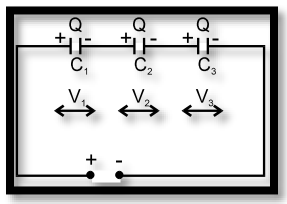
ELECTROSTATIC POTENTIAL AND CAPACITANCE

COMBINATION OF CAPACITORS

COMBINATION OF CAPACITORS:

Series Combination:

(i) When initially uncharged capacitors are connected as shown then the combination is called series combination



(ii) All capacitors will have same charge but different potential difference across them.

(iii) We can say that $V_1 = \frac{Q}{C_1}$

V_1 = potential across C_1

Q = charge on positive plate of C_1

C_1 = capacitance of capacitor similarly

$$V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}; \ldots\ldots$$

(iv) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

We can say that potential difference across capacitor is inversely proportional to its Capacitance in series combination.

$$V \propto \frac{1}{C}$$

Note: In series combination the smallest capacitor gets maximum potential.

(v)

$$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......} V$$

$$V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......} V$$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......} V$$

Where $V = V_1 + V_2 + V_3$

(vi) Equivalent Capacitance: Equivalent capacitance of any combination is that capacitance which when connected in place of the combination, stores same charge and energy that of the combination.

In series: $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

Note: In series combination equivalent capacitance is always less the smallest capacitor of combination.

(vii) Energy stored in the combination

$$U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$U_{combination} = \frac{Q^2}{2C_{eq}}$$

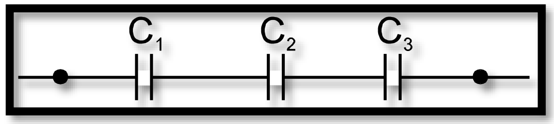
Energy supplied by the battery in charging the combination

$$U_{battery} = Q \times V = Q \, . \, \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$$

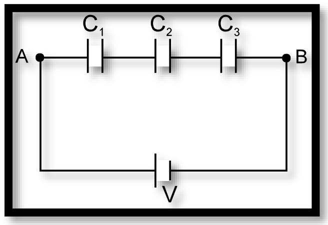
$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

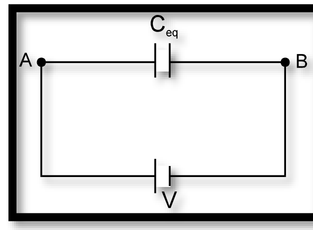
Note: Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (if capacitors are initially uncharged)

Derivation of Formulae:



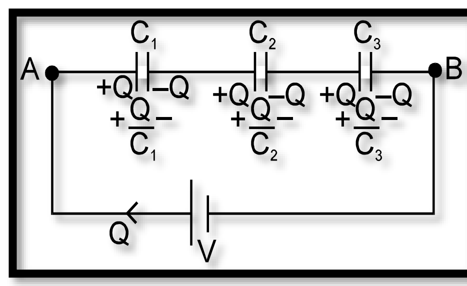
meaning of equivalent capacitor





$$C_{eq} = \frac{Q}{V}$$

Now, initially, the capacitor has no charge. Applying kirchhoff's voltage law

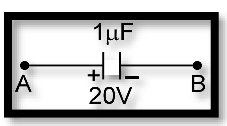


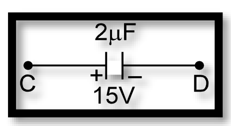
$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0.$$

$$V = Q\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right] \; ; \quad \frac{V}{Q} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} \text{ in general, } \frac{1}{C_{eq}} = \sum_{n=1}^{n}\frac{1}{C_n}$$

Example. Two capacitors of capacitance 1 μF and 2μF are charged to potential difference 20V and 15V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor





Solution: Now applying kirchoff voltage law

$$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$$-40 - 2q - 30 - q = -60$$

$$3q = -10$$

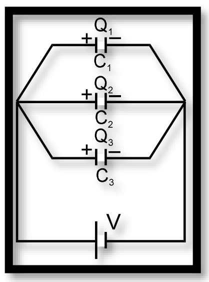
Charge flow $= -10/3\ \mu C$.

Charge on capacitor of capacitance $1\mu F = 20 + q = \frac{50}{3}\ \mu C$

Charge on capacitor of capacitance $2\mu F = 30 + q = \frac{80}{3}\ \mu C$

Parallel Combination:

(i) When one plate of each capacitor (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.



(ii) All capacitors have same potential difference but different charges.

(iii) We can say that:

$Q_1 = C_1V$

Q_1 = Charge on capacitor C_1

C_1 = Capacitance of capacitor C_1

V = Potential across capacitor C_1

(iv) $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

The charge on the capacitor is proportional to its capacitance

$Q \propto C$

(v) $Q_1 = \frac{C_1}{C_1 + C_2 + C_3} Q \quad \Rightarrow \quad Q_2 = \frac{C_2}{C_1 + C_2 + C_3} Q$

$Q_3 = \frac{C_3}{C_1 + C_2 + C_3} Q$

Where $Q = Q_1 + Q_2 + Q_3$ ......

Note: Maximum charge will flow through the capacitor of largest value.

(vi) Equivalent capacitance of parallel combination $C_{eq} = C_1 + C_2 + C_3$

Note: Equivalent capacitance is always greater than the largest capacitor of combination.

(vii) Energy stored in the combination:

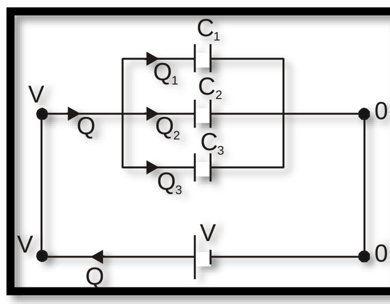
$$V_{combination} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + ... = \frac{1}{2}(C_1 + C_2 + C_3 .....)V^2 = \frac{1}{2}C_{eq}V^2$$

$$U_{battery} = QV = CV^2$$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

Note: Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (If all capacitors are initially uncharged)

Formulae Derivation for parallel combination:



$$Q = Q_1 + Q_2 + Q_3$$

$$\frac{Q}{V} = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$$

$$= C_1 + C_2 + C_3$$

$$C_{eq} = C_1 + C_2 + C_3$$

In general, $C_{eq} = \sum_{n=1}^{n} C_n$

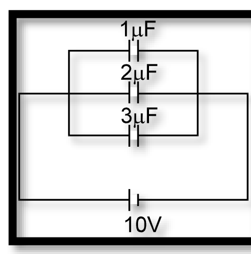
Example. Three initially uncharged capacitors are connected to a battery of 10 V is parallel combination find out following

(i) charge flow from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the 3μF capacitor



Solution: (i) $Q = (30 + 20 + 10)\mu C = 60\ \mu C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) heat produced $= 60 \times 10 - 300 = 300\ \mu J$

(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

Mixed Combination:

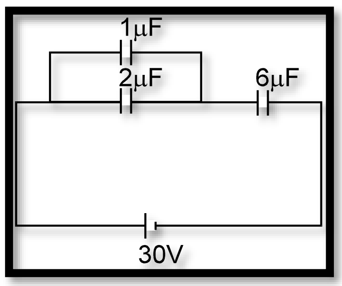
The combination which contains mixing of series parallel combinations or other complex combinations fall in mixed category. There are two types of mixed combinations

Simple

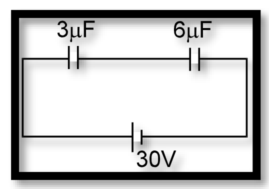
Complex

.

Example. In the given circuit find out charge on 6μF and 1μF capacitor.



Solution: It can be simplified as $C_{eq} = \frac{18}{9} = 2\mu F$



charge flow through the cell = 30 × 2 μC

Q = 60 μC

Now charge on 3μF = Charge on 6μF= 60 μC

Potential difference across 3μF = 60/ 3= 20 V

Charge on 1μF = 20 μC.

.