CURRENT ELECTRICITY
OHM'S LAW

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Using equations (1) and (2)

$$j = nev_d = \frac{ne^2\tau}{2m}E$$

$$j = \sigma E$$

$$\sigma = \frac{ne^2\tau}{2m}$$

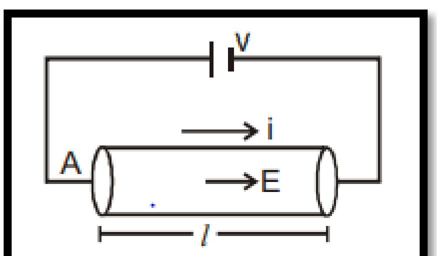
where σ depends only on material of the conductor and its temperature. This constant is called the electrical conductivity of the material. Equation (3) is known as Ohm's law.

The resistivity of a material is defined as

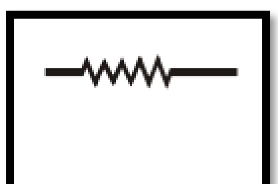
$$\rho = \frac{1}{\sigma} = \frac{2m}{ne^2\tau}$$

Ohm's law tells us that the conductivity (or resistivity) of a material is independent of the electric field existing in the material. This is valid for conductors over a wide range of field

Suppose we have a conductor of length l and uniform cross-sectional area A (figure shown) Let us apply potential difference V between the ends of the conductor. The electric field inside the conductor is $E = \frac{V}{l}$. . If the current in the conductor is i, the current density is $j = \frac{i}{A}$. Ohm's law j = σE then becomes.



(a)


(b)

$$\frac{i}{A} = \sigma \frac{V}{l}$$

$$V = \frac{1}{\sigma}\frac{l}{A}i = \rho\frac{l}{A}i$$

$$V = Ri$$

R is called the resistance of the given conductor. The quantity 1/R is called conductance. is another form of Ohm's law which is widely used in circuit analysis. The unit of resistance is called ohm and is denoted by symbol Ω. An object of conducting material, having a resistance of desired value, is called a resistor.

$$R = \frac{pl}{A}$$

The unit of resistivity ρ is ohm-meter, also written as Ω-m. The unit of conductivity (σ) is (ohm-m) written as mho/m.

$$R = \frac{2ml}{ne^2\tau A} = \frac{pl}{A}$$

where ρ = resistivity $\left(where \rho = \frac{2m}{ne^2\tau} \right)$

λ = length along the direction of current

A = Area of the cross section perpendicular to direction of current

n = no. of free charges per unit volume.

τ = relaxation time

m = mass of electron

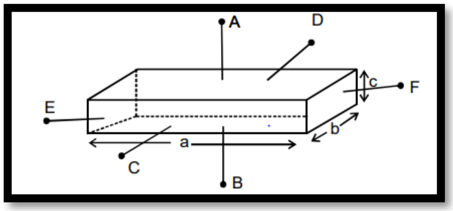
Ex. Calculate the resistance of an aluminum wire of length 50 cm and cross-sectional area 2.0 mm2. The resistivity of aluminum is ρ = 2.6 × 10^{-8} Ω-m

Sol. The resistance is $R = \rho\frac{t}{A}$

$$\frac{2.6\times10^{-8}\Omega - m\times0.50m}{2\times10^{-8}m^2} = 0.0065\Omega$$

We arrived at Ohm's law by making several assumptions about the existence and behavior of the free electrons. These assumptions are not valid for semiconductors, insulators, solutions etc. Ohm's law cannot be applied in such cases.

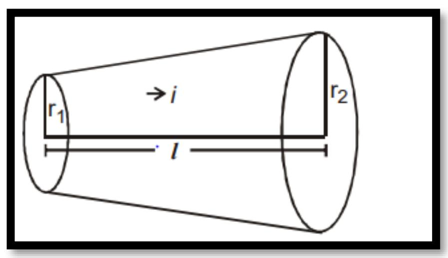
Ex. The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.



Sol. $$R_{AB} = \frac{\rho C}{ab}, R_{CD} = \frac{\rho b}{ac}, R_{EF} = \frac{\rho a}{bc}$$

Ex. A portion of length L is cut out of a conical solid wire. The two ends of this portion have circular cross-sections of radii r_1 and r_2 ($r_2 > r_1$). It is connected lengthwise to a circuit and a current i is flowing in it. The resistivity of the material of the wire is ρ. Calculate the resistance of the considered portion and the voltage developed across it.

Sol. If follows from the figure, that



$$\tan\theta = \frac{r_2 - r_1}{L}$$

$$\therefore r = r_1 + \tan\theta = r_1 + \times\left(\frac{r_2 - r_1}{L}\right) = \frac{r_1 L + \times(r_2 - r_1)}{L}$$

$$\therefore A = \pi r^2 = \frac{\pi}{L^2}\left[r_1 L + (r_2 - r_1)\times\right]^2$$

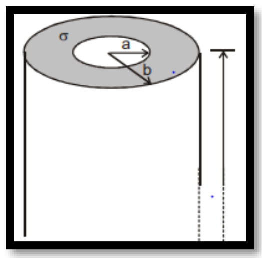
$$dR = \frac{\rho dx}{\pi r^2} = \frac{\rho dx L^2}{\pi\left[r_1 L + (r_2 - r_1)\times\right]^2} \Rightarrow R = \int dR = \frac{\rho L^2}{\pi}\int_0^L \frac{dx}{\left[r_1 L + (r_2 - r_1)\times\right]^2}$$

$$= \frac{\rho L^2}{\pi}\left[\left\{r_1 L + (r_2 - r_1)\times\right\}^{-1}\right]_0^L \left(-\frac{1}{(r_2 - r_1)}\right)$$

$$= \frac{-\rho L}{\pi(r_2 - r_1)}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{\rho L}{\pi(r_1 r_2)}$$

$$\therefore V = IR = \frac{I\rho L}{\pi r\, r_2}$$

Ex. The space between two coaxial cylinders, whose radii are a and b (where a < b as in (figure shown) is filled with a conducting medium. The specific conductivity of the medium is σ. **(For Competitive Exam)**



(a) Compute the resistance along the length of cylinder.

(b) Compute the resistance between the cylinders in the radial direction. Assume that the cylinders are very long as compared to their radii, i.e., L >> b, where L is the length of the cylinders.

Sol. (a) $R = \frac{\rho l}{A} = \frac{l}{\sigma A} = \frac{l}{\sigma(\pi b^2 - \pi a^2)} = \frac{l}{\pi\sigma(b^2 - a^2)}$

(b) From Ohm's law, we have

$$\vec{j} = \sigma\vec{E}$$

Assuming radial current density. $\vec{j}$ Becomes

$$\vec{j} = \frac{I}{2\pi r L}\hat{r} \text{ For } a < r < b$$

And, therefore, $\vec{E} = \frac{I}{2\pi\sigma L}\hat{r}$

Here we have used the assumption that L >> b so that $\vec{E}$ and $\vec{j}$ are in cylindrically symmetric form. The potential drop across the medium is thus:

$$V_{ab} = -\int_b^a \vec{E}(r).\vec{dr} = -\frac{I}{2\pi\sigma L}\int_b^a \frac{dr}{r} = \frac{I}{2\pi\sigma L} in\left(\frac{b}{a}\right)$$

The resistance $$R_{ab} = \frac{V_{ab}}{I} = \frac{In\left(\frac{b}{a}\right)}{2\pi\sigma L}$$

Method 2 We split the medium into differential cylindrical shell elements of width dr, in series. The current flow is cylindrically symmetric (L >> b). The area through which the current flows across a shell of radius r is A(r) = 2πrL. The length the current flows, passing through a shell of radius r is dr. Therefore, the resistance of the shell of radius r is:

$$dR = \frac{1}{\sigma}\frac{dr}{2\pi rL}$$

Since the shells are connected in a series, we have

$$R_{ab} = \int_a^b dR = \frac{In\left(\frac{b}{a}\right)}{2\pi\sigma L}$$

Effect of Temperature on Resistance

(a) Resistance of Pure Metals

(i) We know that

$$R = \left[\frac{2m}{ne^2\tau}\right]\frac{I}{A} \quad R = \left[\frac{2m}{ne^2\tau}\right]\frac{I}{A}$$

For a given conductor, l, A and n are constant, hence R ∝ (1/τ) If λ represents the mean free path (Average distance covered between two successive collisions) of the electron and v_{rms}, the root-mean-square speed, then

$$\tau = \frac{\lambda}{V_{rms}} \text{ Hence } R \propto \frac{V_{rms}}{\lambda}$$

λ decreases with rise in temperature because the amplitude of vibrations of the +ve ions of the metal increases and they create more hindrance in the movement of electrons and,

(b)

(i) v_{rms} increases because $v_{rms} \propto \sqrt{T}$. Therefore, Resistance of the metallic wire increases with rise in temperature. As $\rho \propto R$ and $\sigma \propto (1/\rho)$, hence resistivity increases and conductivity decreases with rise in temperature of the metallic of the metallic wires.

(ii) If R0 and Rt represent the resistances of metallic wire at 0°C and t°C respectively then Rt is given by the following formula:

$$R_t = R_0(1 + \alpha t)$$

where α is called as the Temperature coefficient of resistance of the material of the wire. α depends on material and temperature but generally it is taken as a constant fora particular material for small change.

$$R_t - R_0 = R_0 \alpha t$$

for very small change in temperature $dR = R_0 \alpha dt$

(c) Resistance of semiconductors

(i) There are certain substances whose conductivity lies in between that of insulators and conductors, higher than that of insulators but lower than that of conductors. These are called as semiconductors, e.g., silicon, germanium, carbon etc.

(ii) The resistivity of semiconductors decreases with increase in temperature i.e., α for Semiconductor tors is ñve and high.

(iii) Though at ordinary temperature the value of n (no. of free electrons per unit volume) for these materials is very small as compared to metals, but increases very rapidly with rise in temperature (this happens due to breaking of covalent bonds). Though τ decreases but factor of n dominates. Therefore, the resistance

$R = \dfrac{ml}{ne^2\tau A}$ goes on decreasing with increase in temperature.