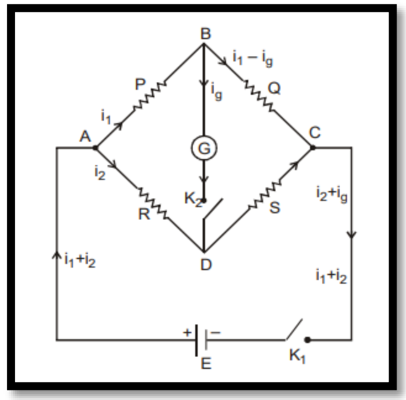
CURRENT ELECTRICITY

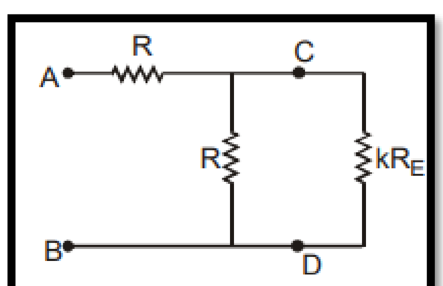
WHEATSTONE BRIDGE

WHEATSTONE'S BRIDGE

1. Wheatstone designed a network of four resistances with the help of which the resistance of a given conductor can be measured. Such a network of resistances is known as Wheatstone's bridge.



2. In this bridge, four resistance P, Q, R and S are so connected so as to form a quadrilateral ABCD. A sensitive galvanometer and key K_2 are connected between diagonally opposite corners B and D, and a cell and key K_1 are connected between two other corners A and C (figure shown)
3. When key K_1 is pressed, a current i flows from the cell. On reaching the junction A, the current i gets divided into two parts i_1 and i_2. Current i_1 flows in the arm AB while i_2 in arm AD. Current i_1, on reaching the junction B gets further divided into two parts (i_1 - i_g) and i_g, along branches BC and BD respectively. At junction D, currents i_2 and i_g are added to give a current ($i_2 + i_g$), along branch DC. (i_2 ñ i_g) and ($i_2 + i_g$) add up at junction C to give a current ($i_1 + i_2$) or i along branch CE. In this way, currents are distributed in the different branches of bridge. In this position, we get a deflection in the galvanometer.
4. Now the resistance P, Q, R and S are so adjusted that on pressing the key K_2, deflection in the galvanism- Eter becomes zero or current i_g in the branch BD becomes zero. In this situation, the bridge is said to be balanced.
5. In this balanced position of bridge, same current i_1 flows in arms AB and BC and similarly same current i_2 in arms AD and DC. In other words, resistances P and Q and similarly R and S, will now be joined in series.



6. **Condition of balance:**

Applying Kirchhoff's 2nd law to mesh ABDA,

$$i_1 P + i_g G - i2R = 0...$$

Similarly, for the closed mesh BCDB,

we get, $(i_1 ñ i_g) Q - (i_2 + i_g)S - i_gG = 0$

$i_1P - i_2R = 0$ or $i_1P = i_2R$ .... (3)

$i_1Q - i_2S = 0$ or $i_1Q = i_2S$.... (4)

Dividing (3) by (4), we have $\frac{P}{Q} = \frac{R}{S}$

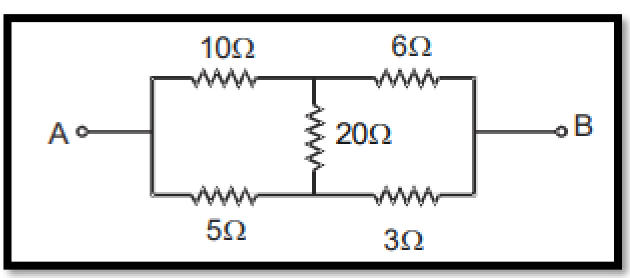
This is called as condition of balanced for Wheatstone's Bridge.

7. It is clear from above equation that if ratio of the resistance P and Q, and the resistance R are known, then unknown resistance S can be determined. This is the reason that arms P and Q are called as ratio arms, arm AD as known arm and arm CD as unknown arm.

8. When the bridge is balanced then on inter-changing the positions of the galvanometer and the cell there is no effect on the balance of the bridge. Hence the arms BD and AC are called as conjugate arms of the bridge.

9. The sensitivity of the bridge depends upon the value of the resistances. The sensitivity of bridge is maximum when all the four resistances are of the same order.

Ex. Find equivalent resistance of the circuit between the terminals A and B.

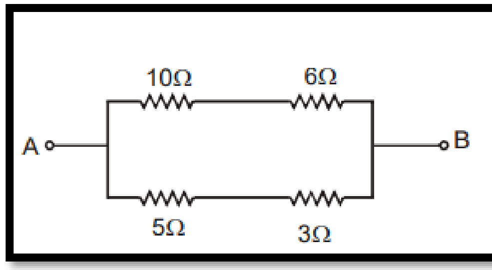


Sol. Since the given circuit is wheat stone bridge and it is in balance condition.

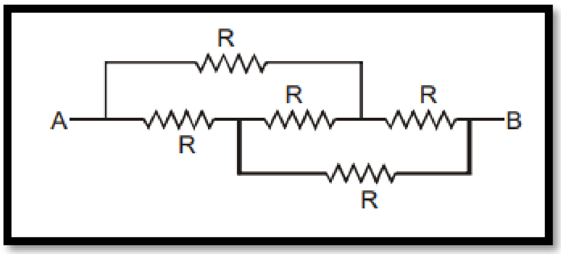
$10 \times 3 = 30 = 6 \times 5$

Hence this is equivalent to

$$R_{eq} = \frac{16 \times 8}{16 + 8} = \frac{16}{3}\Omega$$

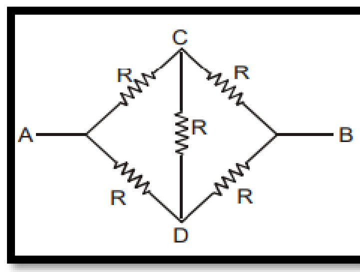


Ex. Find the equivalent resistance between A and B



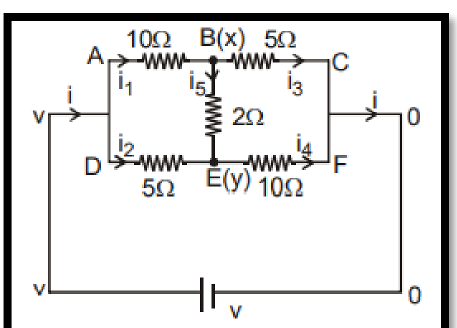
Sol. This arrangement can be modified as shown in figure since it is balanced wheat stone bridge.

$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$



UNBALANCED WHEATSTONE BRIDGE

Ex. Find equivalent resistance?



Sol. Let potential at point B is x and E is Y

$$R_{eq} = \frac{V}{i}$$

Applying KCl at point B

$$\frac{x-v}{10} + \frac{x-y}{2} + \frac{x-0}{5} = 0$$

$$8x - 5y = v \qquad ...(1)$$

Applying KCL at point E

$$\frac{y-v}{5} + \frac{y-x}{2} + \frac{y-0}{10} = 0$$

$$8y - 5x = 2v \qquad ...(2)$$

solving x & y

$x = \frac{6v}{13}$ $\qquad y = \frac{7v}{13}$

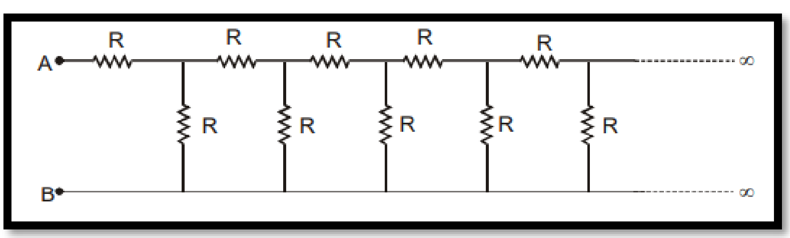
current from branches BC & EF adds up to give total current (i) flowing in the circuit.

$$i = i_3 + i_4 = \frac{x-0}{5} + \frac{y-0}{10} = \frac{19v}{130}$$

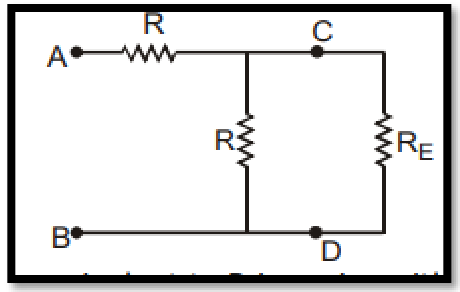
$$i = \frac{v}{R_{eq}} \qquad \therefore \; R_{eq} = \frac{130}{19}$$

LADDER PROBLEM:

Ex. Find the effective resistance between A & B?



Sol. Let the effective resistance between A & B be R_E since the network is infinite long, removal of one cell from the chain will not change the network. The effective resistance between points C & D would also be R_E



The equivalent network will be as shown below

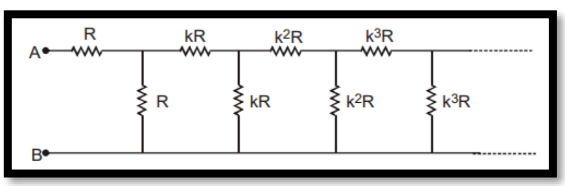
The original infinite chain is equivalent to R in series with R & RE in parallel

$$R_E = R_1 + \frac{RR_E}{R + R_E}$$

$$R_E R + R_E^2 = R^2 + 2RR_E \Rightarrow R_E^2 - RR_E - R^2 = 0$$

$$R_E = \frac{R\left(1+\sqrt{5}\right)}{2}$$

Ex. Find the equivalent resistance between A & B?



Sol. As moving from one section to next one, resistance is

Increasing by k times. Since the network is infinitely long, removal of one section from the chain will bring a little change in the network. The effective resistance between points C & D would be kR_E (where R_E is the effective resistance)

$\therefore$ Effective R between A & B

$$R_E = R + \frac{R\left(KR_E\right)}{KR_E + R}$$

On solving we get

$$R_E = \frac{2KR - R + \sqrt{\left(R - 2KR\right)^2 + 4KR^2}}{2K}$$