NUCLEI

RADIOACTIVITY

RADIOACTIVITY:

Radioactivity, a phenomenon initially observed by Henry Becquerel, refers to the spontaneous emission of radiations, including alpha (α), beta (β), and gamma (γ) particles, from an unstable nucleus. Substances exhibiting this property are termed radioactive substances. The systematic exploration of radioactivity was conducted by Ernest Rutherford. In the process of radioactive decay, an unstable nucleus undergoes the emission of an alpha (α) or beta (β) particle. Following the emission of (α) or (β), the residual nucleus may further release a gamma (γ) particle, ultimately transforming into a more stable nucleus. This intricate process sheds light on the dynamic nature of radioactive decay and the mechanisms by which unstable nuclei strive for greater stability.

α- Particle:

An alpha (α) particle is a helium nucleus carrying a doubly positive charge. Comprising two protons and two neutrons, its mass is equivalent to that of a $_2He^4$ atom minus twice the mass of an electron($2m_e$), resulting in $4m_p$. The charge of an alpha particle is +2e, denoting a double positive charge. In summary, α particle, with its specific composition and charge, plays a significant role in certain nuclear processes such as alpha decay.

β-particle:

a. Beta-minus (β^-) Particle (Electron):

- Mass: m_e
- Charge: $-e$

b. Beta-plus (β^-) Particle (Positron):

- Mass: m_e
- Charge: $+e$

The positron, identified as the antiparticle of the electron, shares identical mass (m_e) but differs in charge, possessing a positive charge ($+e$). Conversely, the beta-minus particle (electron) carries a negative charge ($-e$). Both beta particles play significant roles in certain nuclear processes, contributing to processes like beta decay.

Antiparticle:

An antiparticle is defined as the counterpart of another particle, such that when these two particles collide, they can undergo complete annihilation, converting into energy. Examples of such antiparticle pairs include:

1. Electron $(-e, m_e)$ and Positron $(+e, m_e)$: These particles are considered antiparticles, and their collision can result in annihilation, transforming their masses into energy.

2. Neutrino and Antineutrino: Neutrinos and their corresponding antiparticles, antineutrinos, form another pair of antiparticles. Their collision can lead to annihilation, releasing energy in the process.

In both instances, the concept of antiparticles highlights the potential for particle-antiparticle interactions to result in the conversion of mass into energy, a phenomenon observed in specific particle reactions.

γ-particle:

Photons of high energy, typically on the order of MeV (mega-electronvolts), are referred to as energetic photons. These particles exhibit a rest mass of zero.

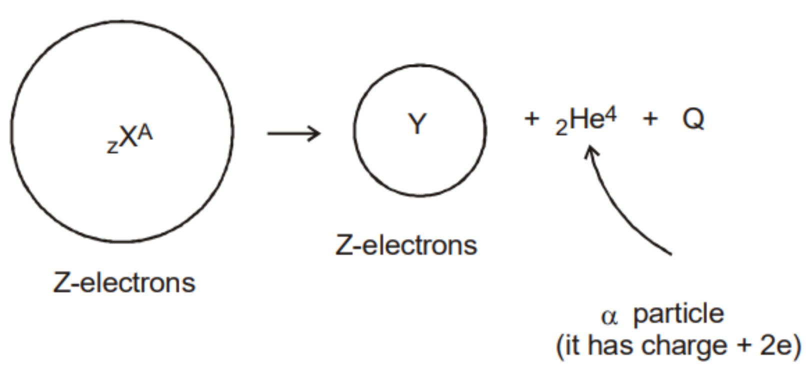
Radioactive decay (displacement law)

α - Decay:

$${}_Z X^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2 He^4 + Q$$

Q value:

The Q value in the context of a decay process is defined as the energy released during that decay. This value is calculated as the difference between the rest mass energy of the reactants and the rest mass energy of the products. The energy released is typically manifested as an increase in the kinetic energy of the resulting products.



Let's denote:

- M_x as the mass of the atom $_ZX^A$,
- M_y as the mass of the atom $_{Z-2}Y^{A-4}$,
- M_{He} As the mass of the atom $_2H^2$.

The Q value can be expressed as follows:

$$Q = [(M_x - Zm_e) - \{(M_y - (Z-2)m_e) + (M_{He} - 2m_e)\}]c^2$$

Simplifying the equation results in:

$$Q = (M_x - M_y - M_{He})c^2$$

Taking into consideration the actual number of electrons in alpha (α -) decay, the Q value can be further refined:

$$Q = [(M_x - (M_y + 2m_e e)) - (M_{He} - 2m_e)]c^2$$

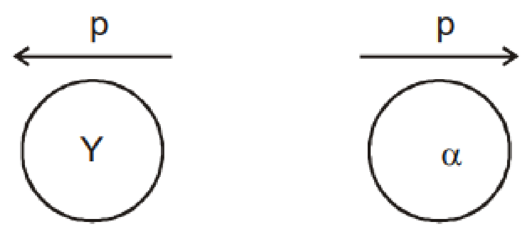
And simplifying once more:

$$Q = (M_x - M_y - M_{He})c^2$$

This expression elucidates the quantitative aspect of the energy released during the decay process, shedding light on the mass differences between the reactants and the products.

Calculation of Kinetic energy of final products

Since atom X was initially at rest, and there are no external forces acting on it, the principle of conservation of momentum dictates that the final momentum must also be zero. Consequently, both particle Y and the alpha (α)-particle will possess equal magnitudes of momentum, but they will travel in opposite directions.



$P_a^2 = P_Y^2$ $\quad$ $2m_a T_a = 2m_Y T_Y$ (Here we are representing T for kinetic energy)

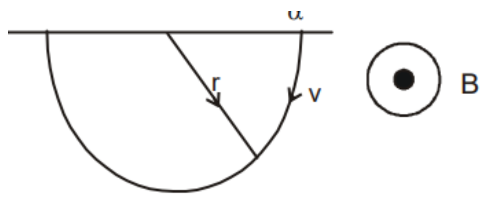
$Q = T_Y + T_a$ $\quad$ $m_a T_a = m_y T_y$

$$T_a = \frac{m_y}{m_a + m_y} Q \quad ; \quad T_Y = \frac{m_a}{m_a + m_y} Q$$

$$T_a = \frac{A-4}{A} Q \quad ; \quad T_Y = \frac{4}{A} Q$$

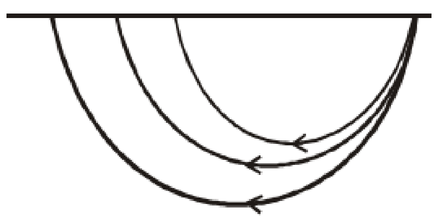
Based on the preceding calculation, it is evident that all emitted alpha (α) particles should exhibit identical kinetic energy. Consequently, if these particles are directed through a region characterized by a uniform magnetic field with its direction perpendicular to the velocity of the particles, they will follow a circular trajectory of uniform radius.

$$r = \frac{mv}{qb} = \frac{mv}{2eB} = \frac{\sqrt{2km}}{2eB}$$



Experimental Observation

Experimental observations have revealed that not all alpha (α)-particles move along circular paths with the same radius. Instead, they exhibit motion in circles with varying radii.



This observation indicates that the emitted alpha (α)-particles possess distinct kinetic energies. However, it is also noted that these particles traverse circular paths with specific, quantized radii. Although the kinetic energy of emitted α -particles is not uniform, it is quantized. The underlying reason for this phenomenon lies in the fact that not all daughter nuclei produced are in their ground state. Some of the daughter nuclei may be generated in excited states, and as they transition to their ground state, they emit photons. This emission of photons in the process contributes to the quantization of the kinetic energy of the emitted α -particles.

$$X \longrightarrow Y^* \text{ (excited state)} + \alpha + Q$$
$$\downarrow$$
$$Y + \text{photon } (\gamma \text{ particle})$$

The distinction between Y and Y* lies in their energy states, with Y* being in an excited state while Y is in its ground state. Let's denote the energy of the emitted gamma (γ)-particles as E.

The equation expressing the energy release (Q) during the process is given by:

$Q = T_\alpha + T_Y + E$

Where $Q = [M_x - M_y - M_{He}]c^2$ represents the difference in rest mass energies.

Now, rearranging the equation, we get:

$T_\alpha + T_Y = Q - E$

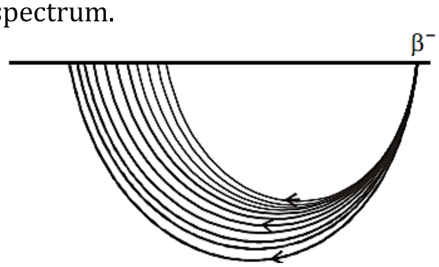
$$T_a = \frac{m_Y}{m_a + m_Y}(Q - E) \quad ; \quad T_Y = \frac{m_a}{m_a + m_Y}(Q - E)$$

β^-- Decay:

The nuclear decay process is represented as ${}_Z X^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q$, where ${}_{-1}e^0$ can also be expressed as ${}_{-1}\beta^0$.

By considering momentum and energy conservation, we obtain the kinetic energies of the electron (e^-) and the antineutrino ($\bar{v}$) as $T_e = \frac{m_Y}{m_e + m_Y}Q$, $\frac{m_e}{m_e + m_Y}Q$. In this equation, as m_e is significantly smaller than m_Y, it can be approximated that all the energy is carried away by the electron.

Despite the theoretical expectation that all emitted β^--particles would have the same energy, yielding identical radii when passed through a region of a perpendicular magnetic field, experimental observations contradict this anticipation. When subjected to a uniform magnetic field perpendicular to their velocity, β^--particles were found to traverse circular paths with different radii, constituting a continuous spectrum.



To address the observed discrepancies, Pauli introduced additional particles known as neutrinos and antineutrinos (the antiparticle counterpart of neutrinos).

Neutrino (v) and Antineutrino ($\bar{v}$) Properties:

1. Similar to photons, they possess zero rest mass and travel at the speed of light (c). Their energy (E) is given by mc^2.

2. They are neutral, carrying no electric charge.

3. Neutrinos and antineutrinos have a spin quantum number.

Taking into account the emission of an antineutrino, the equation for β^--decay can be expressed as follows:

$$_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + Q + \bar{v}$$

The inclusion of the antineutrino in the decay process helps elucidate the continuous spectrum phenomenon. Energy is distributed randomly between the electron and the antineutrino, contributing to the spectrum's variability. Furthermore, this addition aids in maintaining the balance of spin quantum numbers, as each particle involved (proton, neutron, and electron) has a spin quantum number of $s = \pm\frac{1}{2}$.

During β^--decay, a neutron is converted into a proton within the nucleus, accompanied by the emission of an electron and an antineutrino:

$$n \rightarrow p + {}_{-1}e^{0} + \bar{v}$$

Let's denote:
- M_x as the mass of the atom $_{z}X^{A}$,
- M_y as the mass of the atom $_{z+1}Y^{A}$,
- m_e As the mass of the electron.

The Q value for the decay process is given by:

$$Q = [(M_x - Zm_e) - \{(M_Y - (Z+1)m_e) + m_e\}]c^2 = (M_x - M_Y)c^2$$

Taking into consideration the actual number of electrons, the Q value can be further refined:

$$Q = [M_x - \{(M_Y - m_e) + m_e\}]c^2 = (M_x - M_Y)c^2$$

This comprehensive expression addresses the complexities of β^--decay, incorporating the role of antineutrinos and offering insights into the continuous spectrum phenomenon.

K capture:

This phenomenon is a rare process observed in only a few nuclei, where the nucleus captures an atomic electron from the K shell. In this process, a proton within the nucleus combines with the captured electron, transforming into a neutron. Additionally, a neutrino is emitted from the nucleus. The reaction can be represented as follows:

$$p + {}_{-1}e^{0} \rightarrow n + v$$

In the context of atoms X and Y, the reaction is expressed as:

$${}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + v + Q + \text{Characteristic x-rays of Y.}$$

Alternatively, considering X and Y as nuclei, the reaction is written as:

$${}_{Z}X^{A} + {}_{+1}e^{0} \rightarrow {}_{Z-1}Y^{A} + v$$

Key points to note:

1. Nuclei with atomic numbers from Z = 84 to 112 exhibit radioactivity.

2. Nuclei with Z = 1 to 83 are generally stable, with only a few exceptions.

3. The production of a neutron is always accompanied by the production of a neutrino.

4. The conversion of a neutron into a proton is associated with the production of an antineutrino.

RADIOACTIVE DECAY: STATISTICAL LAW

(Proposed by Rutherford and Soddy) The rate of radioactive decay $\left(\frac{dN}{dt}\right)$ is proportional to the number of active nuclei (N), expressed as $\frac{dN}{dt} = -\lambda N$, where N represents the count of active nuclei, and λ is the decay constant of the radioactive substance. Notably, the decay constant (λ) is distinctive for each radioactive substance, yet it remains independent of the quantity of substances and time.

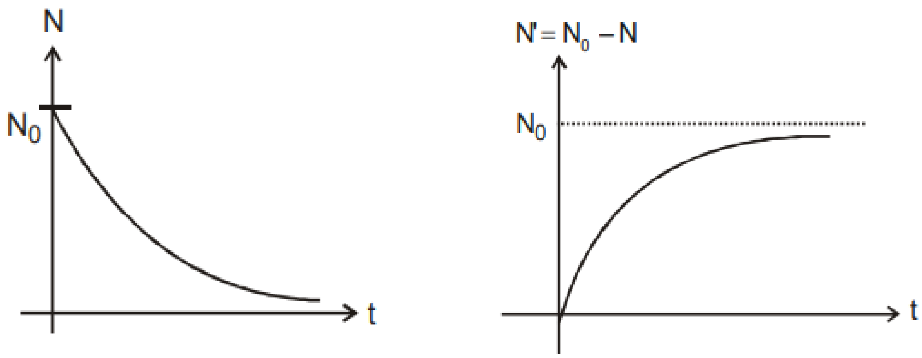
The SI unit of λ is seconds to the power of minus one (s^{-1}). Comparing two substances with decay constants λ_1 and λ_2, if $\lambda_1 > \lambda_2$, the first substance is considered more radioactive, signifying lesser stability.

For a scenario where substance A decays into B with a decay constant λ

$$A \xrightarrow{\lambda} B$$

At $t = 0$, the number of active nuclei of A is N_0, and at $t = t$, the count is N. The rate of radioactive decay is denoted by $-\frac{dN}{dt}$, where n is the constant of proportionality.

$$-\int_{N_0}^{N}\frac{dN}{N}=\int_{0}^{t}\lambda dt \Rightarrow N = N_0 e^{-\lambda t} \text{ (It is exponential decay)}$$



The count of nuclei that have undergone decay, denoted as N', can be expressed as the difference between the initial number of active nuclei (N_0) and the remaining number of active nuclei (N). Mathematically, this can be represented as $N' = N_0 - N$.

Substituting N with $N_0 e^{-\lambda t}$ (the expression for the remaining active nuclei at time t), we get:

$N' = N_0 - N_0 e^{-\lambda t}$

Factoring out N_0 from both terms, we obtain:

$N' = N_0(1 - e^{-\lambda t})$

This equation provides a concise representation of the count of nuclei that have decayed (N') in terms of the initial count of active nuclei (N_0), time (t), and the decay constant (λ).