ELECTROSTATIC POTENTIAL AND CAPACITANCE

POTENTIAL DUE TO A SYSTEM OF CHARGES

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Consider a system of charges q_1, q_2,…. q_n with position vectors r_1 , r_2 ,…, r_n relative to some origin . The potential V1 at P due to the charge q1 is

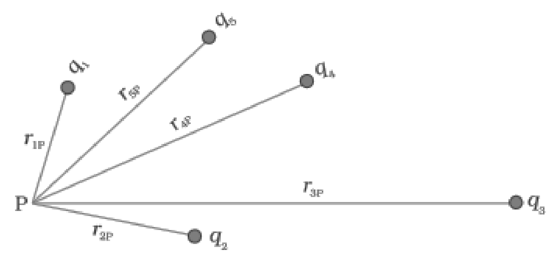
$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1\mathrm{P}}}$$

Where r_{1P} is the distance between q_1 and P.

Similarly, the potential V_2 at P due to q_2 and V_3 due to q_3 are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2\mathrm{P}}}, V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3\mathrm{P}}}$$

Where r_{2P} and r_{3P} are the distances of P from charges q2 and q3, respectively; and so on for the potential due to other charges. By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges



$$V = V_1 + V_2 + \cdots + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}(\frac{q_1}{r_{1\mathrm{p}}} + \frac{q_2}{r_{2\mathrm{p}}} + \cdots \ldots + \frac{q_n}{r_{n\mathrm{p}}})$$

If we have a continuous charge distribution characterised by a charge density ρ (r), we divide it, as before, into small volume elements each of size Δυ and carrying a charge ρΔυ. We then determine the potential due to each volume element and sum (strictly speaking, integrate) over all such contributions, and thus determine the potential due to the entire distribution.

A uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre. Thus, the potential outside the shell is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad (r \geq R)$$

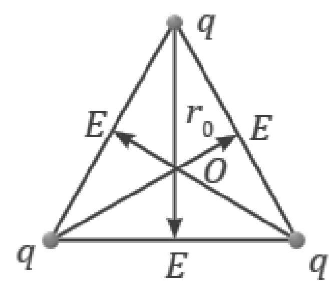
Where q is the total charge on the shell and R its radius. The electric field inside the shell is zero. This implies that potential is constant inside the shell (as no work is done in moving a charge inside the shell), and, therefore, equals its value at the surface, which is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Ex. If the electric field and the electric potential at a point are E and V, respectively, then which of the following statements is/are incorrect?

(A) If $E = 0$, V must be zero. **(B)** If $V = 0$, E must be zero.
(C) If $E \neq 0$, V cannot be zero. **(D)** If $V \neq 0$, E cannot be zero.

Sol. Consider an equilateral triangle with three positive charges at the vertices as shown in the figure.



It is easily seen that the magnitude of all the three electric fields at point O is equal, and the resultant of any two electric field vectors perfectly balances the third electric vector. Therefore, the net electric field is zero at point O

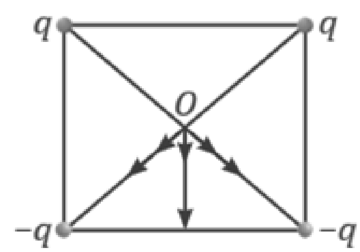
However, if we assume that the distance from the vertices to point O is r0 and try to calculate the electric potential at point O, then that would be,

$$V_o = \frac{3q}{4\pi\varepsilon_0 r_0}$$

Therefore, the statement 'If $E = 0$, V must be zero' is false.

Thus, option (A) is incorrect.

Now, consider a square with two equal positive charges and two equal negative charges as shown in the figure.



In this case, the net electric potential at the centre of the square is zero, but the net electric field is non-zero and its direction is vertically downwards as shown in the figure.

Therefore, the statement 'If $V = 0$, E must be zero' is false.
Thus, option (B) is incorrect.

Consider the statement given in option (C): if $E \neq 0$, V cannot be zero.
From the case of the square, we get,

$$E \neq 0, V = 0$$

Therefore, this statement is also false.
Thus, option (C) is also incorrect.

The discussion about the equilateral triangle also implies that the statement in option (D), 'If $V \neq 0$, E cannot be zero' is also incorrect.

Note:

• We know that $dV = -\vec{E} \cdot d\vec{r}$. So, we might assume that if $E = 0$, $V = 0$. We should be careful. In the expression, $dV = -\vec{E} \cdot d\vec{r}$.if we put $E = 0$, then we get $dV = 0$, which means that the change in potential is zero and this in turn implies that the potential is constant.

The significance of the negative sign in the expression $dV = -\vec{E} \cdot d\vec{r}$ can be understood in the following two ways:

1. along the direction of the electric field, the value of the potential decreases.
2. The direction of the electric field is from a high potential to a low potential.

• For $dV = 0$, the electric field E need not always be zero. This is because if $\vec{E} \perp d\vec{r}$, then $dV = 0$ again. Hence, V becomes constant. This is the concept of equipotential surfaces.