac{\pi}{2}\right)$

When only Capacitor is in An AC circuit

The diagram illustrates an alternating current (AC) circuit configuration comprising a capacitor with capacitance C connected to the terminals of an AC generator. By applying Kirchhoff's loop rule to this particular circuit, we can derive the following equation:

$\varepsilon - V_C = 0$
$V_C = \varepsilon = \varepsilon_0 \sin \omega t$



In this equation, V_C represents the immediate voltage drop across the capacitor. According to the definition of capacitance, V_C can be expressed as $V_C = Q/C$. By substituting this expression for V_C into the equation, we obtain the following result:

$Q = C\,\varepsilon_0 \sin \omega t$

Given that i is equivalent to the rate of change of charge, which can be represented as dQ/dt, differentiating the above equation yields the immediate or instantaneous current within the circuit.

$$i_c = \frac{dQ}{dt} = C\varepsilon_0 \omega \cos \omega t$$

Once more, we observe that the current does not align with the voltage drop across the capacitor, as specified in equation (vi). To rephrase this equation differently, we can use the trigonometric identity cos (ωt) = sin (ωt + π/2). This allows us to present the equation in an alternative form.

$$i_C = \omega C \varepsilon_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{... (vi)}$$

From equation (vii), it becomes evident that the current in the circuit attains its highest value when the cos ωt = 1.

$$i_0 = \omega C \varepsilon_0 = \frac{\varepsilon_0}{X_c}$$

In this context, the term X_C is referred to as capacitive reactance.

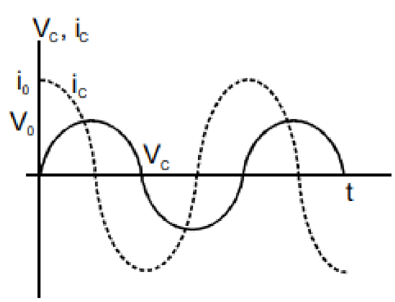
$$X_c = \frac{1}{\omega C}$$

The SI unit of capacitive reactance, denoted as XC, is also expressed in ohms. The formula for the root mean square (rms) current is analogous to the previous equation, but with the substitution of V_0 by V_{rms}.

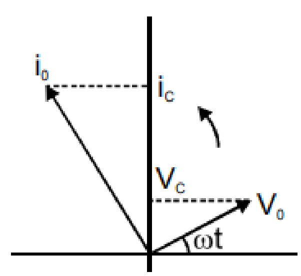
By merging equations (vi) and (vii), we can represent the immediate voltage drop across the capacitor as follows:

$V_C = V_0 \sin \omega t = i_0 X_C \sin \omega t$

When we compare the outcomes of equation (v) with those of equation (vi), it becomes apparent that the current is out of phase with the voltage across the capacitor by π/2 rad or 90º. Visualizing a graph of current and voltage variations over time reveals that the current reaches its peak value one-quarter of a full cycle earlier than the voltage achieves its maximum value. A corresponding phasor diagram, as depicted in Figure (b), illustrates this relationship. Consequently, it is evident that when a sinusoidal electromotive force (emf) is applied, the current consistently leads the voltage across a capacitor by 90º.



Wave diagram
Fig (a)

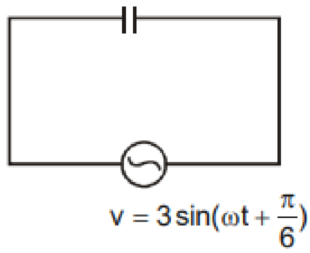


Phasor diagram
Fig.(b)

Here's a brain teaser for you: What is the reactance of a capacitor when it's connected to a constant DC source?

Example.

In the context of scholarly or bookish language: "A capacitor possessing a capacitive reactance of 5 Ω is linked to an alternating current source characterized by a voltage signal represented as V = 3 sin (ωt + π/6). Determine the root mean square voltage, peak voltage, root mean square current, peak current, and the expression for instantaneous current.



Solution.

On comparing with

$v = v_0 \sin(\omega t + \phi) \Rightarrow v0 = 3$

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{3}{\sqrt{2}} \Rightarrow i_{rms} = \frac{V_{rms}}{X_c} = \frac{3}{5\sqrt{2}}$$

$$i_0 = \frac{v_0}{x_0} = \frac{3}{5} \Rightarrow I(t) = I_0 \sin\left(\omega t + \frac{\pi}{6} + \frac{\pi}{2}\right)$$