MOVING CHARGES AND MAGNETISM

MOTION IN COMBINED ELECTRIC AND MAGNETIC FIELDS

MOTION OF A CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELD

When the moving charged particle is subjected simultaneously to both electric field ρE and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F_e} = q\vec{E}$ and magnetic force

$$\vec{F_m} = q\left(\vec{V} \times \vec{B}\right)$$

$$\vec{F} = q\left[\vec{E} + \vec{V} \times \vec{B}\right]$$

which is Lorentz force equation.

Now let us consider two special cases involving the application of above equation

Note

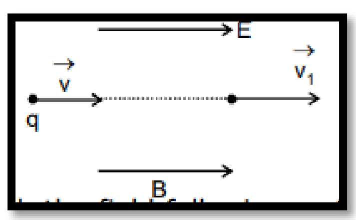
Magnetic force is frame dependent, Electric force is frame dependent but Lorentz force is frame independent

CASEI:

When $\vec{V}$ $\vec{E}$ and $\vec{B}$ all the three are collinear In this situation as the particle is moving parallel or anti-parallel to the field, the magnetic force on it will be zero and only electric force will act, so

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

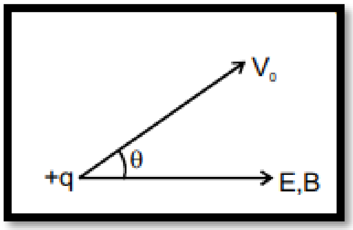
Hence the particle will pass through the field following a straight-line path (parallel to the field) with change in its speed. So in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown in the figure.

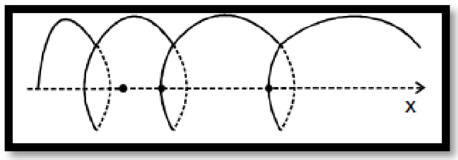


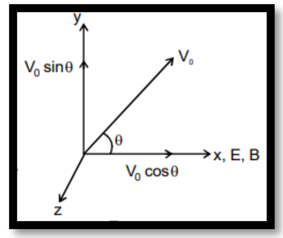
$\vec{V}$, $\vec{E}$, and $\vec{B}$ are collinear.

CASEII:

$\vec{E} \| \vec{B}$ And uniform $\theta \neq 0, 180°$ ($\vec{E}$ and $\vec{B}$ are constant and uniform)



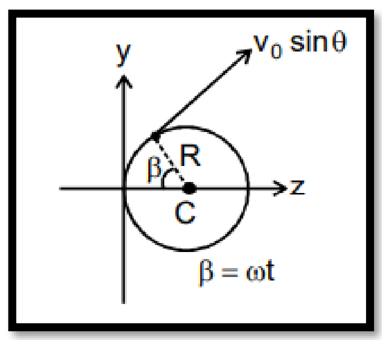




along X axis:

$$\vec{F}_x = qE, a_x = \frac{qE}{m}, v_x = v_0 \cos\theta + a_x t, x = v_0 \cos\theta t + \frac{1}{2} a_x t^2$$

in y z plane:



$qv_0 \sin\theta B = m (v_0 \sin\theta)^2 / R$

$$R = \frac{mv_0 \sin\theta}{qB}$$

$$\omega = \frac{v_0 \sin\theta}{R} = \frac{qB}{m}$$

$$\frac{2\pi}{T} = 2\pi f$$

$$\vec{r} = \{(v_0 \cos\theta =)t + \frac{1}{2}\frac{qE}{m}t^2\}\hat{i} + R\sin\omega t\hat{j} + (R - R\cos\omega t)(-\hat{K})$$

$$\vec{v} = (v_0 \cos\theta + \frac{qE}{m}t)\hat{i} + (V_0 \sin\theta)\cos\omega t\vec{j}$$

$$+v_0 \sin\theta \sin\omega t - \vec{k}$$

$$\vec{a} = \frac{qE}{m}\hat{i} + \omega^2 R[-\sin\beta\hat{j} - \cos\beta\hat{k}]$$

CASEIII:

$\vec{V}$, $\vec{E}$ and B → are mutually perpendicular, $\vec{V}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular. In case situation of $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0$$

Or $$\vec{a} = \left(\frac{\vec{F}}{m}\right) = 0,$$ then the particle will pass through the field with the same velocity. In this situation,

$$F_e = F_m \text{ or, } qE = qvB$$

Or $$v = \frac{E}{B}$$

This principle is used in velocity-selector to get a charged beam having a specific velocity.

Ex. If a charge particle (q) inters into the magnetic field at origin with velocity vi, then find the maximum possible positive x co-ordinate of particle if magnetic field is given as

$$B = B0x$$
$$= 0$$
$$(\text{-k})\ x > 0$$
$$x < 0$$

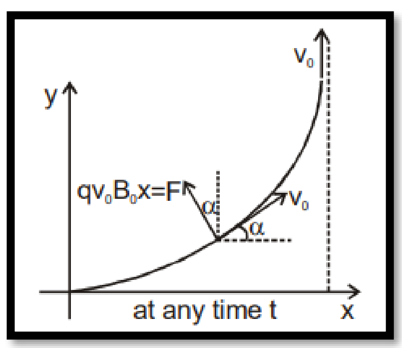
Sol. Because $F = qv_0B_0x$ at any time t at position

$xF_{Fvertical} = qB_0v_0x\cos\alpha$

$$a_y = \frac{dy}{dt} = \frac{qB_0v_0x\cos\alpha}{m}$$

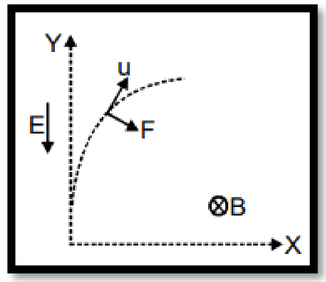
$$\frac{dv_y}{dx}\left(\frac{dx}{dt}\right) = \frac{qB_0x}{m}(v_0\cos\alpha)$$

$$\frac{dx}{dt} = v_0\cos\alpha$$



$$\int_0^{v_0} dV_y = \int_0^{x_{max}} \frac{q\{B_0x\}}{m}dx \Rightarrow V_0 = \frac{qB_0}{m}\cdot\frac{x_{max}^2}{2}$$

Ex. A long, straight wire carries a current i. A particle having a positive charge q and mass m kept at a distance x0 from the wire is projected towards it with a speed v. Find the minimum separation between the wire and the particle



Sol. Let the particle be initially at P(figure). Take the wire as the Y-axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the X-axis. We have, OP = x_0. The magnetic field B at any point to the right of the particle is, therefore, in the X-Y plane. As there is no initial velocity along the Z-axis, the motion will be in the X-Y plane. Also, its speed remains unchanged. As the magnetic field is not uniform, the particle does not go along a circle.

The force at time t is $\vec{F} = q\vec{v} \times \vec{B}$

$$= q(v_x \vec{i} + v_y \vec{j}) \times (-\frac{\mu_0 i}{2\pi x}\vec{k}) = \vec{j} q v_x \frac{\mu_0 i}{2\pi x} - \vec{i} q v_y \frac{\mu_0 i}{2\pi x} \quad \dots (1)$$

Thus $$a_x = \frac{F_x}{m} = -\frac{\mu_0 q i}{2\pi m}\frac{v_y}{x} = -\lambda\frac{v_y}{x} \quad \dots (2)$$

Where $$\lambda = \frac{\mu_0 q i}{2\pi m}$$

Also $$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{v_x dv_x}{dx} \quad \dots (3)$$

As, $$v_x^2 + v_y^2 = v^2$$

Giving $$v_x dv_x = -v_y dv_y \quad \dots (4)$$

From (1), (2) and (3),

$$\frac{v_y dv_y}{dx} = \frac{\lambda v_y}{x}$$

$$\frac{dx}{x} = \frac{dv_y}{\lambda}$$

Initially x = x_0 and $v_y = 0$. At minimum separation from the wire, $v_x = 0$ so that

$$v_y = -v$$

Thus

$$\int_{x_0}^{x}\frac{dx}{x}=\int_{0}^{-y}\frac{dv_y}{\lambda}\ \text{or In}\frac{x}{x_0}=-\frac{v}{\lambda}$$

$$x=x_0e^{-v/\lambda}=x_0e^{\frac{2\pi nw}{\mu 0qi}}$$