ELECTROSTATIC POTENTIAL AND CAPACITANCE

POTENTIAL ENERGY IN AN EXTERNAL FIELD

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Potential energy of a single charge

The source of the electric field was specified – the charges and their locations - and the potential energy of the system of those charges was determined. In this section, we ask a related but a distinct question. What is the potential energy of a charge q in a given field? This question was, in fact, the starting point that led us to the notion of the electrostatic potential but here we address this question again to clarify in what way it is different from the discussion in

The main difference is that we are now concerned with the potential energy of a charge (or charges) in an external field. The external field E is not produced by the given charge(s) whose potential energy we wish to calculate. E is produced by sources external to the given charge(s). The external sources may be known, but often they are unknown or unspecified; what is specified is the electric field E or the electrostatic potential V due to the external sources. We assume that the charge q does not significantly affect the sources producing the external field. This is true if q is very small, or the external sources are held fixed by other unspecified forces. Even if q is finite, its influence on the external sources may still be ignored in the situation when very strong sources far away at infinity produce a finite field E in the region of interest. Note again that we are interested in determining the potential energy of a given charge q (and later, a system of charges) in the external field; we are not interested in the potential energy of the sources producing the external electric field.

The external electric field E and the corresponding external potential V may vary from point to point. By definition, V at a point P is the work done in bringing a unit positive charge from infinity to the point P.

(We continue to take potential at infinity to be zero.) Thus, work done in bringing a charge q from infinity to the point P in the external field is qV. This work is stored in the form of potential energy of q. If the point P has position vector r relative to some origin, we can write:

Potential energy of q at r in an external field = qV(r)

where V(r) is the external potential at the point r.

Thus, if an electron with charge $q = e = 1.6\times10^{-19}$ C is accelerated by a potential difference of $\Delta V = 1$ volt, it would gain energy of $q\Delta V = 1.6 \times 10^{-19}$J. This unit of energy is defined as 1 electron volt or 1eV, i.e., 1 eV=1.6×10^{-19}J. The units based on eV are most commonly used in atomic, nuclear and particle physics, (1 keV = 103 eV = 1.6×10^{-16}J, 1 MeV = 106 eV = 1.6×10^{-13}J, 1 GeV = 109 eV = 1.6×10^{-10}J and 1 TeV = 1012eV = 1.6×10^{-7}J).

Potential energy of a system of two charges in an external field

Next, we ask: what is the potential energy of a system of two charges q_1 and q_2 located at r_1 and r_2, respectively, in an external field? First, we calculate the work done in bringing the charge q_1 from infinity to r_1. Work done in this step is $q_1 V(r_1)$,

Next, we consider the work done in bringing q_2 to r_2. In this step, work is done not only against the external field E but also against the field due to q_1.

Work done on q_2 against the external field = $q_2 V(r_2)$

Work done on q_2 against the field due to q_1 $= \dfrac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

where r_{12} is the distance between q_1 and q_2). By the superposition principle for fields, we add up the work done on q_2 against the two fields (E and that due to q_1):

Work done in bringing q_2 to r_2

$$= q_2 V\left(r_2\right) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Thus,

Potential energy of the system

= the total work done in assembling the configuration

$$q_1 V\left(r_1\right) + q_2 V\left(r_2\right) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Example

a) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and −2 μC (and with no external field) placed at (−9 cm, 0, 0) and (9 cm, 0, 0) respectively.

b) How much work is required to separate the two charges infinitely away from each other?

c) Suppose that the same system of charges is now placed in an external electric field E = A (1/r 2); A = 9 × 10^5 NC^{-1} m^2. What would the electrostatic energy of the configuration be?

Solution

a) $$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = 9\times10^9 \times \frac{7\times(-2)\times10^{-12}}{0.18} = -0.7J.$$

b) W = U_2 – U_1 = 0 – U = 0 – (–0.7) = 0.7 J.

c) The mutual interaction energy of the two charges remains unchanged. In addition, there is the energy of interaction of the two charges with the external electric field. We find,

$$q_1 V(r_1) + q_2 V(r_2) = A\frac{7\mu C}{0.09m} + A\frac{-2\mu C}{0.09m}$$

and the net electrostatic energy is

$$q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} = A\frac{7\mu C}{0.09m} + A\frac{-2\mu C}{0.09m} - 0.7J$$

$$= 70 - 20 - 0.7 = 49.3J$$