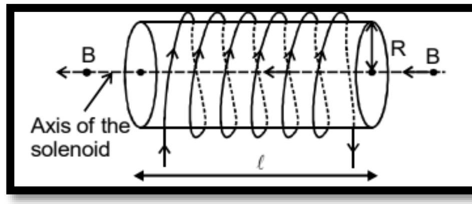
MOVING CHARGES AND MAGNETISM

THE SOLENOID AND THE TOROID

SOLENOID:

1. Solenoid contains large number of circular Loops wrapped around a non-conducting Cylinder. (It may be a hollow cylinder or it may be a solid cylinder.

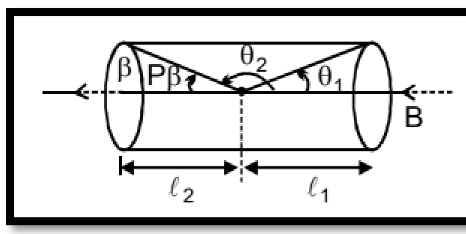


2. The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

3. B on axis (turns should be very close to each other's).

$$B=\frac{\mu_0 ni}{2}\left(\cos\theta_1-\cos\theta_2\right)$$

where n : number of turns per unit length.



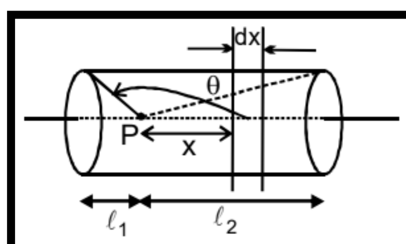
$$\cos\theta_1=\frac{\ell_1}{\sqrt{\ell_1^2+R}}\ ;\quad \cos\beta=\frac{\ell_1}{\sqrt{\ell_2^2+R^2}}=-\cos\theta_2$$

$$B=\frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2+R}}+\frac{\ell_2}{\sqrt{\ell_2^2+R^2}}\right]$$

$$=\frac{\mu_0 ni}{2}\left(\cos\theta_1+\cos\beta\right)$$

Derivation:

Take an element of width dx at a distance x from point P. [point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element dn = ndx where n: number of turns per unit length]



$$dB = \frac{\mu_0 i R^2}{2\left(R^2 + X^2\right)^{3/2}}(ndx)$$

$$B = \int dB = \int_{-\ell_1}^{\ell_2} \frac{\mu_0 i R^2 ndx}{2\left(R^2 + X^2\right)^{3/2}}$$

$$= \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right] = \frac{\mu_0 ni}{2}\left[\cos\theta_1 - \cos\theta_2\right]$$

4. For ideal Solenoid:

Inside (at the midpoint)

$\lambda >>$ R or length is infinite

$\theta 1 \to 0$

$\theta 2 \to \pi$

$$B = \frac{\mu_0 ni}{2}\left[1 - (-1)\right]$$

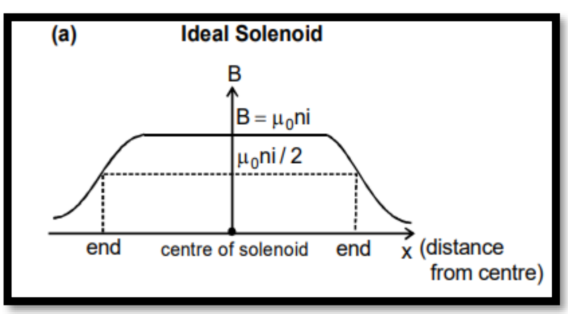
$$B = \mu_0 ni$$

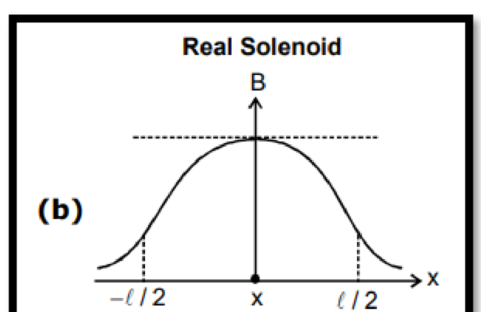
If material of the solid cylinder has relative permeability -μ_rí then $B = \mu_0 \mu_r ni$

At the ends

$$B = \frac{\mu_0 ni}{2}$$

5. Comparison between ideal and real solenoid:

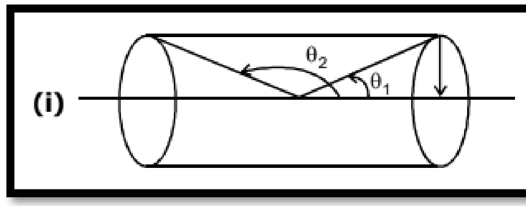




Ex. A solenoid of length 0.4m and diameter 0.6m consists of a single layer of 1000 turns of fine wire carrying a current of 5.0 × 10ñ3 ampere. Find the magnetic field on the axis at the middle and at the ends of the solenoid. **(For Competitive Exam)**

(Gives $\mu_0 = 4\pi \times 10^{-7} \dfrac{V-s}{A-m}$)

Sol. $B = \frac{1}{2}\mu_0 ni\left[\cos\theta_1 - \cos\theta_2\right]$



$\Rightarrow \; n = \dfrac{100}{0.4} = 2500$ per meter

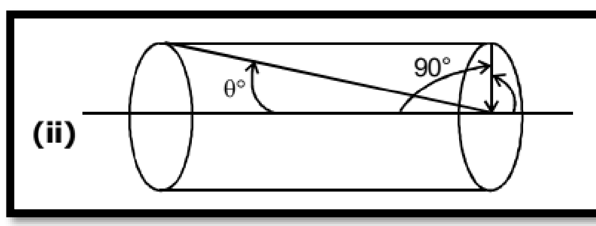
$i = 5\times10^{-3} A.$

$.\cos\theta_1 = \dfrac{0.2}{\sqrt{(0.3)^2+(0.2)^2}} = \dfrac{0.2}{\sqrt{0.13}}$

$\cos\theta_2 = \dfrac{-0.2}{\sqrt{0.13}}$

$\Rightarrow B = \frac{1}{2}\times\left(4\times\pi\times10^{-7}\right)\times2500\times5\times10^{-3}\,\dfrac{2\times0.2}{\sqrt{0.13}} = \dfrac{\pi\times10^{-5}}{\sqrt{13}}T$

At the end



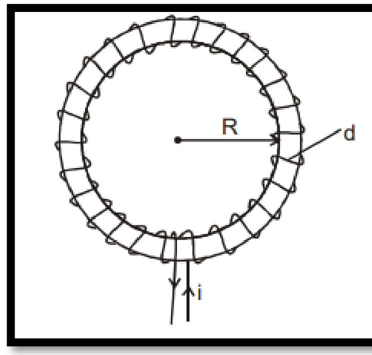
$\cos\theta_1 = \dfrac{0.4}{\sqrt{(0.3)^2+(0.4)^2}} = 0.8$

$\cos\theta_2 = \cos 90^\circ = 0$

$\Rightarrow B = \frac{1}{2}\times\left(4\times\pi\times10^{-7}\right)\times2500\times5\times10^{-3}\times0.8$

$\Rightarrow B = 2\pi\times10^{-6} Wb/m^2$

Toroid:



It is on hollow circular tube have windings of conducting wire closely attached to each other circularly on it (as shown below)

for ideal Toroid d << R

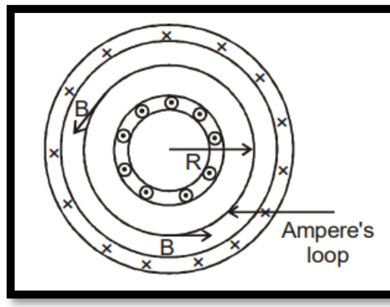
Magnetic field in Toroid

Let N = Total No. of turns Now from Amperes circuital law

$\oint B.d\ell = \mu_0 i_{in}$

$B.2\pi R = \mu_0 i_{in} = \mu_0 Ni$

$\Rightarrow B = \dfrac{\mu_0 Ni}{2\pi R}$



$n = \dfrac{N}{2\pi R} =$ No of turns per unit length

So $B = \mu_0 ni$