ELECTROMAGNETIC INDUCTION

ROTATING EMF

INDUCED E.M.F DUE TO ROTATION

Let's explore the concept of induced emf resulting from the rotation of a conducting rod in a uniform magnetic field:

Rotation of the Rod:

- Imagine a conducting rod of length 'l' that is rotating within a uniform magnetic field.
- To understand the emf induced in a small segment of length 'dr' of the rod, we can consider the following:
- The emf induced in the small segment dr is given by the product of the rod's linear velocity 'v,' the magnetic field strength 'B,' and the length 'dr,' which is expressed as vBdr.
- To account for the entire rod, we integrate this expression from 0 to 'l' to encompass the entire length of the rod, resulting in the emf induced in the entire rod as:

  emf = $\int$ (0 to l) vBdr
  emf = (1/2) $B\omega l^2$, where ω is the angular velocity of the rod.

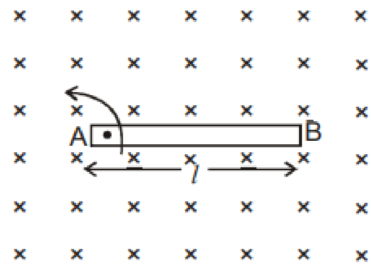
Equivalent Representation:

- An equivalent representation of the emf induced due to the rod's rotation can be expressed using the rate of change of magnetic flux (Φ) with respect to time (t). This representation is derived as follows:
- emf (ε) is equal to the change in magnetic flux ($d\phi$) through the area swept by the rod in a time interval (dt), divided by dt, represented as $d\phi/dt$.
- To calculate this, we consider the flux through the area swept by the rod in a time interval dt, which is equal to (1/2) $B\omega l^2 dt$.
- Therefore, emf (ε) is given by (1/2) $B\omega l^2$.

In summary, when a conducting rod rotates within a uniform magnetic field, emf is induced in the rod. The emf induced in a small segment of the rod can be calculated using the product of the linear velocity, magnetic field strength, and the length of the segment. Integrating this over the entire length of the rod yields the total emf induced in the rod. Alternatively, the emf can be expressed as the rate of change of magnetic flux through the area swept by the rod over a given time interval. Both methods lead to the same result, which is (1/2) $B\omega l^2$.

Example.

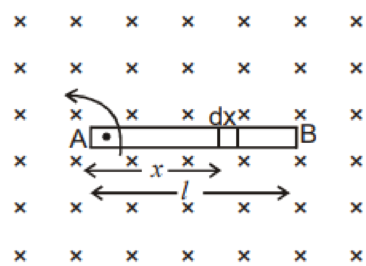
Calculate the electric potential difference existing between points A and B.



Solution.

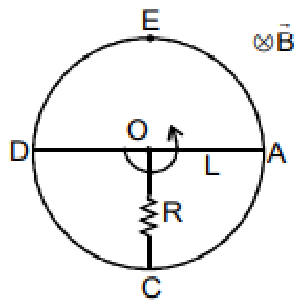
$$\int dE = \int_0^{\ell} B\omega x dx$$

$$V_A - V_B = \frac{B\omega\ell^2}{2}$$



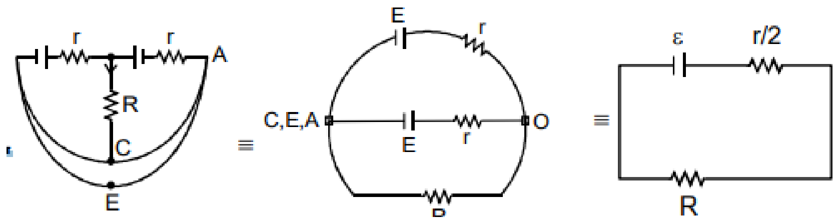
Example.

Address the previous inquiry considering a modified scenario in which the rod's length has extended to 2L, and its resistance has increased to 2r. Additionally, the rod is currently in a rotational motion around its center, and both of its ends make contact with the conducting ring. Determine the solution in this context.



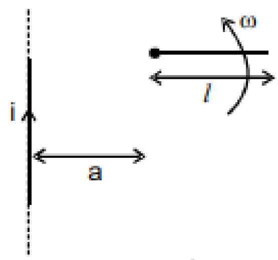
Solution.

$$i = \frac{\varepsilon}{R+\frac{r}{2}} = \frac{\frac{1}{2}B\omega\ell^2}{R+\frac{r}{2}}$$



Example.

Find out the electric voltage generated in the rod when one of its ends, at a distance -a from a very long wire carrying an electric current i, is rotating in a circular motion with a certain speed ω. This refers to the specific moment shown in the diagram.



Solution.

Consider a small segment of rod of length dx, at a distance x from one end of the rod. Emf induced in the segment

$$dE = \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx$$

$$E = \int_0^{\ell} \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx = \frac{\mu_0 i\omega}{2\pi}\left[\ell - a\ln\left(\frac{\ell+a}{a}\right)\right]$$