ELECTROMAGNETIC INDUCTION

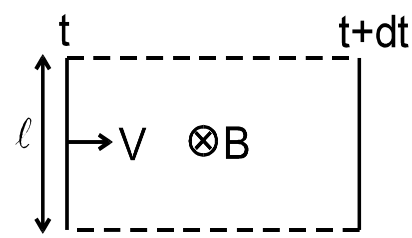
MOTIONAL ELECTROMOTIVE FORCE

Motional emf

To determine the electromotive force (emf) induced in a rod as it moves through a magnetic field, we can consider the number of magnetic field lines it cuts per second. Let's break down this concept:

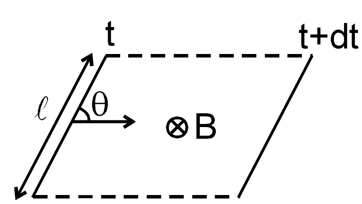
Emf calculation:

- Imagine a rod of length 'λ' moving with velocity 'v' in a magnetic field 'b.' as the rod sweeps through the magnetic field, it covers an area per unit time equal to λv. Consequently, it cuts through 'bλv' magnetic field lines per unit time.
- Therefore, the emf induced between the ends of the rod is given by: emf = bvλ.
- This emf calculation is based on the rate of change of magnetic flux, where emf (ε) is equal to the change in flux (φ) with respect to time (t). In this case, φ represents the flux passing through the area swept by the rod. The rod sweeps an area equal to λv dt in a time interval dt, leading to φ = bλv.



Changing orientation:

If the rod's movement and orientation differ from the previous scenario, the emf can still be determined. In this case, if the rod sweeps an area per unit time equal to 'vλ' and makes an angle 'θ' with the magnetic field, the emf remains: emf = bvλsin(θ).



Explanation of induced emf based on magnetic force:

- When a rod moves at velocity 'v' within a magnetic field 'b,' the free electrons in the rod experience a magnetic force directing them downward. This accumulation of electrons at the lower end results in an excess of positive charge at the upper end.

- These charge imbalances at the ends of the rod create an electric field, which exerts an upward force on the electrons. Over time, a sufficient number of charges accumulate at the ends to balance the magnetic and electric forces, resulting in an equilibrium where e = vb.

Equivalent circuit:

The moving rod can be conceptually represented as an electrical circuit, shown in the diagram. The voltage difference (VP - VQ) between point's P and Q is equal to the voltage generated by the moving rod, VBλ.

Closed coil in a uniform magnetic field:

When a closed coil ABCA moves in a uniform magnetic field b with a velocity v, the flux passing through the coil remains constant, resulting in an induced emf of zero for the entire coil.

Emf in rod ab:

- Consider a part of the coil, rod ab, as shown. The emf induced in this part is given by Emf = Blv.

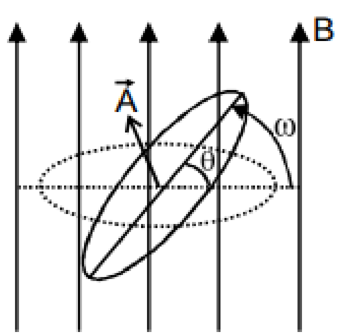
- Since the emf in the entire coil is zero, the emf in part ACB must be equal and opposite of that in part BA. This leads to the equation: E+ vBL = 0, which simplifies to E = vBL.

- Therefore, the emf induced in any path joining points A and B is the same, provided the magnetic field is uniform. The equivalent emf between A and B is Blv. These two emfs are in parallel.

Changing the angle:

- When the magnetic field strength and the area of the coil remain constant, rotating the coil relative to the field direction induces a current. This current continues as long as the coil is in motion. The induced emf can be calculated using the formula: $\varepsilon = BA \sin(\theta)$, where 'θ' is the angle between the magnetic field and the area vector.

- In summary, the emf induced in a moving rod is a result of cutting magnetic field lines and can be calculated based on the rate of change of magnetic flux. The orientation and angle between the rod and the magnetic field also affect the magnitude of the induced emf.



Example.

Calculate the electromotive force (emf) induced in a rod of length 'l' that is positioned parallel to an extensive wire with a steady electric current 'i.' the rod is in motion, moving away from the wire at a velocity 'v.' the objective is to determine the emf induced in the wire when the distance between the rod and the long wire is 'x.'

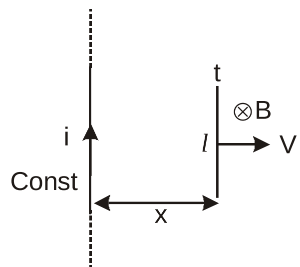
Solution.

$$E = B\,l V = \frac{\mu_0 i l v}{2\pi x}$$

Or

Emf is equal to the rate with which magnetic field lines are cut. In dt time the area swept by the rod is l v dt. The magnetic field lines cut in dt time

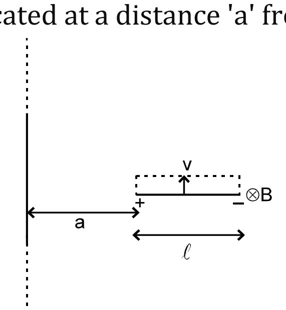
$=B\,l v\,dt = \frac{\mu_0 i l v dt}{2\pi x}$.



The rate with which magnetic field lines are cut $= \frac{\mu_0 i l v}{2\pi x}$

Example.

Calculate the electromotive force (emf) induced in a rod, which has a length 'l' and is positioned at a right angle to a lengthy wire conducting a current 'i.' The rod is being displaced parallel to the wire at a velocity 'v.' The objective is to determine the emf when the nearest end of the rod is located at a distance 'a' from the wire.



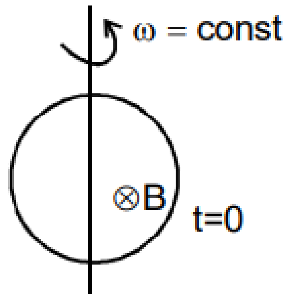
Solution.

Consider a segment of rod of length dx, at a distance x from the wire. Emf induced in the segment

$$dE = \frac{\mu_0 i}{2\pi x} dx.v \qquad \therefore \quad E = \int_a^{a+\ell} \frac{\mu_0 i v dx}{2\pi x} = \frac{\mu_0 i v}{2\pi} \ell n\left(\frac{\ell + a}{a}\right)$$

Example.

Calculate the electromotive force (emf) induced in a rotating ring with a fixed angular velocity (ω). The axis of rotation is situated within the ring's plane and passes through the ring's center. A uniform magnetic field (B) remains perpendicular to the plane of the ring. Determine the time-dependent emf generated in the ring under these conditions, with the initial time marked as t=0.



Solution.

At any time, t, $\phi = BA\cos\theta = BA\cos\omega t$

Now induced emf in the loop

$$e = \frac{-d\phi}{dt} = BA\omega \sin \omega t$$

If there are N turns

emf = BAωN sin ωt

BA ωN is the amplitude of the emf e = em sin ωt

$$i = \frac{e}{R} = \frac{e_m}{R} \sin \omega t = i_m \sin \omega t$$

$$i_m = \frac{e_m}{R}$$

The rotating coil thus produces a sinusoidally varying current or alternating current. This is the principle which is always used in generator.