ELECTROSTATIC POTENTIAL AND CAPACITANCE

CAPACITORS AND CAPACITANCE

INTRODUCTION

A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.

Definition of capacitance:

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

Important points about the capacitance of an isolated conductor:

(i) It is a scalar quantity.

(ii) Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1} L^{-2} I^2 T^4$

(iii) **1 Farad:** 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

(iv) **Capacitance of an isolated conductor depends on following factors:**

- (a) **Shape and size of the conductor:** On increasing the size, capacitance increases.
- (b) **On surrounding medium:**
  With increase in dielectric constant K, capacitance increases
- (c) **Presence of other conductors:**
  When a neutral conductor is placed near a charged conductor, capacitance of conductors increases.

(v) **Capacitance of a conductor do not depend on**

- (a) Charge on the conductor
- (b) Potential of the conductor
- (c) Potential energy of the conductor.

CAPACITOR:

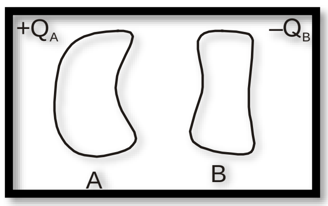
A capacitor or condenser consists of two conductors separated by an insulator or dielectric.

(i) When uncharged conductor is brought near to a charged conductor, the charge on conductors remains same but its potential decreases resulting in the increase of capacitance.

(ii) In capacitor two conductors have equal but opposite charges.

(iii) The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.

(iv) Formulae related with capacitors



(a)

$$Q = CV \qquad \Rightarrow \qquad C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}$$

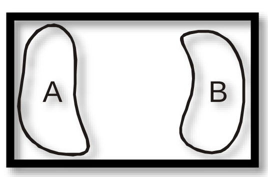
Q = Charge of positive plate of capacitor.

V = Potential difference between positive and negative plates of capacitor

C = Capacitance of capacitor.

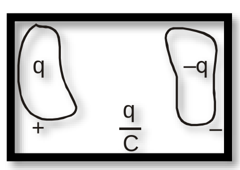
(b)

Energy stored in the capacitor



Initially charge = 0 0

Intermediate



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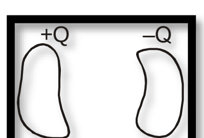

$$W = \int dW = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

$\therefore$ Energy stored in the capacitor $= U = \dfrac{Q^2}{2C} = \dfrac{1}{2} CV^2 = \dfrac{1}{2} QV.$

(v) This energy is stored inside the capacitor in its electric field with energy density

$$\frac{dU}{dV} = \frac{1}{2} \in E^2 \text{ or } \frac{1}{2} \in_0 \in_r E^2.$$

The capacitor is represented as following:

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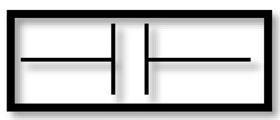
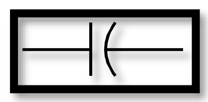




(vi) Based on shape and arrangement of capacitor plates there are various types of capacitors.

(a) Parallel plate capacitor.

(b) Spherical capacitor.

(c) Cylindrical capacitor

(vii) Capacitance of a capacitor depends on

(a) Area of plates.

(b) Distance between the plates.

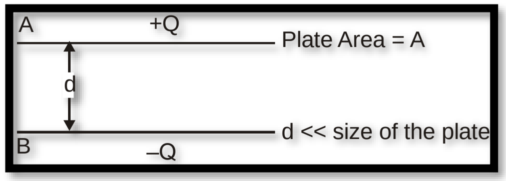
(c) Dielectric medium between the plates

(viii) Electric field intensity between the plates of capacitors (air filled) $E = \sigma/\in_0$ V/d

(ix) Force experienced by any plate of capacitor $F = q^2/2A\in_0$

Example. Find out the capacitance of parallel plate capacitor of plate area A and plate separation d.

Solution: $mE = \dfrac{Q}{A \in_0} \Rightarrow V_A - V_B = E.d. = \dfrac{Qd}{A \in_0} = \dfrac{Q}{C} \Rightarrow C = \dfrac{\in_0 A}{d}$

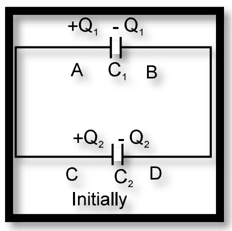


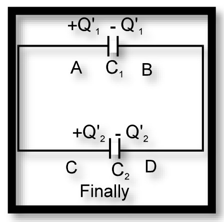
where A = area of the plates.

d = distance between plates.

Distribution of charges on connecting two charged capacitors:

When two capacitors are C_1 and C_2 are connected as shown in figure





Before connecting the capacitors

Parameter	Ist Capacitor	IInd Capacitor
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the capacitors

Parameter	Ist Capacitor	IInd Capacitor
Capacitance	C_1	C_2
Charge	Q'_1	Q'_2
Potential	V	V

(a) Common potential:

By charge conservation of plates A and C before and after connection.

$Q_1 + Q_2 = C_1V + C_2V$

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $Q_1' = C_1V = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2) \Rightarrow \quad Q_2' = C_2V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$

(c) Heat loss during redistribution:

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

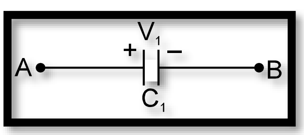
The loss of energy is in the form of Joule heating in the wire.

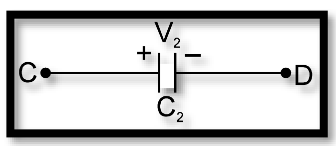
Note:

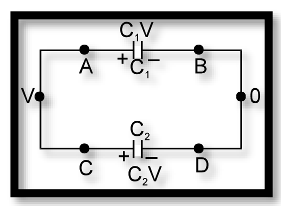
I. When plates of similar charges are connected with each other (+with +and-with-)then put all values (Q1 Q2 V1 V2) with positive sign

II. When plates of opposite polarity are connected with each other (+with-)then take charges and potential of one of the plate to be negative.

Derivation of above formulae:







Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V

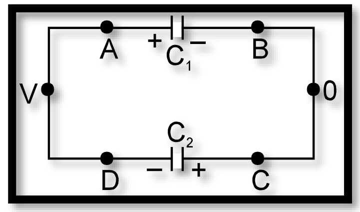
$C_1V + C_2V = C_1V_1 + C_2V_2$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \Rightarrow H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

$$\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$\frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right] = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

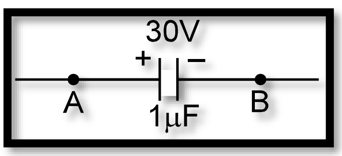


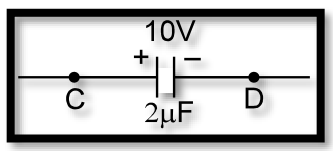
When oppositely charge terminals are connected then

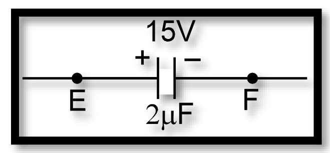
$C_1V + C_2V = C_1V_1 - C_2V_2$

$V = \frac{C_1V_1 - C_2V_2}{C_1+C_2}$ and $H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 + V_2)^2$

Example. Three capacitors as shown of capacitance 1μF, 2μF and 2μF are charged upto potential difference 30 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.



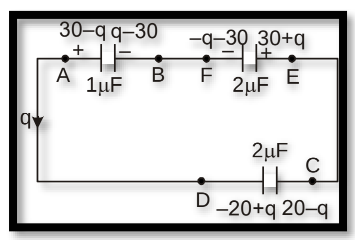




Solution:

Let charge flow is q. Now applying kirchhoff's voltage low

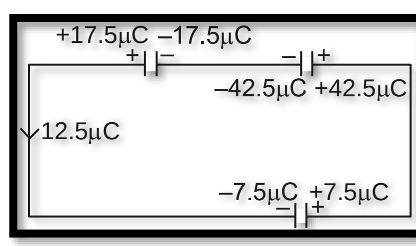
$$-\frac{(q-20)}{2} - \frac{(30+q)}{2} + \frac{30-q}{1} = 0$$



$-2q = -25$

$q = 12.5\ \mu C$

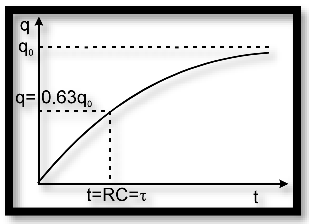
Final charges on plates

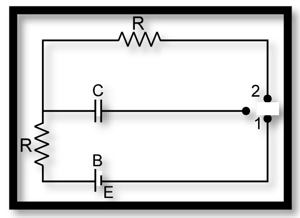


CHARGING AND DISCHARGING OF A CAPACITOR:

Charging of a condenser:

(i) In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0[1 - e^{-(t/RC)}]$Where q_0 = maximum final value of charge at $t = \infty$. According to these equations the quantity of charge on the condenser increases exponentially with increase of time.





(ii) If $t = RC = \tau$ then

$$q = q_0\,[1 - e^{-(RC/RC)}] = q_0\left[1-\frac{1}{e}\right]$$

or $q = q_0\,(1 - 0.37) = 0.63\,q_0 = 63\%$ of q_0

(iii) Time $t = RC$ is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

(iv) The potential difference across the condenser plates at any instant of time is given by

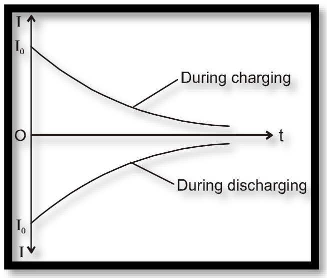
$V = V_0[1 - e^{-(t/RC)}]$ volt

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$I = I_0[e^{-(t/RC)}]$ ampere

According to this equation the current falls in the circuit exponentially.

(vi) If $t = RC = \tau$ = Time constant

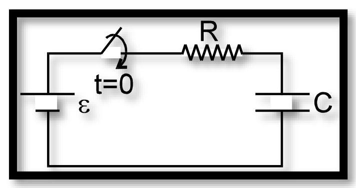


$$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\, I_0$$

$= 37\%$ of I_0

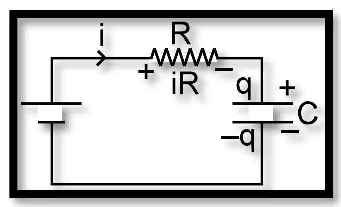
i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.

Derivation of formulae for charging of capacitor



it is given that initially capacitor is uncharged. Let at any time charge on capacitor is q

Applying Kirchhoff voltage law.

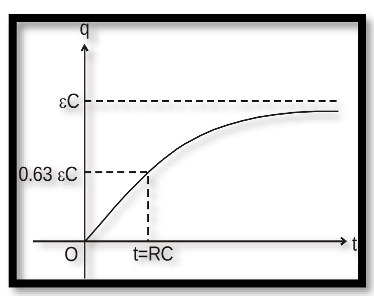


$$\varepsilon - iR - \frac{q}{C} = 0 \qquad \Rightarrow iR = \frac{\varepsilon C - q}{C}$$

$$i = \frac{\varepsilon C - q}{CR} \qquad \Rightarrow \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \qquad \Rightarrow \frac{CR}{\varepsilon C - q}\,.\, dq = dt.$$

$$\int_O^q \frac{dq}{\varepsilon C - q} = \int_O^t \frac{dt}{RC} \quad \Rightarrow -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$

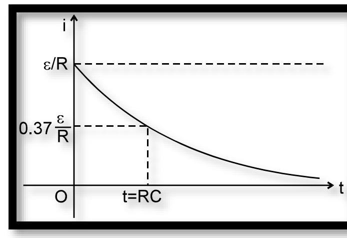


$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC} \;;\; \varepsilon C - q = \varepsilon C\,.\, e^{-t/RC}$$

$$q = \varepsilon C(1 - e^{-t/RC})$$

After one time constant

$q = \varepsilon C\left(1-\frac{1}{e}\right) = \varepsilon C\ (1 - 0.37) = 0.63\ \varepsilon C.$



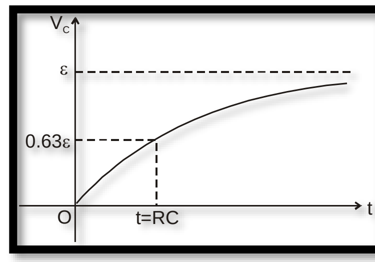
Current at any time t

$$i = \frac{dq}{dt} = \varepsilon C\ \left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right)$$

$$= \frac{\varepsilon}{R}e^{-t/RC}$$

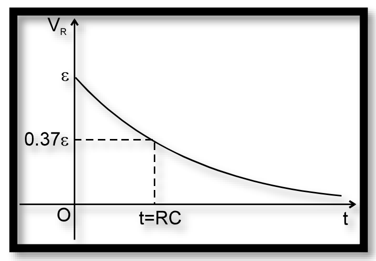
Voltage across capacitor after one time constant V = 0.63 ε

$Q = CV\ ;\quad V_C = \varepsilon(1 - e^{-t/RC})$



Voltage across the resistor

$V_R = iR\ = \varepsilon e^{-t/RC}$



By energy conservation,

Heat dissipated = work done by battery – $\Delta U_{capacitor}$

$= C\varepsilon(\varepsilon) - (\frac{1}{2}\ C\varepsilon^2 - 0) = \frac{1}{2}C\varepsilon^2$

Example A capacitor is connected to a 36 V battery through a resistance of 20Ω. It is found that the potential difference across the capacitor rises to 12.0 V in 2μs. Find the capacitance of the capacitor.

Solution The charge on the capacitor during charging is given by $Q = Q_0(1 - e^{-t/RC})$.

Hence, the potential difference across the capacitor is

$V = Q/C = Q_0/C\ (1 - e^{-t/RC})$.

Here, at t = 2 μs, the potential difference is 12V whereas the steady potential difference is

$Q_0/C = 36V$. So, $\Rightarrow$ $12V = 36V(1 - e^{-t/RC})$

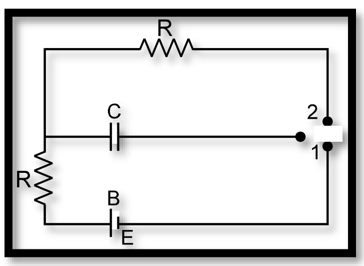
or, $1 - e^{-t/RC} = \frac{1}{3}$ or, $e^{-t/RC} = \frac{2}{3}$

or, $\frac{t}{RC} = \ell n\left(\frac{3}{2}\right) = 0.405$ or, $RC = \frac{t}{0.405} = \frac{2\ \mu s}{0.45} = 4.936\ \mu s$

or, $C = \frac{4.936\mu s}{20\Omega} = 0.25\ \mu F.$

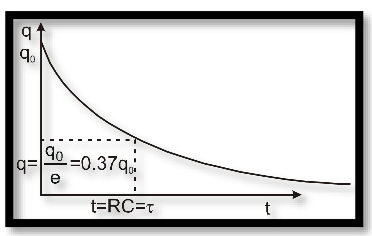
Discharging of a condenser:

(i) In the above circuit (in article 10.1) if key 1 is opened and key 2 is closed then the condenser gets discharged.



(ii) The quantity of charge on the condenser at any instant of time t is given by

$q = q_0\ e^{-(t/RC)}$



i.e. the charge falls exponentially.

here q_0 = initial charge of capacitor

(iii) If $t = RC = \tau$ = time constant, then $q = \frac{q_0}{e} = 0.37q_0 = 37\%$ of q_0

i.e., the time constant is that time during which the charge on condense plates in discharge process, falls to 37%

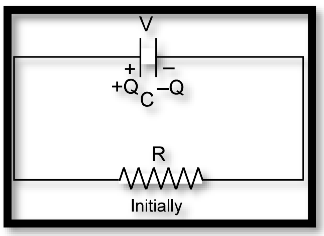
(iv) The dimensions of RC are those of time i.e. $M^0L^0T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

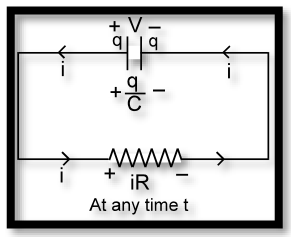
(v) The potential difference across the condenser plates at any instant of time t is given by $V = V_0e^{-(t/RC)}$ Volt.

(vi) The transient current at any instant of time is given by $I = -I_0e^{-(t/RC)}$ ampere.

i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current. (– ive only means that direction of current is opposite to that at charging current)

Derivation of equation of discharging circuit:





Applying K.V.L.

$$+\frac{q}{C} - iR = 0$$

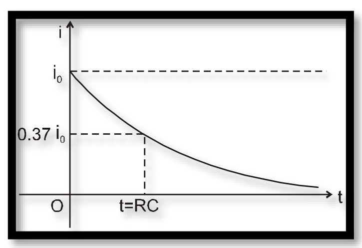
$$i = \frac{q}{CR}$$

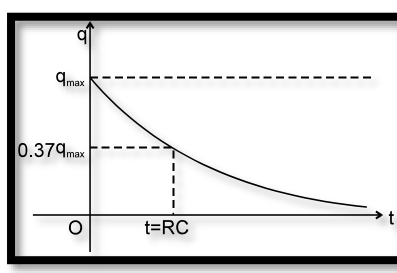
$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR}$$

$$-\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q\,.\,e^{-t/RC}$$

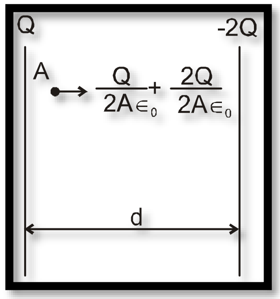
$$i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC} = i_0\,e^{-t/RC}$$





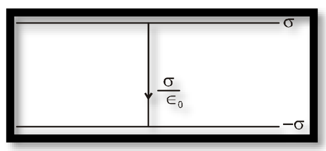
Example Two parallel conducting plates of a capacitor of capacitance C containing charges Q and –2Q at a distance d apart. Find out potential difference between the plates of capacitors.

Solution: Capacitance = C



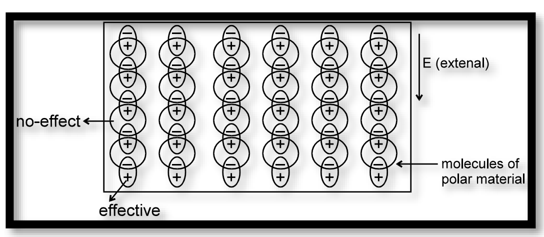
Electric field = $\frac{3Q}{2A\in_0}$; $V = \frac{3Qd}{2A\in_0} \Rightarrow \quad V = \frac{3Q}{2C}$

CAPACITORS WITH DIELECTRIC



(i) In absence of dielectric $E = \frac{\sigma}{\in_0}$

(ii) When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.

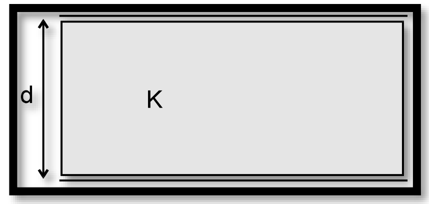


σ_b = induced charge density (called bound charge because it is not due to free electrons).

∗ For polar molecules dipole moment ≠ 0

∗ For non-polar molecules dipole moment = 0

(iii) Capacitance in the presence of dielectric



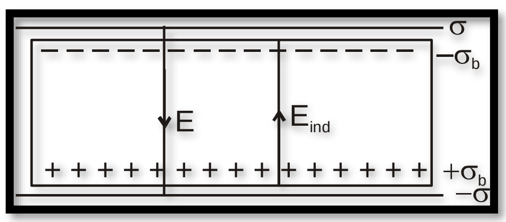
$$C = \frac{\sigma A}{V} = \frac{\sigma \quad A}{\frac{\sigma}{K \in_0}.d} = \frac{AK \in_0}{d} = \frac{AK \in_0}{d}$$

Here capacitance is increased by a factor K.

$$C = \frac{AK \in_0}{d}$$

(iv) Polarization of material: When nonpolar substance is placed in electric field then dipole moment is induced in the molecule. This induction of dipole moment is called polarization of material. The induced charge also produces electric field. σ_b = induced (bound) charge density.

$$E_{in} = E - E_{ind} = \frac{\sigma}{\in_0} - \frac{\sigma_b}{\in_0}$$

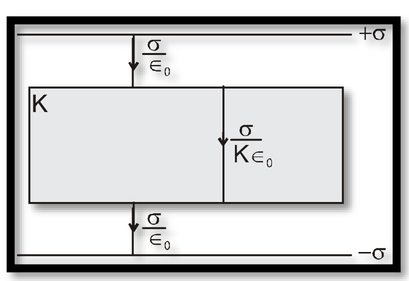


It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by ε_r or k.

$$E_{in} = \frac{\sigma}{K \in_0} \qquad \Rightarrow \sigma_b = \sigma\left(1 - \frac{1}{K}\right)$$

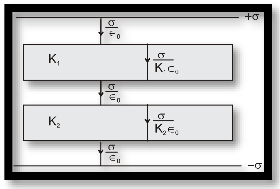
(v) If the medium does not filled between the plates completely then electric field will be as shown in figure

Case: (1):

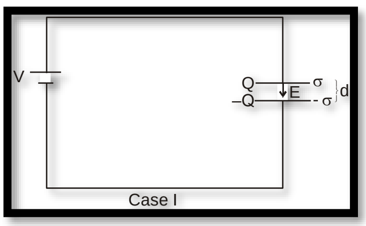


The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other.

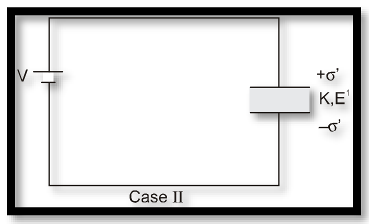
Case: (2)



(vi) Comparison of E (electric field), σ (surface char ges density), Q (charge), C (capacitance) and before and after inserting a dielectric slab between the plates of a parallel plate capacitor.



Case I



Case II

$$C = \frac{\epsilon_0 A}{d}$$

$$C' = \frac{A \epsilon_0 K}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$C' = \frac{A \epsilon_0 K}{d}$$

$$Q = CV$$

$$Q' = C'V$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{CV}{A \epsilon_0}$$

$$= \frac{V}{d}$$

$$E' = \frac{\sigma}{K \epsilon_0} = \frac{CV}{A \epsilon_0}$$

$$E' = \frac{V}{d}$$

Here potential difference between the plates, the

$$Ed = V$$

$$E = \frac{V}{d}$$

Here potential difference between plates

$$E'd = V$$

$$E' = \frac{V}{d}$$

$$\frac{V}{d} = \frac{\sigma}{\epsilon_0} \qquad\qquad \frac{V}{d} = \frac{\sigma'}{K\epsilon_0}$$

Equating both

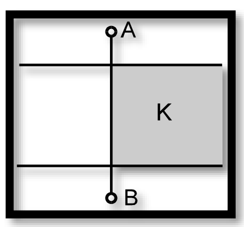
$$\frac{\sigma}{\epsilon_0} = \frac{\sigma'}{K\epsilon_0}$$

$\sigma' = K\sigma$

In the presence of dielectric, i.e. in case II capacitance of capacitor is more.

(vii) Energy density in a dielectric $= \frac{1}{2}\epsilon_0\epsilon_r E^2$

Example. A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be-



(A) $\frac{C}{2}$

(B) $\frac{C}{2K}$

(C) $\frac{C}{2}$ $[1 + K]$

(D) $\frac{2[1+K]}{C}$

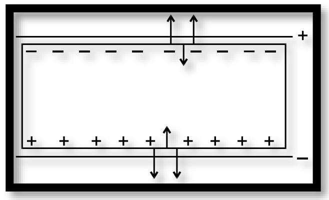
Solution: This system is equivalent to two capacitors in parallel with area of each plate $\frac{A}{2}$.

$$C' = C_1 + C_2 = \frac{\epsilon_0 A/2}{d} + \frac{\epsilon_0 (A/2)K}{d} = \frac{\epsilon_0 A}{2d}[1+K] = \frac{C}{2}[1+K]$$

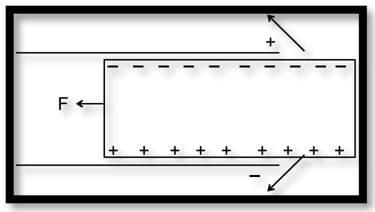
Hence the correct answer will be (C).

(viii) Force on a dielectric due to charged capacitor:

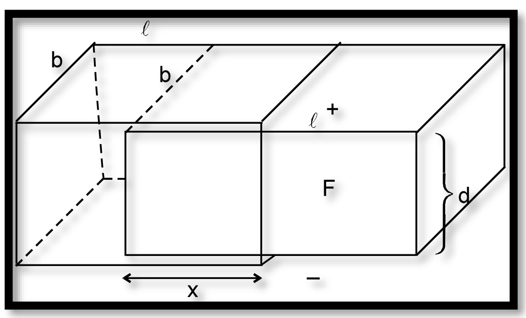
(a) If dielectric is completely inside the capacitor, then force is equal to zero.



(b) If dielectric is not completely inside the capacitor.



Case-I: Voltage source remains connected



V = constant.

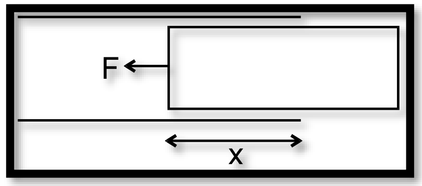
$$U = \frac{1}{2} CV^2$$

$$F = \left(\frac{dU}{dx}\right) = \frac{V^2}{2}\frac{dC}{dx} \text{ where } C = \frac{xb \in_0 K}{d} + \frac{\in_0 (\ell - x)b}{d} \Rightarrow C = \frac{\in_0 b}{d}[Kx + \lambda - x]$$

$$\frac{dC}{dx} = \frac{\in_0 b}{d}(K - 1)$$

$$\therefore F = \frac{\in_0 b(K-1)V^2}{2d} = \text{constant (does not depend on x)}$$

Case II : When charge on capacitor is constant

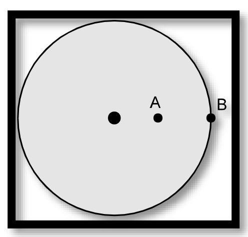


$$C = \frac{xb \in_0 K}{d} + \frac{\in_0 (\ell - x)b}{d}, \quad U = \frac{Q^2}{2C}$$

$$F = \left(\frac{dU}{dx}\right) = \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \quad \left[\text{where, } \frac{dC}{dx} = \frac{\in_0 b}{d}(K-1)\right]$$

$$= \frac{Q^2}{2C^2} \cdot \frac{dC}{dx}$$ (here force 'F' depends on x)

Example What is potential at a distance r (<R) in a dielectric sphere of uniform charge density ρ, radius R and dielectric constant ε_r.



Solution $V_A = V_B + \frac{W_{B\to A}}{q}$

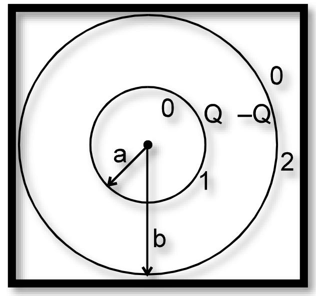
$$V = \frac{Q}{4\pi \in_0 R} + \int_R^r \frac{\rho r}{3 \in_0 \in_r}(-dr) = \frac{Q}{4\pi \in_0 R} + \frac{\rho(R^2 - r^2)}{3 \in_0 \in_r}$$

$$V_{outiside} = \frac{KQ}{r}$$

OTHER TYPES OF CAPACITORS

Spherical capacitor:

This arrangement is known as spherical capacitor.



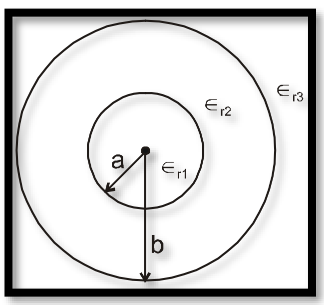
$$V_1 - V_2 = \left[\frac{KQ}{a} - \frac{KQ}{b}\right] - \left[\frac{KQ}{b} - \frac{KQ}{b}\right] = \frac{KQ}{a} - \frac{KQ}{b}$$

$$C = \frac{Q}{V_1 - V_2} = \frac{Q}{\frac{KQ}{a} - \frac{KQ}{b}} = \frac{ac}{K(b-a)} = \frac{4\pi \in_0 ab}{b-a}$$

$$C = \frac{4\pi \in_0 ab}{b-a}$$

If b >> a then

$C = 4\pi\in_0 a$ (Like isolated spherical capacitor

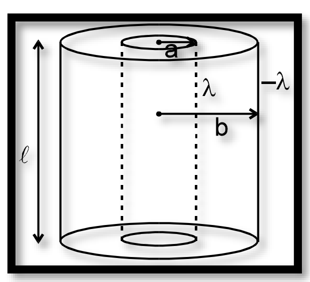


If dielectric mediums are filled as shown then : $C = \frac{4\pi \in_0 \in_{r_2} ab}{b-a}$

Cylindrical capacitor

There are two co-axial conducting cylindrical surfaces where

$\lambda >> a$ and $\lambda >> b$, where a and b is radius of cylinders.

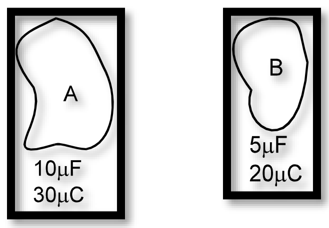


Capacitance per unit length $C = \frac{\lambda}{V}$

$$= \frac{\lambda}{2K\lambda \ell n\frac{b}{a}} = \frac{4\pi \in_0}{2\ell n\frac{b}{a}} = \frac{2\pi \in_0}{\ell n\frac{b}{a}}$$

Capacitance per unit length $= \dfrac{2\pi \epsilon_0}{\ell n \frac{b}{a}}$ F/m

Problem. When two isolated conductors A and B are connected by a conducting wire positive charge will flow from.



(A) A to B

(B) B to A

(C) will not flow

(D) cannot say.

Solution: Charge always flows from higher potential body to lower potential body

Hence, $V_A = \dfrac{30}{10} = 3V \Rightarrow V_B = \dfrac{20}{5} = 4$ V As $V_B > V_A$

(B) is correct Answer.