MOVING CHARGES AND MAGNETISM

TORQUES ON CURRENT LOOP, MAGNETIC DIPOLE

CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

MAGNETICMOMENT

According to magnetic effects of current, in case of current-carrying coil for axial point,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{\left(R^2 + x^2\right)^{3/2}}$$

When x >> R, $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{x^3}$

If we compare this result with the field due to a small bar magnet for a distant axial point, i.e.,

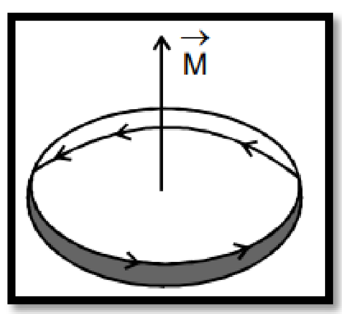
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{x^3}$$

where M is magnetic moment of the bar magnet.

We find that a current-carrying coil for a distant point behaves as a magnetic dipole of moment

$$\vec{M} = Ni\pi R^2 = NiA$$

Where A is area of the loop. So the magnetic moment of a current carrying coil is defined as the product of current in the coil with the area of coil in the vector form. Magnetic moment of a current loop is a vector quantity and direction is perpendicular to the plane of the loop. Its dimensions are $[L^2A]$ and units are A-m^2



Magnetic moment is case of a charged particle having charge q and moving in a circle of radius R with speed v is given by $\frac{1}{2}qvR$ as we know, the equivalent current

$$i = qf = q\frac{v}{2\pi R} \text{ And}$$

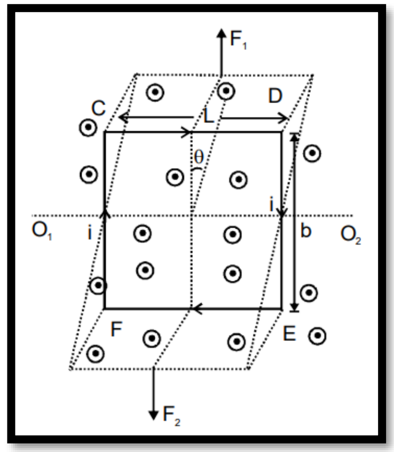
$$|A| = \pi R^2$$

$$\therefore\ M = i\left|\vec{A}\right| = \frac{1}{2}qvR$$

TORQUE ON A CURRENT LOOP

Consider a rectangular coil CDEF of length L and width b is placed vertically, while a uniform magnetic induction B passes normally through it as shown. The coil is capable of rotation about an axis O_1O_2.

If the loop is oriented in the magnetic field such that the normal to the plane of the coil makes an angle θ with the direction of $\vec{B}$ then the torque experienced by the loop



$$\tau = \frac{b}{2}(iLB)\sin\theta + \frac{b}{2}(iLB)\sin\theta$$

$$i.e.,\quad \tau = iLbB\sin\theta = iAB\sin\theta$$

where A = Lb is the area of the loop.

The maximum torque experienced is τ = iAB, when θ = 90°

and for a coil of N turns

$$\tau = NiAB$$

Here NiA = M = Magnetic moment of the loop.

In vector notation

$$\vec{\tau} = \overrightarrow{M} \times \vec{B}$$

This result holds good for plane loops of all shapes rectangular, circular or otherwise.

WORK DONEIN ROTATING A CURRENTLOOP

When a current loop is rotated in a uniform magnetic field through an angle θ about an axis then work done will be

$$\int_0^w dW = \int \tau d\theta = \int_0^0 MB\sin\theta d\theta$$

$$W = -\left[MB\cos\theta\right]_0^\theta = MB(1-\cos\theta)$$

Ex. For a given length L of a wire carrying a current i, how many circular turns would produce the maximum magnetic moment and of what value?

Sol. For a circular coil having N turns, magnetic moment

$$M = \pi R^2 iN$$

Now, length of wire $L = (2\pi R) N$

$$\therefore \quad R = \frac{L}{2\pi N}$$

Substituting the above value of R in equation (i), we get

$$M = \tau N i \times \frac{L^2}{4\pi^2 N^2} \text{ or, } M = \frac{iL^2}{4\pi N}$$

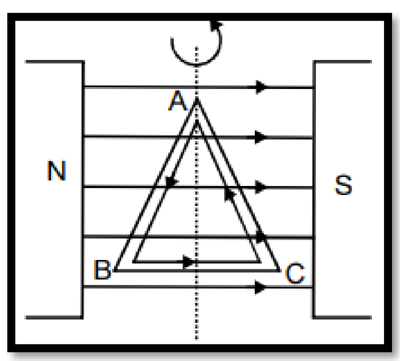
From equation (ii), it is clear that M will be maximum when N = minimum = 1, i.e., the coil has only one turn and

$$(M)_{\max} = \frac{1}{4\pi} iL^2$$

Ex. A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical in plane magnetic field of 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 ampere is passed through it and the magnetic field is parallel to its plane

Sol As the coil is in the form of an equilateral triangle, its area

$$A = \frac{1}{2} \times L \times L \sin 60^\circ$$



$$= \frac{1}{2} \times (0.02)^2 \times \frac{\sqrt{3}}{2} = \sqrt{3} \times 10^{-4} m^2$$

So its magnetic moment

$$M = iA = 0.1 \times \sqrt{3} \times 10^{-4}$$

$$= \sqrt{3} \times 10^{-5} A - m^2$$

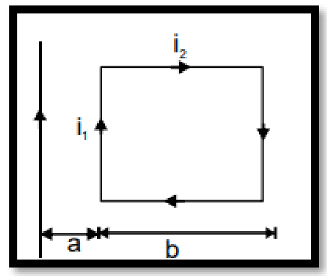
Now, the couple on a current-carrying coil in a magnetic field is given by τ = MB sin θ since the plane of the coil is parallel to the magnetic field, the angle between M and $\vec{B}$ will

be 90° and hence τ = MB sin 90° = MB

Ex. The arrangement is as shown below **(For Competitive Exam)**

(a) Find the potential energy of the loop.

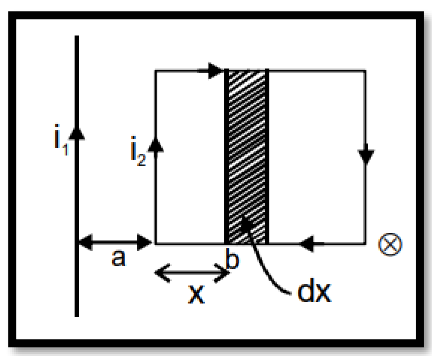
(b) Find the work done to increase the spacing between the wire and the loop from a to 2a.



Sol. (a) Magnetic moment of a small element of the loop.

dM = i_2 Ldx

The direction of the magnetic moment is perpendicular to the plane of paper pointing inwards. dU = $-d\vec{M}\vec{B} =$ - dM B, where B is the magnetic field at the position of this element.



i.e., $B = \frac{\mu_0}{4\pi}\frac{2i_1}{a+x}$

$\therefore$ dU= $-\frac{\mu_0}{4\pi}2i_1i_2l\left(\frac{dx}{a+x}\right)$

$\therefore \quad U = -\frac{\mu_0}{4\pi}2i_1i_2l\int_0^b\frac{dx}{a=x} = \frac{\mu_0}{4\pi}2i_1i_2l\log_e\left(\frac{a+b}{a}\right)$

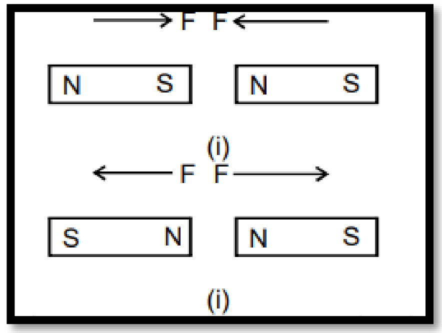
(b) $U_1 = -\frac{\mu_0}{4\pi}2i_1i_2l\log_e\left(\frac{a+b}{a}\right)$

$U_f = -\frac{\mu_0}{4\pi}2i_1i_2l\left(\log_e\frac{a+b}{a}\right)$

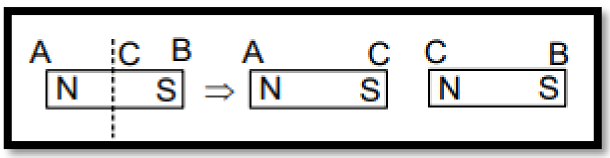
$\therefore \quad W = \Delta U = U_f - U_i = \frac{\mu_0}{4\pi}2i_1i_2l\log\left(\frac{2a+b}{2a+b}\right)_e$

Pole strength, magnetic dipole and magnetic dipole moment:

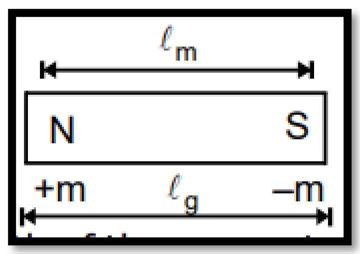
A magnet always has two poles Ní and Sí and like poles of two magnets repel other and the unlike poles of two magnets attract each other they form action reaction pair.



The poles of the same magnet do not comes to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The othe end becomes pole of opposite nature. SO, ëNí and ëSí always exist together. ∴ they are



Known as +ve and -ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as -ve pole (or -ve magnetic charge). They are quantitatively represented by their ìPOLE STRENGTHî + m and ñ m respectively (just like we have charge +q and ñq in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

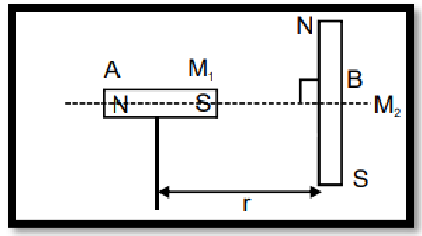


A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges -q and +q). It is called MAGNETIC DIPOLE and it has a direction is from -m to +m that means from Sí to Ní).

M = m.l_m here l_m = magnetic length of the magnet. l_m is slightly less than lg (it is geometrical length of the magnet = end to end distance). The Ní and Sí are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$

[Actually ℓ_m / ℓ_g -~ 0.84] he units of m and M will be mentioned afterwards where you can remember and understand.

Ex. Two short magnets A and B of magnetic dipole moments M1 and M2 respectively are placed as shown. The axis of Aí and the equatorial line of Bí are the same. Find the magnetic force on one magnet due to the other.



Sol.

$$F = 3\left(\frac{\mu_0}{4\pi}\right)\frac{M_2 M_1}{r^4}$$

upwards on M_1
down wards on M_2