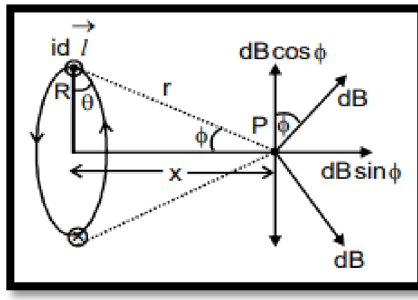
MOVING CHARGES AND MAGNETISM

MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP

MAGNETIC FIELD AT AN AXIAL POINT OF A CIRCULAR COIL

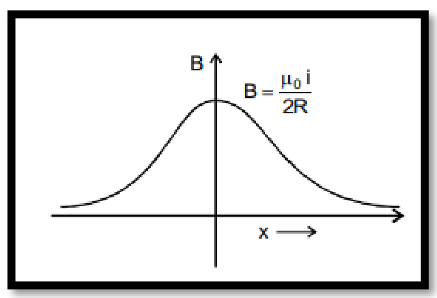
Consider a circular loop of radius R and carrying a steady current i. We have to find out magnetic field at the axial point P, which is at distance x from the center of the loop.



Consider an element i $\vec{dl}$ of the loop as shown in figure, and the distance of point P from current element is r. The magnetic field at P due to this current element from the equation (1) can be given by

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\vec{dl}\times\vec{r}}{r^3}$$

In case of point on the axis of a circular coil, as for every current element there is a symmetrically situated opposite element, the component of the field perpendicular to the axis cancel each other while along the axis add up.



$$B = \int dB\sin\phi = \frac{\mu_0}{4\pi}\int \frac{dB\sin\phi}{r^2}$$

Here, θ is angle between the current element id $\vec{l}$ and $\vec{r}$, which is $\frac{\pi}{2}$ everywhere and

$$\sin\phi\frac{R}{r} = \frac{R}{\sqrt{R^2+x^2}}$$

$$B = \frac{\mu_0}{4\pi}\frac{iR}{(R^2+x^2)^{3/2}}\int_0^{2\pi R} dL$$

$$B = \frac{\mu_0}{4\pi}\frac{iR}{(R^2+x^2)^{3/2}}(2\pi R)$$

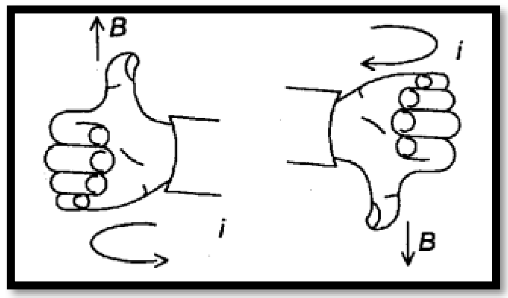
$$B = \frac{\mu_0}{4\pi}\frac{2\pi i R^2}{(R^2+x^2)^{3/2}}$$

If the coil has N turns, then

$$B = \frac{\mu_0}{4\pi}\frac{2\pi i R^2}{(R^2+x^2)^{3/2}}$$

Direction of $\vec{B}$:

Direction of magnetic field at a point the axis of a circular coil is along the axis and its orientation can be obtained by using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field.



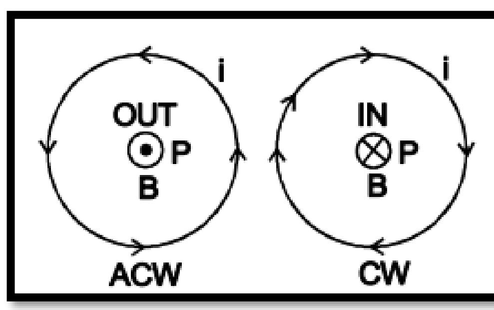
Magnetic field will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure given. Now consider some special cases involving the application of equation

CASE I:

Field at the centre of the coil

In this case distance of the point P from the centre (x) = 0, the magnetic field

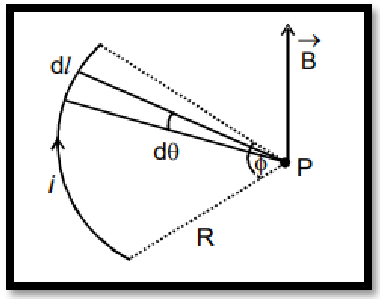
$B = \frac{\mu_0}{4\pi}\frac{2\pi i}{R} = \frac{\mu_0}{2}\frac{i}{R}$



CASE II: Field at a point far away from the centre

It means x >> R, $B = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{X^3}$

FIELD AT THE CENTRE OF A CURRENT ARC



Consider an arc of radius R carrying current i and subtending an angle ϕ at the centre. According to Biot-Savart Law, the magnetic field induction at the point P is given by

$$B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{idl}{R^2}$$

Here, $dl = Rd\theta$

$$B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{iRd\theta}{R^2}$$

$$B = \frac{\mu_0}{4\pi}\frac{i\phi}{R}$$

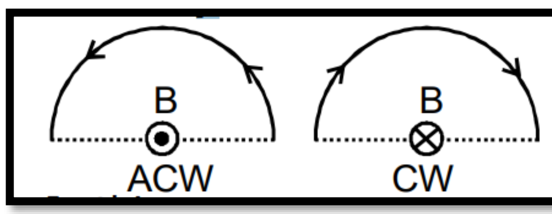
It lí is the length of the circular arc, we have

$$B = \frac{\mu_0}{4\pi}\frac{il}{R^2}$$

Consider some special cases involving the application of equation

CASE I:

If the loop is semicircular



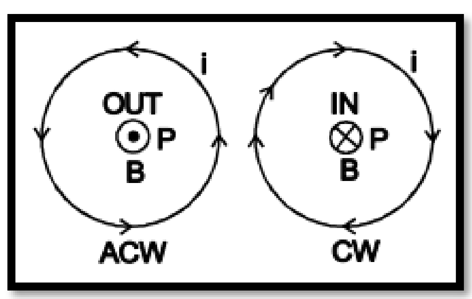
In this case $\phi = \pi$, so

$$B = \frac{\mu_0}{4\pi}\frac{\pi i}{R}$$

And will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure

CASE II.

If the loop is a full circle with N turns

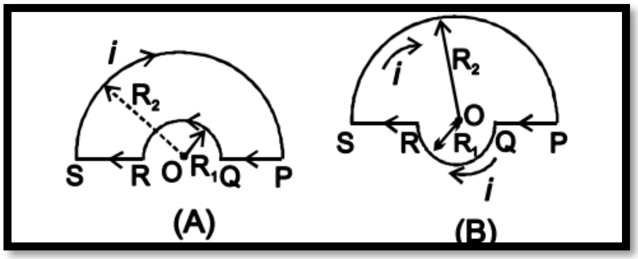


In this case $\phi = 2\pi$, s

$$B = \frac{\mu_0}{4\pi}\frac{2\pi Ni}{R}$$

And will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure.

Ex. Two wire loop PQRSP formed by joining two semicircular wires of radi R_1 and R_2 carries a current i as shown in the figure given below. What is the magnetic field induction at the center O in cases (A) and (B) ?



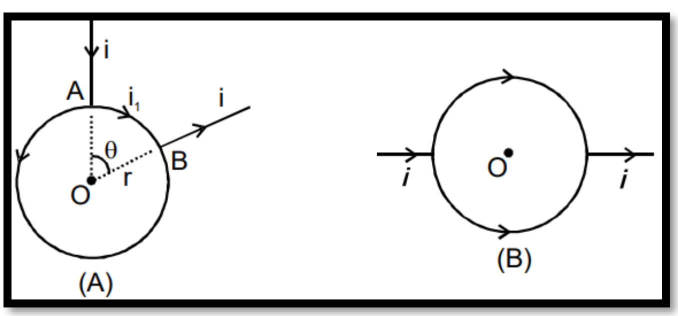
Sol. (a) As the point O is along the length of the straight wires, so the field at O due to them will be zero and hence magnetic field is only due to semicircular portions

$$|\vec{B}| = \frac{\mu_0}{4\pi}\left[\frac{\pi i}{R_2} \otimes + \frac{\pi i}{R_1} \odot\right]$$

$$|\vec{B}| = \frac{\mu_0}{4\pi} \pi i\left[\frac{1}{R_2} - \frac{1}{R_1}\right] \text{ Out of the page}$$

(b) $$|\vec{B}| = \frac{\mu_0}{4\pi} i\pi\left[\frac{1}{R_2} + \frac{1}{R_1}\right] \text{ Into the page}$$

Ex. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R as shown in the figure given below. One of the arcs AB of the ring subs tends an angle θ at the central. What is the value of the magnetic field at the central due to the current in the ring?



Sol. (a) As the field due to arc at the centre is given by

$$B = \frac{\mu_0}{4\pi}\frac{i\phi}{r}$$

$$B = \frac{\mu_0}{4\pi}\frac{i1\phi}{r} \otimes + \frac{\mu_0}{4\pi}\frac{2_2(2\pi-\theta)}{r}$$

$$(V_A - V_B) = i_1 R_1 = i_2 R_2$$

Or, $$i_2 = i_1 \frac{R_1}{R_2} = i_1 \frac{L_1}{L_2} [\therefore R \alpha L]$$

$$i_2 = i_1 \frac{\theta}{(2\pi-\theta)} [\therefore L = r\theta]$$

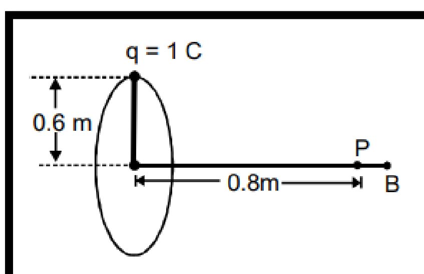
$$B_R = \frac{\mu_0}{4\pi}\frac{i_1\theta}{r} \otimes + \frac{\mu_0}{4\pi}\frac{i_1\theta}{r} \odot = 0$$

i.e., the field at the centre of the coil is zero and is independent of θ

Ex. A charge of one coulomb is placed at one end of a nonconducting rod of length 0.6m. The rod is rotated in a vertical plane about a horizontal axis passing through the other end of the rod with angular frequency 104π rad/s. Find the magnetic field at a point on the axis of rotation at a distance of 0.8 m from the centre of the path. Now half of the charge is removed from one end and placed on the other end. The rod is rotated in a vertical plane about horizontal axis passing through the mid-point of the rod with the same angular frequency. Calculate the magnetic field at a point on the axis at a distance of 0.4 m from the centre of the rod.

Sol. As the revolving charge q is equivalent to a current

$$i = qf = q \times \frac{\omega}{2\pi} = 1 \times \frac{10^4\pi}{2\pi} = 5 \times 10^3 \text{ A}$$

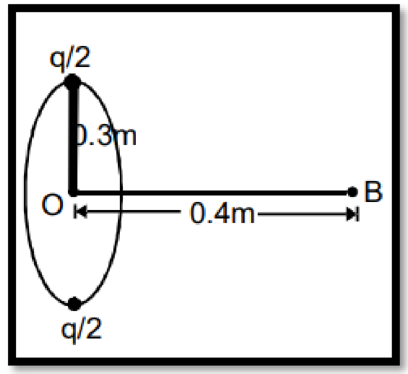


$$B = \frac{\mu_0}{4\pi}\frac{2\pi i R^2}{(R^2+x^2)^{3/2}}$$

$$B = 10^{-7} \times \frac{2\pi \times 5 \times 10^3 (0.6)^2}{[(0.6)^2+(0.8)^2]^{3/2}} = 1.13 \times 10^{-3} \text{ T}$$

If half of the charge is placed at the other end and the rod is rotated at the same frequency, the equivalent current.

$i = (\frac{q}{2})f + (\frac{q}{2})f = qf = i = 5 \times 10^3$ A



In this case, R' = 0.3 m and x' = 0.4 m

$$B = 10^{-7} \times \frac{2\pi \times 5 \times 10^3 \times (0.3)^2}{[(0.3)^2 + (0.4)^2]^{3/2}} = 2.3 \times 10^{-3}\ \text{T}$$