CURRENT ELECTRICITY

CELLS IN SERIES AND IN PARALLEL

COMBINATIONS OF CELLS:

A cell is used to maintain current in an electric circuit. We cannot obtain a strong current from a single cell. Hence need arises to combine two or more cells to obtain a strong current. Cells can be combined in three possible ways:

(A) In series,

(B) In parallel, and

(C) In mixed grouping.

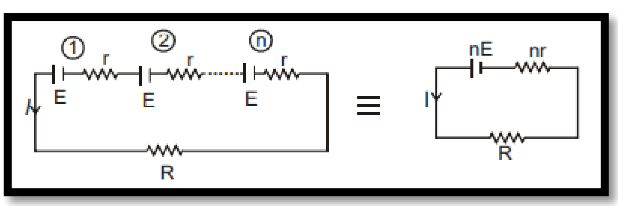
(A) Cells in Series

In this combination, cells are so connected that -ve terminal of each cell is connected with the +ve terminal of next and so on. Suppose n cells are connected in this way. Let e.m.f and internal resistance of each cell are E and r respectively.Net e.m.f of the cells = nE. Total internal resistance = nr. Hence total

Resistance of the circuit = nr +R.

If total current in the circuit is I, then

$$I = \frac{net\ e.m.f}{Total\ Resistance} = \frac{nE}{nr = R} \qquad ...(1)$$



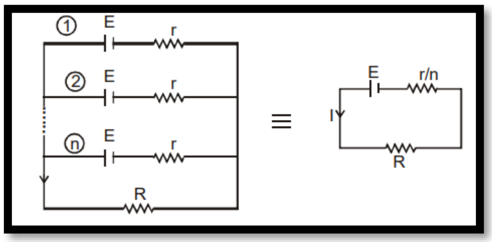
Case (i):

If nr < < R, then I ≅ n E/R i.e., if total internal resistance of the cells is far less than external resistance, then current obtained from the cells is approximately equal to n times the current obtained from a single cell. Hence cells, whose total internal resistance is less than external resistance, just be joined in series to obtain strong current

Case (ii):

If nr >> R, then $I \cong \frac{nE}{nr} = \frac{E}{r}$ i.e., if total internal resistance of the cells is much greater than the external resistance, then current obtained from the combination of n cells is nearly the same as obtained from a single cell. Hence there is no use of joining such cells in series.

(B) Cells in Parallel



(I) When E.M.F's and internal resistance of all the cells are equal :

In this combination, positive termi- nals of all the cells are connected at one point and negative terminals at other point. Figure shown such cells connected in parallel across some external resistance R. Let e.m.f and internal resistance of each cell are E and r respectively. Because all the cells are connected in parallel between two points, hence e.m.f of battery = E. Total internal resistance of the combination of n cells = r/nBecause external resistance R is connected in series with internal Resistance, hence total resistance of the circuit = (r/n) + R

If current in external resistance, is I, then

$$I = \frac{net\ e.m.f}{Total\ Resistance} = \frac{E}{(r/n)+R} = \frac{nE}{r+nR}$$

Case (I):

If r << R, the $I \cong \frac{nE}{nr} = \frac{E}{R} =$ i.e., if internal resistance of the cells is much less than external resistance, then total current obtained from combination is nearly equal to current given by one cell only. Hence there is no use of joining cells of low internal resistance in parallel.

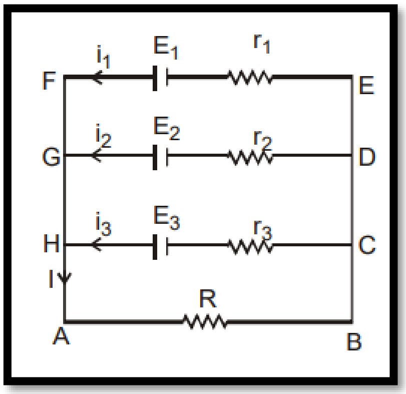
Case (II):

If r >> R, then $I \cong \frac{nE}{r}$ i.e., if the internal resistance of the cells is much higher than the external resistance, then total current is nearly equal to n times the current given by one cell. Hence cells of high internal resistance must be joined in parallel to get a strong current.

Case (III):

When emf's and internal resistance of all the cells connected in parallel are different. In this case, total current in external resistance is obtained with the help of Kirchhoff's laws. Figure shows three cells of e.m.f E_1, E_2 and E_3 and internal resistances r_1, r_2 and r_3 connected in parallel across some external resistance R. Suppose currents given by three cells are i_1, i_2 and i_3. Hence according to Kirchhoff's first law, total current I in external resistance R, is given by

$$I = i_1 + i_2 + i_3 \quad \text{... (1)}$$



Applying Kirchhoff's 2nd law to closed mesh ABEF we get

$$IR + i_1 r_1 = E \text{ or } i_1 = \left[\frac{(E_1 - IR)}{r_1}\right] \quad \text{... (2)}$$

Similarly, for closed meshes ABDG and ABCH, we get

$$i_2 = \frac{E_2 - 1R}{r_2} \quad \text{... (3)}$$

Or $$i_3 = \frac{E_2 - 1R}{r_3} \quad \text{... (4)}$$

Substituting eq. (2), (3) and (4) in eq. (1), we have

$$I = \frac{E_1 - IR}{r_1} + \frac{E_2 - IR}{r_2} + \frac{E_3 - IR}{r_3} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} - IR\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

$$I\left[1 + R\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)\right] = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}$$

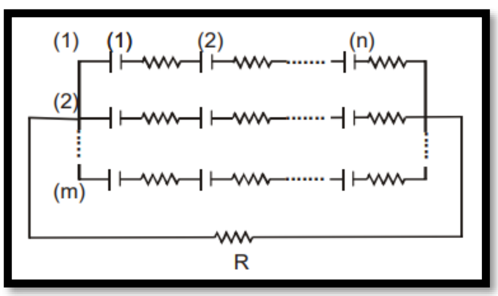
$$I = \frac{(E_1/r_1) + (E_2/r)_2 + (E_3/r_3)}{1 + R(1/r_1 + 1/r_2 + 1/r_3)}$$

If n cells are joined in parallel, then

$$I = \frac{\sum_i \frac{E_i}{r_i}}{1 + R\sum_i \frac{1}{r_1}} \quad \text{and} \quad E_{eq} = \frac{\sum \frac{E_i}{r_i}}{\sum \frac{1}{r_1}} \ , \ r_{eq} = \frac{1}{\sum \frac{1}{r_1}}$$

(C) CELLS IN MIXED GROUPING

In this combination, a certain number of cells are joined in series in various rows, and all such rows are then connected in parallel with each other. Suppose n cells, each of e.m.f E and internal resistance r, are connected in series in every row and much rows are connected in parallel across some external resistance R, as shown in figure.



Total number of cells in the combination = mn. As e.m.f. of each row = nE and all the rows are connected in parallel, hence net e.m.f of battery = nE. Internal resistance of each row = nr. As m such rows are connected in parallel, hence total internal resistance of battery $= \left(\frac{nr}{m}\right)$

Hence total resistance of the circuit $= \left[\left(\frac{nr}{m}\right) + R\right]$

If the current in external resistance is I, then

$$I = \frac{net\ e.m.f}{Total\ Resistance} = \frac{nE}{(nr/m)+R} = \frac{mnE}{nr+mR}$$

$$= \frac{mnE}{\left(\sqrt{nr}-\sqrt{mr}\right)^2 + 2\sqrt{nmrR}}$$

It is clear from above equation that I will be maximum when

[(√nr ñ √mR) 2 + 2 √ nmrR] is minimum

This will be possible when the quantity [√nr ñ √mR] 2 is minimum. Because this quantity is in square, it cannot be negative, hence its minimum value will be equal to zero, i.e.,

mR = nr or $R = \frac{nr}{m}$

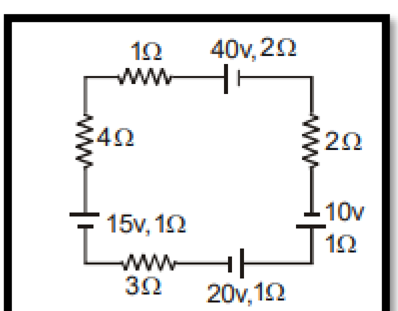
i.e., In mixed grouping of cells, current in external resistance will be maximum when total inter- nal resistance of battery is equal to external resistance.

Because power consumed in the external resistance or load = I 2R, hence
when current in load is maximum, consumed power in it is also maximum,
Hence consumed power in the load will
also be maximum when

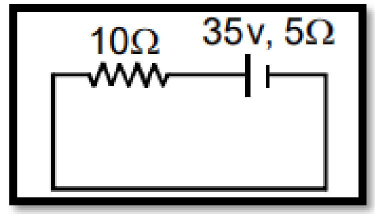
$R = \frac{nr}{m}$.

$I_{max} = \frac{mnE}{2mR}$ or $\frac{mnE}{2nr} = \frac{nE}{2R}$ or $\frac{mE}{2r}$

Ex. Find the current in the loop



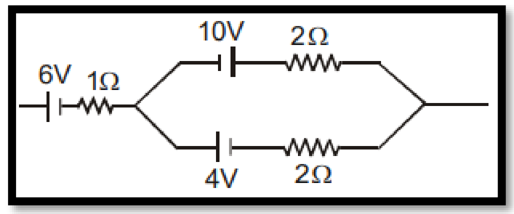
Sol. The given circuit can be simplified as



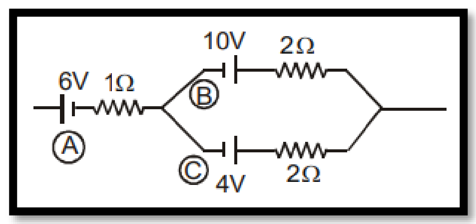
$$i=\frac{35}{10+5}=\frac{35}{15}$$

$$=\frac{7}{3}A \Rightarrow I=\frac{7}{3}A$$

Ex. Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



Sol. Battery (B) and (C) are in parallel combination with opposite polarity. So, their equivalent



$$\in_{BC}=\frac{\frac{10}{2}+\frac{-4}{2}}{\frac{1}{2}+\frac{1}{2}}=\frac{5-2}{1}3V$$

$$r_{BC}=1\Omega$$

NOW $\in_{ABC}=6-3=3V$

$r_{ABC}=2\Omega$ **Ans.**