CURRENT ELECTRICITY

CELLS, EMF, INTERNAL RESISTANCE

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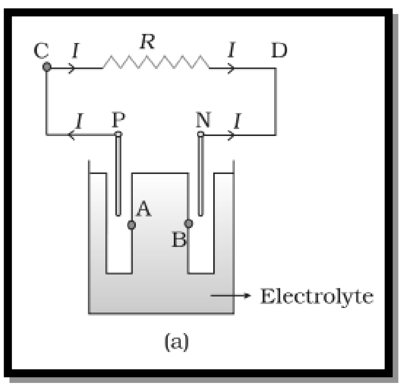
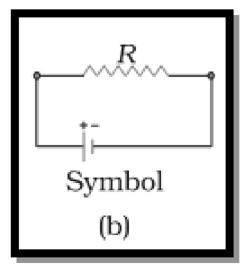
We have already mentioned that a simple device to maintain a steady current in an electric circuit is the electrolytic cell. Basically, a cell has two electrodes, called the positive (P) and the negative (N). They are immersed in an electrolytic solution. Dipped in the solution, the electrodes exchange charges with the electrolyte. The positive electrode has a potential difference V_+ ($V_+ > 0$) between itself and the electrolyte solution immediately adjacent to it marked A in the figure. Similarly, the negative electrode develops a negative potential $-(V_-)$ ($V_- \geq 0$) relative to the electrolyte adjacent to it, marked as B in the figure. When there is no current, the electrolyte has the same potential throughout, so that the potential difference between P and N is $V_+ - (-V_-) = V_+ + V_-$. This difference is called the electromotive force (emf) of the cell and is denoted by ε.
Thus $\varepsilon = V_+ + V_- > 0$



(a) Sketch of an electrolyte cell with positive terminal P and negative terminal N. The gap between the electrodes is exaggerated for clarity. A and B are points in the electrolyte typically close to P and N.



(b) The symbol for a cell, + referring to P and – referring to the N electrode. Electrical connections to the cell are made at P and N.

Note that ε is, actually, a potential difference and not a force. The name emf, however, is used because of historical reasons, and was given at a time when the phenomenon was not understood properly. To understand the significance of ε, consider a resistor R connected across the cell. A current I flow across R from C to D. As explained before, a steady current is maintained because current flows from N to P through the electrolyte. Clearly, across the electrolyte the same current flows through the electrolyte but from N to P, whereas through R, it flows from P to N.

The electrolyte through which a current flow has a finite resistance r, called the internal resistance. Consider first the situation when R is infinite so that $I = V/R = 0$, where V is the Potential difference between P and N. Now,

V = Potential difference between P and A
+ Potential difference between A and B
+ Potential difference between B and N
=ε

Thus, emf ε is the potential difference between the positive and negative electrodes in an open circuit, i.e., when no current is flowing through the cell.
If, however R is finite, I is not zero. In that case the potential difference between P and N is

$$V = V_{+} + V_{-} - I\,r$$

$$= \varepsilon - I\,r$$

Note the negative sign in the expression (I r) for the potential difference between A and B. This is because the current I flows from B to A in the electrolyte.
In practical calculations, internal resistances of cells in the circuit may be neglected when the current I is such that ε >> I r. The actual values of the internal resistances of cells vary from cell to cell. The internal resistance of dry cells, however, is much higher than the common electrolytic cells.

We also observe that since V is the potential difference across R, we have from Ohm's law

$$V = I\,R$$

Combining and we get

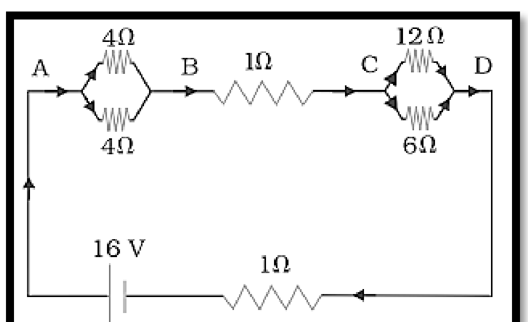
$$I\,R = \varepsilon - I\,r$$

$$I = \frac{\varepsilon}{R = r}$$

The maximum current that can be drawn from a cell is for R = 0 and it is $I_{max} = \varepsilon/r$. However, in most cells the maximum allowed current is much lower than this to prevent permanent damage to the cell.

Example A network of resistors is connected to a 16 V battery with internal resistance of 1Ω, **(For Competitive Exam)**

(a) Compute the equivalent resistance of the network.
(b) Obtain the current in each resistor.
(c) Obtain the voltage drops V_{AB}, V_{BC} and V_{CD}



Solution (a) The network is a simple series and parallel combination of resistors. First the two 4Ω resistors in parallel are equivalent to a resistor = [(4 × 4)/(4 + 4)] Ω = 2 Ω. In the same way, the 12 Ω and 6 Ω resistors in parallel are equivalent to a resistor of [(12 × 6)/(12 + 6)] Ω = 4 Ω. The equivalent resistance R of the network is obtained by combining these resistors (2 Ω and 4 Ω) with 1 Ω in series, that is, R = 2 Ω + 4 Ω + 1 Ω = 7 Ω.

(b) The total current I in the circuit is

$$I = \frac{\varepsilon}{R = r} = \frac{16V}{(7+1)\Omega} = 2\text{A}$$

Consider the resistors between A and B. If I1 is the current in one of the 4 Ω resistors and I 2 the current in the other,

$$I_1 \times 4 = I_2 \times 4 \text{ that is, } I_1 = I_2,$$

Which is otherwise obvious from the symmetry of the two arms. But

$$I_1 + I_2 = I = 2 \text{ A. Thus, } I_1 = I_2 = 1 \text{ A}$$

Thus, $I_1 = I_2 = 1$ A

That is, current in each 4 Ω resistor is 1 A. Current in 1 Ω resistor between B and C would be 2 A. Now, consider the resistances between C and D. If I_3 is the current in the 1₂ Ω resistor, and I_4 in the 6 Ω resistor,

$$I_3 \times 12 = I_4 \times 6, \text{ i.e., } I_4 = 2I_3$$

But, $I_3 + I_4 = I = 2$ A

Thus $I_3 = (\frac{2}{3})A I_4 = (\frac{4}{3})A$

That is, the current in the 12 Ω resistor is (2/3) A, while the current in the 6 Ω resistor is (4/3) A.

(c) The voltage drop across AB is

$$V_{AB} = I_1 \times 4 = 1 \text{ A} \times 4\ \Omega = 4 \text{ V},$$

This can also be obtained by multiplying the total current between A and B by the equivalent resistance between A and B, that is,

$$V_{AB} = 2 \text{ A} \times 2\ \Omega = 4 \text{ V}$$

The voltage drop across BC is

$$V_{BC} = 2\text{ A} \times 1\ \Omega = 2\text{ V}$$

Finally, the voltage drop across CD is

$$V_{CD} = 12\ \Omega \times I_3 = 12\ \Omega \times \frac{2}{3}\text{A} = 8\text{ V}.$$

This can alternately be obtained by multiplying total current between C and D by the equivalent resistance between C and D, that is,

$$V_{CD} = 2\text{ A} \times 4\ \Omega = 8\text{ V}$$

Note

That the total voltage drop across AD is 4 V + 2 V + 8 V = 14 V. Thus, the terminal voltage of the battery is 14 V, while its emf is 16 V. The loss of the voltage (= 2 V) is accounted for by the internal resistance 1 Ω of the battery [2 A × 1 Ω = 2 V