ELECTROSTATIC POTENTIAL AND CAPACITANCE

EFFECT OF DIELECTRIC ON CAPACITANCE

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With the understanding of the behavior of dielectrics in an external field developed, let us see how the capacitance of a parallel plate capacitor is modified when a dielectric is present. As before, we have two large plates, each of area A, separated by a distance d. The charge on the plates is $\pm$Q, corresponding to the charge density $\pm\sigma$ (with $\sigma = Q/A$). When there is vacuum between the plates,

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

and the potential difference V_0 is

$$V_0 = E_0 d$$

The capacitance C_0 in this case is

$$C_0 = \frac{Q}{V_0} = \varepsilon_0 \frac{A}{d}$$

Consider next a dielectric inserted between the plates fully occupying the intervening region. The dielectric is polarized by the field and, as explained, the effect is equivalent to two charged sheets (at the surfaces of the dielectric normal to the field) with surface charge densities σ_p and $-\sigma_p$. The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$. That is,

$$E = \frac{\sigma - \sigma_p}{\varepsilon_0}$$

so that the potential difference across the plates is

$$V = Ed = \frac{\sigma - \sigma_p}{\varepsilon_0} d$$

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e., to σ. Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write

$$\sigma - \sigma_p = \frac{\sigma}{K}$$

where K is a constant characteristic of the dielectric. Clearly, $K > 1$. We then have

$$V = \frac{\sigma d}{\varepsilon_0 K} = \frac{Qd}{A\varepsilon_0 K}$$

The capacitance C, with dielectric between the plates, is then

$$C = \frac{Q}{V} = \frac{\varepsilon_0 KA}{d}$$

The product $\varepsilon_0 K$ is called the permittivity of the medium and is denoted by ε

$\varepsilon = \varepsilon_0 K$

For vacuum $K = 1$ and $\varepsilon = \varepsilon_0$; ε_0 is called the permittivity of the vacuum. The dimensionless ratio

$$K = \frac{\varepsilon}{\varepsilon_0}$$

Is called the dielectric constant of the substance. it is clear that K is greater than 1.

$$K = \frac{C}{C_0}$$

Thus, the dielectric constant of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor. Though we arrived at
the case of a parallel plate capacitor, it holds good for any type of capacitor and can, in fact, be viewed in general as a definition of the dielectric constant of a substance

Example A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness (3/4)d, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?

Solution Let $E_0 = V_0/d$ be the electric field between the plates when there is no dielectric and the potential difference is V_0. If the dielectric is now inserted, the electric field in the dielectric will be $E = E_0/K$. The potential difference will then be

$$V = E_0\left(\frac{1}{4}d\right) + \frac{E_0}{K}\left(\frac{3}{4}d\right)$$

$$= E_0 d\left(\frac{1}{4} + \frac{3}{4K}\right) = V_0\,\frac{K+3}{4K}$$

The potential difference decreases by the factor (K + 3)/4K while the free charge Q0 on the plates remains unchanged. The capacitance thus increases

$$C = \frac{Q_0}{V} = \frac{4K}{K+3}\frac{Q_0}{V_0} = \frac{4K}{K+3}C_0$$