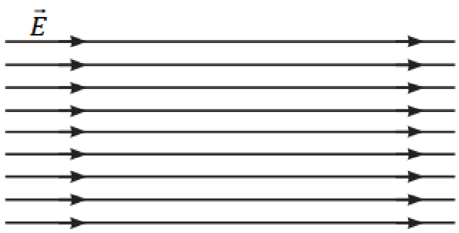
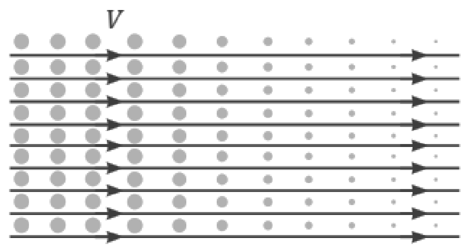
ELECTROSTATIC POTENTIAL AND CAPACITANCE

ELECTROSTATIC POTENTIAL

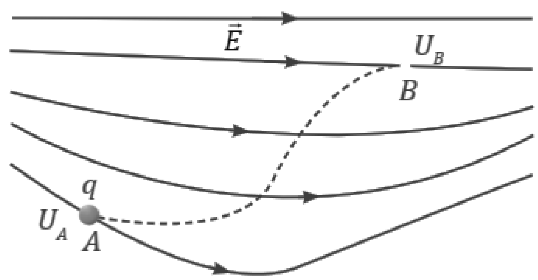
Electric Potential:

The electric field in a region of space is described by assigning a vector quantity $\vec{E}$ at each point. Pictorially, the uniform electric field can be described as equispaced, parallel electric lines of force, and the direction of the electric field is denoted by the arrowheads as shown in the figure.
The same field can also be described by assigning a scalar quantity V at each point known as electric potential.





Assume that a particle of charge +q is moving from point A to point B in a non-uniform electric field and the potential energy at point A and point B is U_A and U_B, respectively, with respect to the reference position fixed at any point in space.



If V_A and V_B are the electric potentials at points A and B, respectively, then the change in the electric potential is defined as,

$$V_B - V_A = \frac{U_B - U_A}{q}$$

Therefore, the change in potential is defined as the change in potential energy per unit charge. Hence, the change in potential energy is,

$$U_B - U_A = q(V_B - V_A) = -W_{el}$$

Now, suppose that we choose some point (P) at infinity where the electric potential is, $V_P = 0$
It means that we chose the datum of the electric field at infinity. However, there are no restrictions to choose the datum of the electric potential at infinity. Rather, it can be chosen at any point in space, depending on the given scenario. This is also true for the electric potential energy.

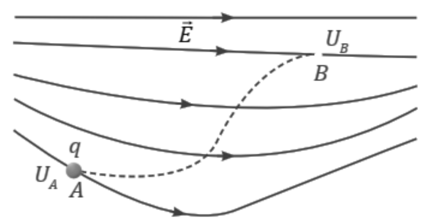
If a particle of charge +q is brought from point P (at infinity) to point A, then the change in electric
potential is,

$$V_A - V_P = V_A = \frac{U_A - U_P}{q}$$

1) If a particle of charge +q is moving from point A to point B in an electric field and the potential energy at point A and point B is U_A and U_B, respectively, then the change in potential energy is, $U_B - U_A = q(VB - VA) = -W_{el}$.

2) If a particle of charge +q is moving from point A to point B in an electric field under the action of an external force and the potential energy at point A and point B is U_A and U_B, respectively, then the change in potential energy is $U_B - U_A = q(VB - VA) = W_{ext}$, provided that Δ(K.E.) = 0

Ex. The kinetic energy of a charged particle decreases By 10 J as it moves from a point at potential 100 V To a point at potential 200 V. Find the charge of the Particle.



Sol. It is given that the kinetic energy decreases by 10 J as the charged particle moves from point B to point A. Therefore, Δ (K.E.) = $K_f - K_i = -10$

Since the potential energy is a scalar quantity, path-independent, and a state function, it depends only on the initial and final positions. Similarly, electric potential is a scalar quantity and path-independent.

Given,

The electric potential at point A is, $V_A = 100$ V.
The electric potential at point B is, $V_B = 200$ V.
By applying the work-energy theorem, we get,

$$W_{ext} + W_{el} = \Delta(K.E.)$$

$0 + (-\Delta U) = -10$ {Since there is no external agent, the external work done is zero.}

$$\Delta U = 10$$

$$U_f - U_i = 10$$

$$q(V_f - V_i) = 10$$

$$q(V_A - V_B) = 10$$

$$q(200 - 100) = 10$$

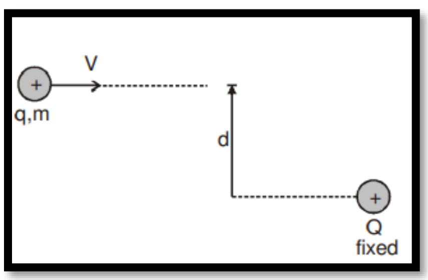
$$q = +0.1C$$

Therefore, the charge of the particle is +0.1 C.

Motion of a Charge Particle and Angular Momentum Conservation:

We know that a system of particles when no external torque acts, the total angular momentum of system remains conserved. Consider following examples which explains the concept for moving charged particles.

Ex. Figure shows a charge +Q fixed at a position in space. From a large distance another charge particle of charge +q and mass m is thrown toward +Q with an impact parameter d as shown with speed v. find the distance of closest approach of the two particles.



Here we can see that as +q moves toward +Q, a repulsive force acts on –q radially outward +Q. Here as the line of action of force passes through the fix charge, no torque act on +q relative to the fix point charge +Q, thus here we can say that with respect to +Q, the angular momentum of +q must remain constant. Here we can say that +q will be closest to +Q when it is moving perpendicularly to the line joining the two charges as shown.

If the closest separation in the two charges is r_{min}, from conservation of angular momentum we can write

$$mvd = mv_0 r_{\min}$$

Now from energy conservation, we have

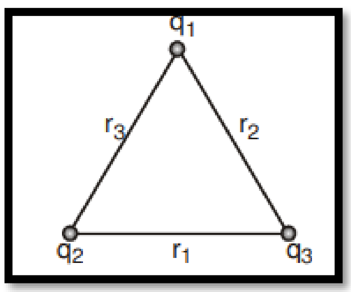
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \frac{KqQ}{r_{\min}}$$

Here we use from equation (1) $\quad v_0 = \frac{vd}{r_{\min}}$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv^2\frac{d^2}{r^2_{\min}} + \frac{kqQ}{r_{\min}}$$

Solving equation (2) we'll get the value of $r_{\min}$.

Potential Energy for a System of charged Particles:



When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energy of all the pairs of particles. For example, if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure. The total interaction energy of this system can be given as

$$U = \frac{Kq_1q_2}{r_3} + = \frac{Kq_1q_2}{r_2} + = \frac{Kq_1q_2}{r_3}$$

Derivation for a system of point charges:

(i) Keep all the charges at infinity. Now bring the charges one by one to its Corresponding position and find work required. PE of the system is algebraic sum of all the works.

Let

W_1 = work done in bringing first charge

W_2 = work done in bringing second charge against force due to 1st charge

W_3 = work done in bringing third charge against force due to 1st and 2nd charge.

$P_E = W_1 + W_2 + W_3 + \ldots\ldots\ldots\ldots$ (This will contain $\frac{n(n-1)}{2} = {}^nc_2$ terms)

(ii) Method of calculation (to be used in problems) U = sum of the Interaction energies of the charges.

$= (U_{12} + U_{13} + \ldots\ldots + U_{1n}) + (U_{23} + U_{24} + \ldots\ldots\ldots + U_{2n}) + (U_{34} + U_{35} + \ldots\ldots U_{3n})\ldots\ldots$

(iii) Method of calculation useful for symmetrical point charge systems. Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges.

$$= (U_{12} + U_{13} + \ldots\ldots + U_{1n})$$

U_2 = PE of second charges due to all other charges.

$$= (U_{21} + U_{23} + \ldots\ldots + U_{2n})$$

U=PE of the system $= \frac{U_1 + U_2 + \ldots\ldots}{2}$

Capacitance;

Introduction;

A capacitor can store energy in the form of potential energy in an electric field. In this Chapter we'll discuss the capacity of conductors to hold charge and energy.

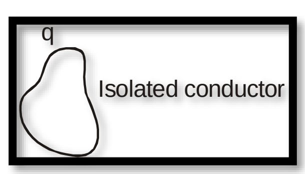
Capacitance of an isolated conductor:

When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.

q = charge on conductor

V = potential of conductor

$q \propto V$



$\Rightarrow q = CV$

Where C is proportionality constant called capacitance of the conductor.

Definition of capacitance:

Capacitance of conductor is defined as charge required to increase the potential of conductor By one unit.

Important points about the capacitance of an isolated conductor:

(i) It is a scalar quantity.

(ii) Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1} L^{-2} I^{2} T^{4}$

(iii) **1 Farad:** 1 Farad is the capacitance of a conductor for which 1 coulomb charge Increases potential by 1 volt.

$$\text{Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

$$\mu F = 10^{-6}\ F,\ 1nF = 10^{-9}\ F \quad \text{or} \quad 1\ pF = 10^{-12}\ F$$

(iv) **Capacitance of an isolated conductor depends on following factors:**

a. **Shape and size of the conductor:**

On increasing the size, capacitance increases.

b. **On surrounding medium:**

With increase in dielectric constant K, capacitance increases.

c. **Presence of other conductors:**

When a neutral conductor is placed near a charged conductor, capacitance of Conductors increase.

(v) Capacitance of a conductor do not depend on

a. Charge on the conductor

b. Potential of the conductor

c. Potential energy of the conductor.

Potential energy or self-energy of an isolated conductor:

Work done in charging the conductor to the charge on it against its own electric field or Total energy stored in electric field of conductor is called self-energy or self-potential Energy of conductor.

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C}$$

$$W = U = \frac{q^2}{2C} = \frac{1}{2} CV^2 = \frac{qV}{2}.$$

q = Charge on the conductor

V = Potential of the conductor

C = Capacitance of the conductor.

Self-energy is stored in the electric field of the conductor with energy density (Energy per Unit volume)

$$\frac{dU}{dV} = \frac{1}{2} \in_0 E^2 \text{ [The energy density in a medium is } \frac{1}{2} \in_0 \in_r E^2]$$

Where E is the electric field at that point.

In case of charged conductor energy stored is only outside the conductor but in case of Charged insulating material it is outside as well as inside the insulator.

Capacitance of an isolated spherical conductor:

The capacitance of an isolated spherical conductor of radius R.

Let there is charge Q on sphere.

$$\text{Potential } V = \frac{KQ}{R}$$

Hence by formula: Q = CV

$$Q = \frac{CKQ}{R}$$

$$C = 4\pi\in_0 R$$

Capacitance of an isolated spherical conductor

$$C = 4\pi\in_0 R$$

1. If the medium around the conductor is vacuum or air.

$C_{Vacuum} = 4\pi\in_0 R$

R = Radius of spherical conductor. (May be solid or hollow.)

2. If the medium around the conductor is a dielectric of constant K from surface of Sphere to infinity.

$C_{medium} = 4\pi\in_0 KR$

3. $\frac{C_{medium}}{C_{air / vaccum}} = K =$ dielectric constant.