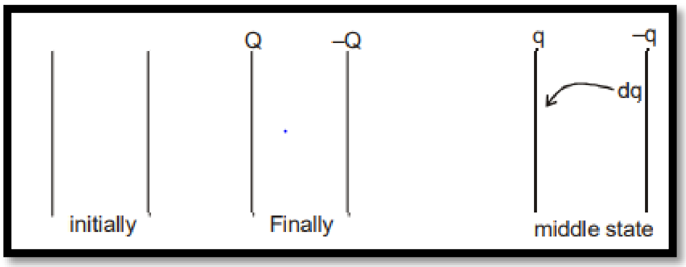
ELECTROSTATIC POTENTIAL AND CAPACITANCE

ENERGY STORED IN A CAPACITOR

ENERGY STORED IN A CHARGED CAPACITOR:



Work has to be done in charging a conductor against the force of repulsion by the already existing charges on it. The work is stored as a potential energy in the electric field of the conductor. Suppose a conductor of capacity C is charged to a potential V0 and let q0 be the charge on the conductor at this instant. The potential of the conductor when (during charging) the charge on it was q (< q0)

$$V = \frac{q}{c}$$

Now, work done in bringing a small charge dq at this potential is,

$$dW = Vdq = \left(\frac{q}{c}\right)dq$$

$\therefore$ total work done in charging it from 0 to q_0 is,

$$W = \int_0^{q_0} dW = \int_0^{q_0} \frac{q}{C} dq = \frac{1}{2}\frac{{q_0}^2}{C}$$

This work is stored as the potential energy

$$U = \frac{1}{2}\frac{q_0^2}{C}$$

Further by using $q_0 = CV_0$ we can write this expression also

$$U = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

In general, if a conductor of capacity C is charged to a potential V by giving it a charge q, Then

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

ENERGY DENSITY OF A CHARGED CAPACITOR:

This energy is localized on the charges or the plates but is distributed in the field. Since in case of a parallel plate capacitor, the electric field is only between the plates, i.e., in a volume (A × d), the energy density.

$$U_E = \frac{U}{volume} = \frac{\frac{1}{2}CV^2}{A \times d} = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\frac{V^2}{Ad}$$

Or $$U_E = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right) = \frac{1}{2}\varepsilon_0 E^2\left[\frac{V}{d} = E\right]$$

CALCULATION OF CAPACITANCE:

The method for the calculation of capacitance involves integration of the electric field between two conductors or the plates which are just equipotential surfaces to obtain the potential difference V_{ab}. Thus

$$V_{ab} = -\int_b^a \vec{E}.\vec{dr}$$

$$C = \frac{q}{v_{ab}} = \frac{q}{-\int_b^a \vec{E}.\vec{dr}}$$

HEAT GENERATED:

(1) Work done by battery

W = QV

Q = charge flow in the battery

V = EMF of battery

(2) W = +Ve (When Battery discharging)

W = ñVe (When Battery charging)

(3) Θ Q = CV (C = equivalent capacitance)

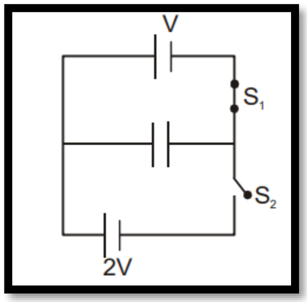
so W = CV ×V=CV^2

Now energy on the capacitor $=\frac{1}{2}CV^2$

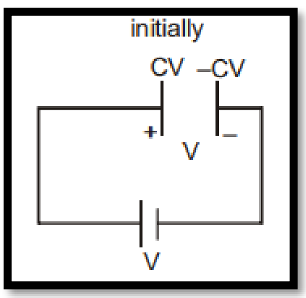
Energy dissipated in form of heat (due to resistance)

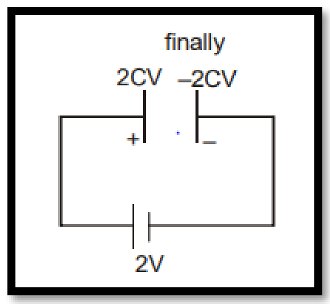
H = Work done by battery ñ {final energy of capacitor - initial energy of capacitor}

Ex. At any time S1 switch is opened and S2 is closed then find out heat generated in Circuit.



Sol.





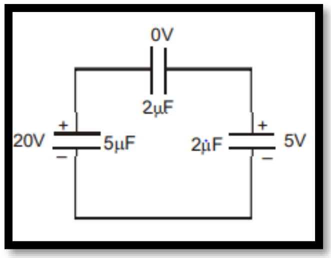
Charge flow through battery = $Q_f - Q_i = 2CV - CV = CV$

$$H=(CV\times 2V)-\left\{\frac{1}{2}C(2V)^2-\frac{1}{2}CV^2\right\}$$

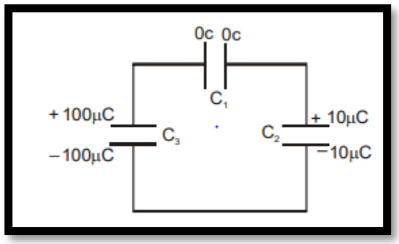
$$=2CV^2-\left\{2CV^2-\frac{1}{2}CV^2\right\}$$

$$H=\frac{1}{2}CV^2$$

Ex. (a) Find the final charge on each capacitor if they are connected as shown in the



Sol. Initially



Finally let q charge flows clockwise then

Now applying KVL

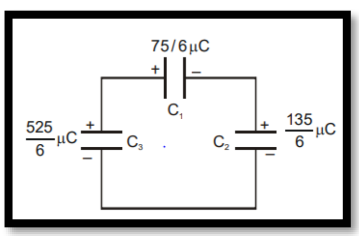
$$\frac{+q}{C_1}+\frac{(10+q)}{C_2}-\frac{(100-q)}{C_3}=0$$

$$\Rightarrow \frac{q}{2}+\frac{10+q}{2}-\frac{100-q}{5}=0$$

$$5q+50+5q-200+2q=0$$

$$12q-150=0 \Rightarrow q=\frac{75}{6}\,\mu C$$

so final



(b) Find heat loss in the above circuit.

ΔH = Energy [initially ñ finally] on capacitor

$$=\left[\left\{\frac{1}{2}\times 5\times(20)^2+\frac{1}{2}\times 2\times(5)^2\right\}-\left\{\frac{1}{2}\times\left(\frac{525}{6}\right)^2\times\frac{1}{5}+\frac{1}{2}\times\left(\frac{135}{6}\right)^2\times\frac{1}{2}\right\}\right]\times 10^{-6}\,J$$