ELECTROSTATIC POTENTIAL AND CAPACITANCE

POTENTIAL ENERGY OF A SYSTEM OF CHARGES

ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

(This concept is useful when more than one charges move.)

It is the work done by an external agent against the internal electric field required to make a system of charges in a particular configuration from infinite separation without accelerating it.

Types of system of charge

(i) Point charge system

(ii) Continuous charge system.

Derivation for a system of point charges:

(i) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebraic sum of all the works. Let

W_1 = work done in bringing first charge

W_2 = work done in bringing second charge against force due to 1st charge.

W_3 = work done in bringing third charge against force due to 1st and 2nd charge.

PE = $W_1 + W_2 + W_3 + \ldots\ldots$. (This will contain $\frac{n(n-1)}{2} = {}^nC_2$ terms)

(ii) Method of calculation (to be used in problems)

U = sum of the interaction energies of the charges.

$= (U_{12} + U_{13} + \ldots\ldots + U_{1n}) + (U_{23} + U_{24} + \ldots\ldots + U_{2n}) + (U_{34} + U_{35} + \ldots\ldots + U_{3n})$ ..

(iii) Method of calculation useful for symmetrical point charge systems.

Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges.

$= (U_{12} + U_{13} + \ldots\ldots + U_{1n})$

U_2 = PE of second charge due to all other charges.

$= (U_{21} + U_{23} + \ldots\ldots.. + U_{2n})$ then U = PE of the system = $\frac{U_1 + U_2 + \ldots .U_n}{2}$

Example. Find out potential energy of the two point charge system having q_1 and q_2 charges separated by distance r.

Solution. Let both the charges be placed at a very large separation initially.

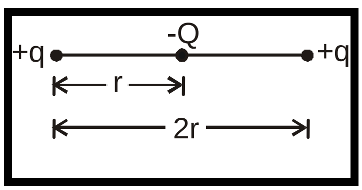
Let

W_1 = work done in bringing charge q_1 in absence of $q_2 = q(V_f - V_i) = 0$

W_2 = work done in bringing charge q_2 in presence of $q_1 = q(V_f - V_i) = q_1(Kq_2/r - 0)$

$PE = W_1 + W_2 = 0 + Kq_1q_2 / r = Kq_1q_2$ /

Example. Figure shows an arrangement of three point charges. The total potential energy of this arrangement is zero. Calculate the ratio $\frac{q}{Q}$.



Solution. $U_{sys} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qQ}{r} + \frac{(+q)(+q)}{2r} + \frac{Q(-q)}{r}\right] = 0 \Rightarrow -Q + \frac{q}{2} - Q = 0$ or $2Q = \frac{q}{2}$ or

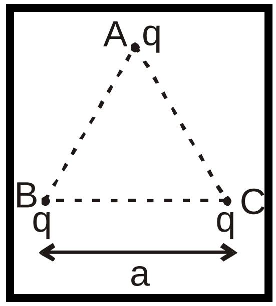
$\frac{q}{Q} = \frac{4}{1}$.

Example Three equal charges q are placed at the corners of an equilateral triangle of side a.

(i) Find out potential energy of charge system.

(ii) Calculate work required to decrease the side of triangle to a/2.

(iii) If the charges are released from the shown position and each of them has same mass m then find the speed of each particle when they lie on triangle of side 2a.



Solution.

(i) Method I (Derivation)

Assume all the charges are at infinity initially.

work done in putting charge q at corner A

$W_1 = q\,(v_f - v_i) = q\,(0 - 0)$

Since potential at A is zero in absence of charges, work done in putting q at corner B in presence of charge at A:

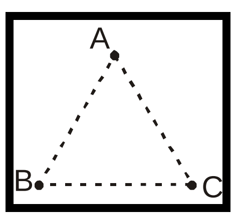
$$W_2 = \left(\frac{Kq}{a} - 0\right) = \frac{Kq^2}{a}$$

Similarly work done in putting charge q at corner C in presence of charge at A and B.

$$W_3 = q(v_f - v_i) = q\left[\left(\frac{Kq}{a} + \frac{Kq}{a}\right) - 0\right]$$

So net potential energy PE = $W_1 + W_2 + W_3$

$$= 0 + \frac{Kq^2}{a} + \frac{2Kq^2}{a} = \frac{3Kq^2}{a}$$



Method II (using direct formula)

$$U = U_{12} + U_{13} + U_{23} = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a}$$

(ii) Work required to decrease the sides $W = U_f - U_i = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a}$

(iii) Work done by electrostatic forces = change is kinetic energy of particles.

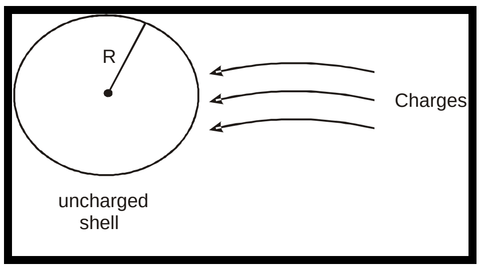
$$U_i - U_f = K_f - K_i \Rightarrow \quad \frac{3Kq^2}{a} - \frac{3Kq^2}{2a} = 3\left(\frac{1}{2}mv^2\right) - 0 \Rightarrow v = \sqrt{\frac{Kq^2}{am}}$$

Derivation of electric potential energy for continues charge system:

This energy is also known as self-energy.

(i) **Finding P.E. (Self Energy) of a uniformly Charged spherical shell:-**

For this, let's use method 1. Take an uncharged shell Now bring charges one by one from infinite to the surface fo the shell. The work required in this process will be stored as potential Energy.



Suppose we have given q charge to the sphere and now we are giving extra dq charge to it.

Work required to bring dq charge from infinite to them shell is

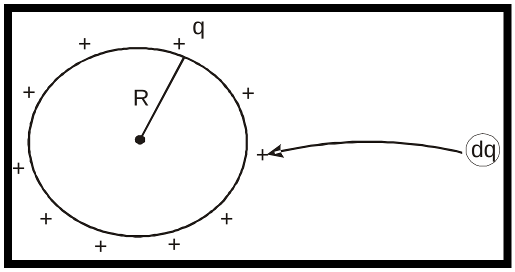
$dw = (dq)\ (V_f - V_i)$

$dW = (dq)\ \left(\frac{Kq}{R} - 0\right) = \frac{Kq}{R}\,dq$

total work required to give Q charge is $W = \int_{q=o}^{q=Q} \frac{Kq}{R}\,dq = \frac{KQ^2}{2R}$

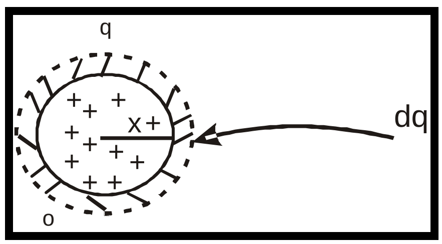
This work will stored as a form of P.E. (self energy)

So P.E. of a charged spherical shall $U = \frac{KQ^2}{2R}$



(ii) Self energy of uniformly charged solid sphere:

In this case we have to assemble a solid charged sphere. So as we bring the charges one-by-one from infinite to the sphere, the size of me sphere will increase. Suppose we have given q charge to the sphere, and its radius becomes ‘x’ .Now we are giving extra dq charge to it, which will increase its radius by ‘dx’ work required to bring dq charge from infinite to the sphere



$= dq\,(V_f - V_i) = (dq)\left(\frac{Kq}{x} - o\right) = \frac{Kqdq}{x}$

Total work required to give Q charge: $W = \int \frac{Kqdq}{x}$ $\qquad q = \rho\left(\frac{4}{3}\pi x^3\right)$

$dq = \rho\,(4\,\pi x^2\,dx) \qquad \Rightarrow \qquad W = \int_{x=o}^{x=R} K\frac{\rho\left(\frac{4}{3}\pi x^3\right)\rho(4\pi x^2 dx)}{x}$

Solving well get $\qquad W = \frac{3}{5}\frac{KQ^2}{R} = U_{self}$ for a solid sphere

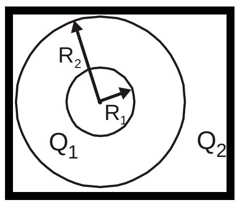
Example A spherical shell of radius R with uniform charge q is expanded to a radius 2R. Find the work performed by the electric forces and external agent against electric forces in this process (slow process).

Solution. $W_{ext} = U_f - U_i = \frac{q^2}{16\pi\varepsilon_0 R} - \frac{q^2}{8\pi\varepsilon_0 R} = -\frac{q^2}{16\pi\varepsilon_0 R}$

$W_{elec} = U_i - U_f = \frac{q^2}{8\pi\varepsilon_0 R} - \frac{q^2}{16\pi\varepsilon_0 R} = \frac{q^2}{16\pi\varepsilon_0 R}$

Example Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out total energy of the system.

Solution. $U_{total} = U_{self\ 1} + U_{self\ 2} + U_{Interaction} = \frac{Q_1^2}{8\pi\varepsilon_0 R_1} + \frac{Q_2^2}{8\pi\varepsilon_0 R_2} + \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_2}$



Energy density:

Def:

Energy density is defined as energy stored in unit volume in any electric field. Its mathematical formula is given as following: Energy density = $\frac{1}{2}\varepsilon E^2$

where E = electric field intensity at that point ;$\varepsilon = \varepsilon_0\varepsilon_r$ electric permittivity of medium

Example. Find out energy stored in an imaginary cubical volume of side a in front of a infinitely large nonconducting sheet of uniform charge density σ.

Solution. Energy stored $U = \int \frac{1}{2}\varepsilon_0 E^2\, dV$ where dV is small volume

$.= \frac{1}{2}\varepsilon_0 E^2 \int dV$ Θ E is constant $= \frac{1}{2}\varepsilon_0 \frac{\sigma^2}{4\varepsilon_0^2} \cdot a^3 = \frac{\sigma^2 a^3}{8\varepsilon_0}$