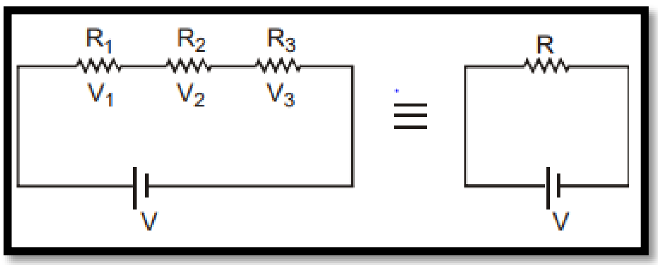
CURRENT ELECTRICITY

COMBINATION OF RESISTORS – SERIES AND PARALLEL

COMBINATION OF RESISTANCE:

A number of resistances can be connected in a circuit and any complicated combination can be, in general, reduced essentially to two different types, namely series and parallel combinations.

(a) Resistance in Series



1. In this combination the resistance are joined end to end. The second end of each resistance is joined to first end of the next resistance and so on. A cell is connected between the first end of first resistance and second end of last resistance. Figure shows three resistances R_1, R_2 and R_3 connected in this way.
   Let V_1, V_2 and V_3 are the potential differences across these resistances.

2. In this combination current flowing through each resistance will be same and will be equal to current supplied by the battery

3. As resistances are different and current flowing through them is same, hence potential differ- emcees across them will be different. Applied potential difference will be distributed among three resistances directly in their ratio As i is constant, hence $V \propto R$
   i.e., $V1 = iR_1$, $V_2 = iR_2$, $v_3 = iR_3$

4. If the potential difference between the points A and D is V, then

$$V = V_1 + V_2 + V_3 = i\,(R_1 + R_2 + R_3)$$

5. If the combination of resistances between two points is replaced by a single resistance R such that there is no change in the current of the circuit in the potential difference between those two points, then the single resistance R will be equivalent to combination and $V = i\,R$ i.e.,

$$iR = i\,(R_1 + R_2 + R_3) \text{ or } R = R_1 + R_2 + R_3$$

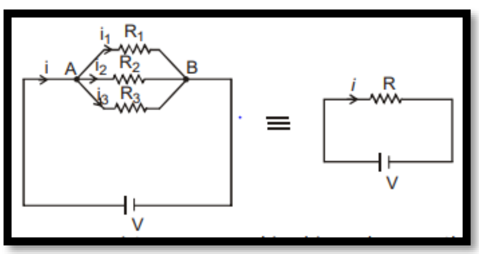
6. Thus, in series combination of resistances, important conclusion are
   (a) Equivalent Resistance > highest individual resistance
   (b) Current supplied by source = Current in each resistance

$$\frac{V}{R_1 + R_2 + R_3} = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3}$$

   (c) The total potential difference V between points A and B is shared among The three resistances directly in their ratio

$$V_1 : V_2 : V_3 = R_1 : R_2 : R_3$$

(b) Resistance in Parallel



1. When two or more resistance are combined in such a way that their first ends are connected tone terminal of the battery while other ends are connected to other terminal, then they are said to be connected in parallel. Figure shows three resistances R_1, R_2 and R_3 joined in parallel between two points A and B. Suppose the current flowing from the battery is i. This current gets divided into three parts at the junction A. Let the currents in three resistance R_1, R_2 andR_3, are i_1, i_2, i_3 respectively.

2. Suppose potential difference between points A and B is V. Because each resistance is con- nested between same two points A and B, hence potential difference across Each resistance will be same and will be equal to applied potential difference V.

3. Since potential difference across each resistance is same, hence current approaching the junction A is divided among three resistances reciprocally in their ratio.

As V is constant, hence $i \propto (1/R)$ i.e.,

$i_1 = \frac{V}{R}$ $\quad i_2 = \frac{V}{R_2}$ $\quad$ or $\quad i_3 = \frac{V}{R_3}$

4. Because i the main current which is divided into three parts i_1, i_2 and i_3 at the junction A.

Hence $$i = i_1 + i_2 + i_3 = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

If the equivalent resistance between the points A and B is R, then $i = \frac{V}{R}$

Thus, $\frac{V}{R} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$ $\quad$ or $\quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

5. Thus, in parallel combination of resistance important conclusion are:

(a) Equivalent resistance < lowest individual resistance

(b) Applied potential difference = Potential difference across each resistance.

or $\quad iR = i_1 R_1 = i_2 R_2 = i_3 R_3$

(c) Current approaching the junction A = Current leaving the junction B and current is shared among the three resistances in the inverse ratio of resistances

$$i_1 : i_2 : i_3 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

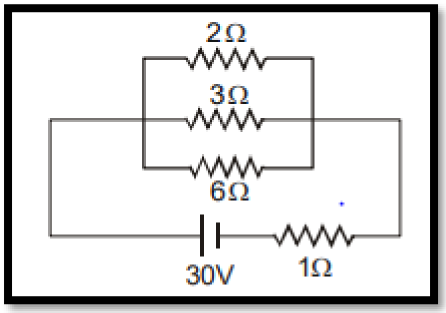
(i) If two or more resistance are joined in parallel then $i_1R = i_1R_2 = i_3R_3$ ………… i.e., iR = constant i.e., a low resistance joined in parallel always draws a higher current.

(ii) When two resistance R_1 and R_2 are joined in parallel, then

$$\frac{i_1R_1}{i_2R_2} = 0 \text{ or } \frac{i_1^2R_1^2}{i_2^2R_2^2} = 1 \text{ or } \frac{i_1^2R_1t}{i_2^2R_2t} = \frac{R_2}{R_1} \text{ or } \frac{H_1}{H_2} = \frac{R_2}{R_1}$$

i.e., heat produced will be maximum in the lowest resistance.

Ex. Find current which is passing through battery

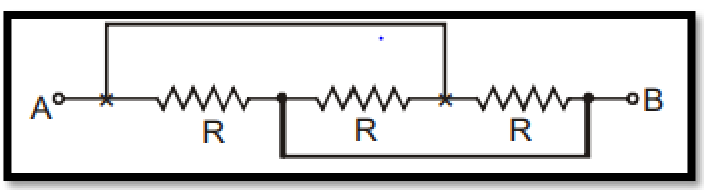


Sol. Here potential difference across each resistor is not 30 V Θ battery has internal resistance here the concept of combination of resistors is useful.

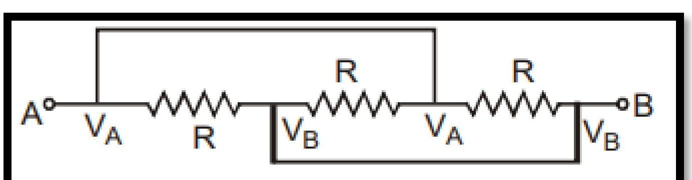
$R_{eq} = 1 + 1 = 2\Omega$

$$i = \frac{30}{2} = 15A$$

Ex. Find equivalent Resistance

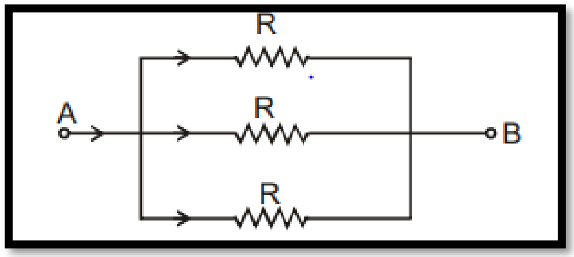


Sol.

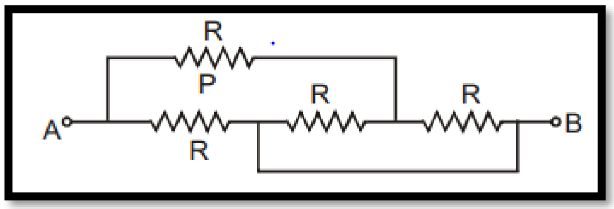


Here all the Resistance are connected between the terminals A and B. So, Modified circuit is

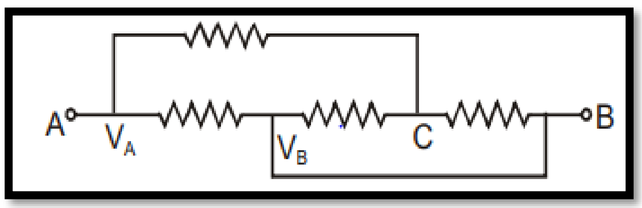
So $$R_{eq} = \frac{R}{3}$$



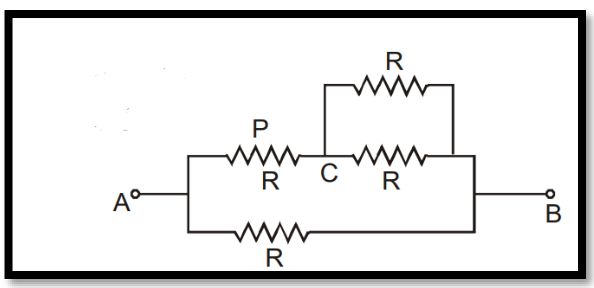
Ex. Find the current in Resistance P if voltage supply between A and B is V volts



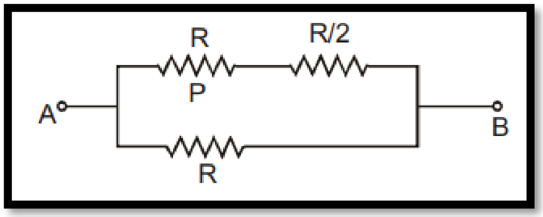
Sol. $$R_{eq} = \frac{3R}{5}$$



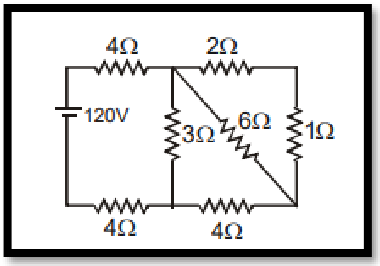
$$I = \frac{5V}{3R}$$ Modified circuit



Current in $$P = \frac{R \times \frac{5V}{3R}}{1.5R + R} = \frac{2V}{3R}$$



Ex. Find the current in 2 Ω resistance



Sol. 2Ω, 1Ω in series = 3Ω

3Ω, 6Ω in parallel = $\frac{18}{9} = 2\Omega$

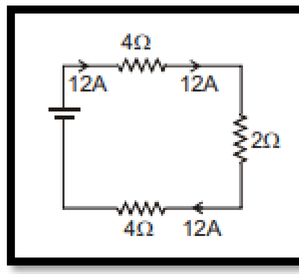
2Ω, 4Ω in series = 6Ω

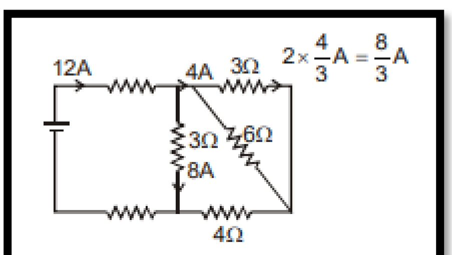
6Ω, 3Ω is parallel = 2Ω

$R_{eq} = 4 + 4 + 2 = 10\ \Omega$

$i = \frac{120}{10} = 12A$

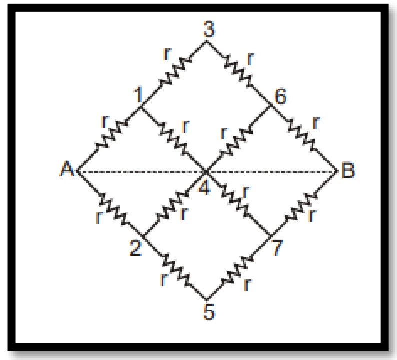
So current in 2Ω Resistance = $\frac{8}{3}A$



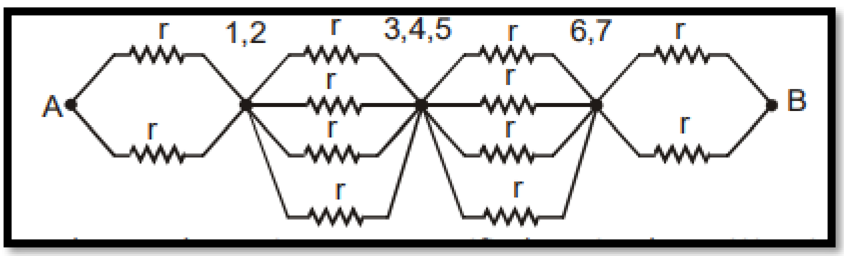


SPECIAL PROBLEMS

We wish to determine equivalent resistance between A and B. In figure shown points (1,2) (3, 4, 5) and (6, 7) are at same potential Equivalent circuit can be redrawn as in figure shown. The equivalent resistance of this series combination is

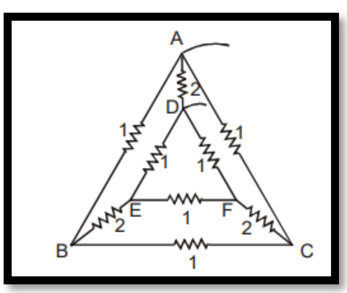


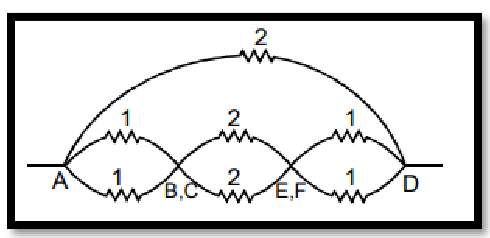
$$R_{eq}=\frac{r}{2}+\frac{r}{4}+\frac{r}{4}+\frac{r}{2}=\frac{3r}{2}$$



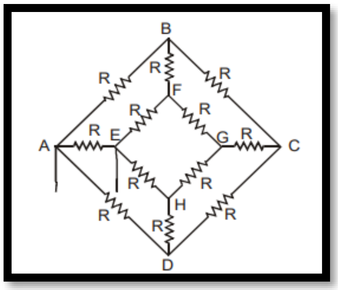
In the figure shown, the resistances specified are in ohms. We wish to determine the equivalent resistance between point A and D. Point B and C, E and F are the the same potential so the circuit can be redrawn as in figure shown.

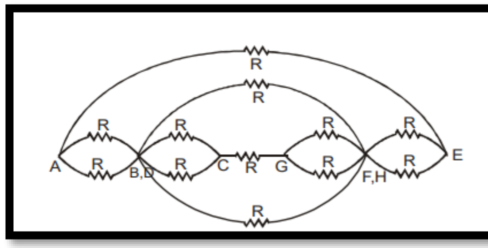
Thus, the equivalent resistance is 1 Ω.





In the network shown in figure shown all the resistances are equal, we wish to determine equivalent resistance between A and E. Point B and D have same potential, similarly F and H have same potential. The equivalent circuit is shown in figure shown. The equivalent resistance of network is 7R/2





Ex. In the circuit shown in figure. (a) find the current flowing through the 100 Ω resistor connecting points U and S.

Sol. Figure (b) shows simplified circuit. The battery is directly attached to resistor 90Ω hence current in it is 2 A, see figure (c), The total resistance of second branch is also 90 Ω, hence current divides equally. Now current through 45 Ω resistor is 2 A and it is a combination of two equal 90 Ω resistors. Once again current divides equally. 90 Ω resistor is a series combination of 40 Ω and 50 Ω, hence current through them is equal, i.e.,

1 A. As 50 Ω resistor is a parallel combination of two equal 100 Ω resistors, they must have the same current i.e., 0.5 A