ALTERNATING CURRENT

AC VOLTAGE APPLIED TO A SERIES LCR CIRCUIT

SERIES L-C-R CIRCUIT

Let's examine an alternating current (AC) circuit comprising a series connection of a resistor with resistance R, a capacitor with capacitance C, and an inductor with inductance L, all connected to an AC source generator. In this context, let's assume that in a phasor diagram, the current is represented along the positive x-direction.

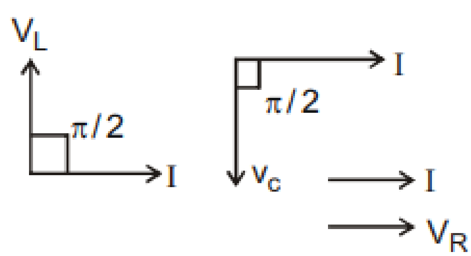
Under these conditions, the phasor diagram depicts the following orientations:

- The voltage across the resistor, VR, aligns with the positive x-direction.
- The voltage across the inductor, VL, aligns with the positive y-direction.
- The voltage across the capacitor, VC, aligns with the negative y-direction.

This arrangement reflects the phase relationships in the circuit. Specifically, the potential difference across an inductor leads the current by 90° in phase, which is why VL aligns with the positive y-direction. Conversely, the potential difference across a capacitor lags the current by 90°, explaining why VC aligns with the negative y-direction in the phasor diagram.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

L - R - C circuit



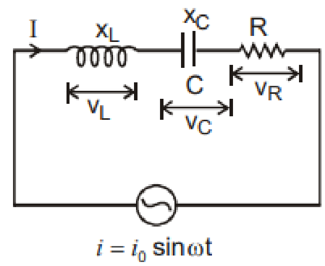
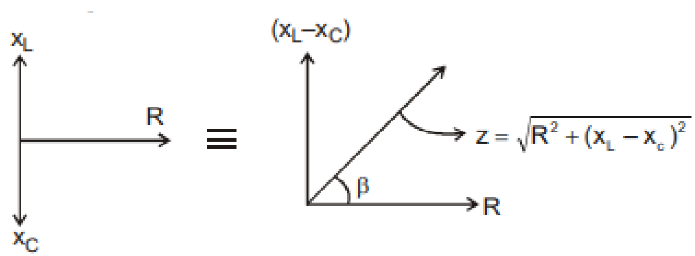


Fig : A series L-C-R circuit

IMPEDANCE PHASOR OF ABOVE CIRCUIT



Impedance triangle

$$Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

Here B is phase angle by triangle tan $\beta=\frac{X_L-X_C}{R}$

Power factor $\cos\phi=\frac{R}{Z}$

Suppose at a given moment we have a series circuit with a flowing current, denoted as I.

Certainly! In the given expression:

1. The voltage $V(t)=V_0\sin(\omega t+\beta)=i_0 z\sin(\omega t+\beta)$ is described. Here, V_0 represents the peak voltage, ω is the angular frequency, t is time, and β is the phase angle. The expression also includes the current i_0, impedance z, and the same phase angle β.

Additionally:

- $V_0=i_0 z$, indicating that the peak voltage (V_0) is equal to the product of the peak current i_0 and the impedance (z).

- $V_{rms}=i_{rms}z$, signifying that the root mean square voltage (V_{rms}) is equivalent to the product of the root mean square current (i_{rms}) and the impedance (z).

In summary, the expression provides a mathematical representation of voltage in terms of peak values, time, and phase angle, while also establishing relationships between peak voltage, peak current, root mean square voltage, root mean square current, and impedance in the context of the given circuit.

2. $v(t)=V\sin\left(\frac{t}{2}\right)$, the voltage v is described as a sinusoidal function with amplitude V and a frequency represented by $\omega=\frac{\pi}{2}$.

Additionally, it is stated that the voltage V_L across the inductance is leading the current I in phase by $\frac{\pi}{2}$ radians. This phase relationship signifies that the voltage waveform across the inductance reaches its peak value V_{0L} a quarter-cycle earlier than the corresponding peak in the current waveform I_0.

So, $V_{0L} - I_0 X_i$ when V_{0L} is the peak voltage across the inductance, I_0 is the peak current, and X_L is the inductive reactance, represents the instantaneous phase difference between the voltage across the inductance and the current, indicating that the voltage leads the current by $\frac{\pi}{2}$ radians in the given circuit.

3. When $X_L = X_C$ or $V_L = V_C$, , and $\beta = 0$, it implies that the inductive reactance (X_L) is equal to the capacitive reactance (X_C), and the voltage across the inductor (V_L) is equal to the voltage across the capacitor (V_C)In this scenario, the phase angle (β) becomes zero, indicating that the electromotive force (emf) and the current in the circuit are in perfect alignment. This condition characterizes the series LCR circuit as purely resistive.

It's worth noting that under these conditions:

- $I_0 = \frac{\varepsilon_0}{Z}$, where (I_0) the peak is current, ϵ_0 is the peak emf, and Z is the impedance of the circuit.
- $\frac{I_0}{\sqrt{2}} = \frac{\varepsilon_0}{\sqrt{2}Z}$, and $I_{Rms} = \frac{R_{Rms}}{Z}$, highlighting relationships between peak current, peak emf, impedance, and root mean square values.

Additionally, the terms "Susceptance" and "Admittance" are introduced:

- Susceptance is defined as the reciprocal of the reactance in an AC circuit.
- Admittance is defined as the reciprocal of the impedance in an AC circuit.

Example.

In a series circuit, there's a 15-ohm resistor, a 0.08-henry inductor, and a 30-micro farad capacitor. The voltage applied to this circuit has a frequency of 500 radians per second. Now, we want to know if the current in the circuit is ahead (leads) or behind (lags) the applied voltage, and by how much.

Solution.

Here ωL = 500 × 0.08 = 40 ohm

$$and \quad \frac{1}{\omega C} = \frac{1}{500 \times \left(30 \times 10^{-6}\right)}$$

$$\tan\phi = \frac{\left[\omega L - \left(1/\omega C\right)\right]}{R} = \frac{40 - 66.7}{15} = -1.78$$

$$\phi = -60.67^\circ$$

Thus, the current leads the applied voltage by 60.67°

Brain teaser

Is it possible for the peak voltage across the inductor in an LCR circuit to surpass the peak voltage of the source? In other words, could the maximum voltage experienced by the inductor be higher than the maximum voltage provided by the source in the circuit?