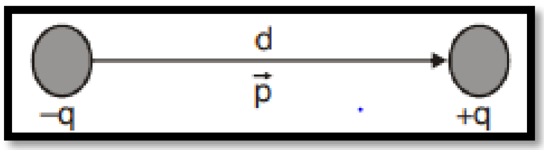
ELECTROSTATIC POTENTIAL AND CAPACITANCE

POTENTIAL DUE TO AN ELECTRIC DIPOLE

Electric Dipole:

A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges is given as.

$$\vec{P} = q\overleftarrow{d}$$

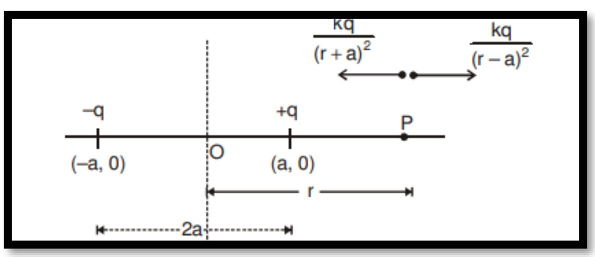


Dipole moment is a vector quantity and conventionally its direction is given from negative pole to positive pole.

(a) Electric field due to a dipole

(1) At an axial point

Figure shows an electric dipole placed on x-axis at origin



Here we wish to find the electric field at point P having coordinates (r, o) (where r >> 2a). Due to positive charge of dipole electric field at P is in outward direction & due to negative charge it is in inward direction.

$$\therefore\ E_{net} \text{ at } P = \frac{Kq}{(r-a)^2} - \frac{Kq}{(r+a)^2} \frac{4Kqar}{(r^2-a^2)^2}$$

As $\vec{p} = 2aq$

$$\therefore\ E_{net} \text{ at } \ p = \frac{2Kpr}{(r^2-a^2)^2}$$

As r >> 2a

we can neglect a w.r.t. r

$$E_{net} \text{ at } \ p = \frac{2Kp}{r^3}$$

As we can observe that for axial point direction of field is in direction of dipole moment

$$\therefore \text{Vectorially, } \vec{E} = \frac{2k\vec{p}}{r^3}$$

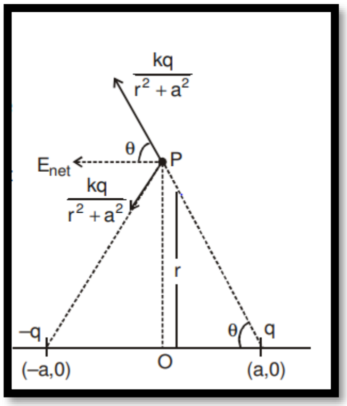
(2) At an equatorial point.

Again, we consider the dipole placed along the x-axis & we wish to find, electric field at point P which is situated equatorially at a distance r (where r >> 2a) from origin. Vertical component of the electric field vectors cancel out each other.

$$E_{net} \text{ at } \ P = 2E\cos\theta \ \left[where E = \frac{Kq}{r^2+a^2} \right]$$

$$E_{net} \text{ at } \ P = \frac{2kq}{r^2+a^2} \bullet \frac{a}{\sqrt{r^2+a^2}} \left[\cos\theta = \frac{a}{\sqrt{r^2+a^2}} \right]$$

$$E_{net} = \frac{2kqa}{\left(r^2+a^2\right)^{3/2}} = \frac{kq}{\left(r^2+a^2\right)^{3/2}}$$



As we have already stated that r > > 2a

$$\therefore\ E_{net} \ \text{ at } \ P = \frac{kp}{r^3}$$

We can observe that the direction of dipole moment & electric field due to dipole at P are in opposite direction.

$$\therefore \text{Vectorially } \ \vec{E} = \frac{-k\vec{P}}{r^3}$$

(b) Electric field at a general Point due to a dipole

Figure shows a electric dipole place on x-axis at origin & we wish to find out the electric field at point P with coordinate (r, θ)

E_{net} at

$$E_{net} = \sqrt{\left(\frac{2kp\cos\theta}{r^3}\right)^2 + \left(\frac{kp\sin\theta}{r^3}\right)^2}$$

$$= \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$

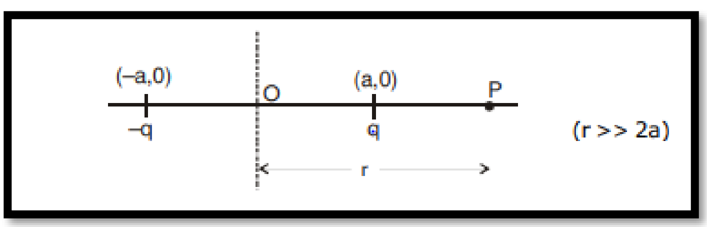
$$\tan\alpha \frac{\frac{kp\sin\theta}{r^3}}{\frac{2kp\cos\theta}{r^3}}$$

$$\tan\alpha = \frac{\tan\theta}{2}$$

$$\alpha = \tan^{-1}\left[\frac{\tan\theta}{2}\right]$$

(c) Electric potential due to a dipole.

1. At an axial point



We wish to find out potential at P due to dipole (with p = 2aq)

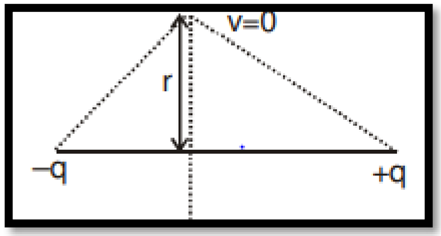
$$V_{net} = \frac{kq}{(r-a)} - \frac{kq}{(r+a)}$$

$$V_{net} = \frac{2kap}{(r^2-a^2)}$$

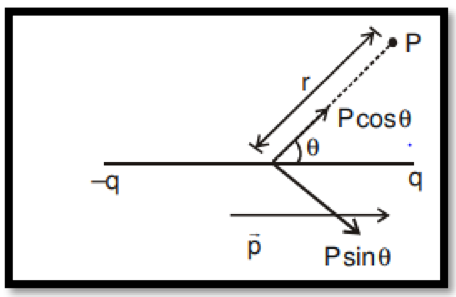
$$V_{net} = \frac{kp}{r^2} \text{ (As P = 2aq)}$$

2. **At a point on perpendicular bisector**

At an equatorial point, electric potential due to dipole is always zero because potential due to +ve charge is cancelled by –ve charge.



(3) **Potential due to dipole at a general Point**



Potential at P due to dipole $= \dfrac{Kp\cos\theta}{r^2}$

- **Basic torque concept**

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- If the net transactional force on the body is zero then the torque of the forces may or may not be zero but net torque of the forces about each point of universe is same

- If we have to prove that a body is in equilibrium then first, we will prov F_{net} is equal to zero & after that we will show τ_{net} about any point is equal to zero.

- If the body is free to rotate then it will rotate about the axis passing through center of mass & parallel to torque vector direction & of the body is hinged then it will rotate about hinged axis.